1 Model

Let $Y = \{y_{ij},\ i,j = 1,\ldots,n;\ i \neq j\}$ a set of response where $y_{ij} = 1$ if the player i beats the player j and 0 otherwise. Consider $X = (x_{ij}),\ i = 1,\ldots,n;\ j = 1,\ldots,p$ the matrix of covariates associated to the fixed effects $\beta = (\beta_1,\ldots,\beta_p)'$ and $U_i,\ i = 1,\ldots,n$ the random effects supposed independent and identically distributed with normal with mean 0 and covariance σ^2.

The Bradley-Terry model is defined by:

$$logit\left(P(y_{ij} = 1 \mid U_i, U_j)\right) = \sum_{r=1}^{p}(x_{ir} - x_{jr})\beta_r + U_i - U_j = z'_{ij}\beta + U_i - U_j \quad . \tag{1}$$

From now on, denote by $U = (U_1,\ldots,U_n)$ the vector of the random effects with a multivariate normal distribution $N(0,\sigma^2 I_n)$.
The interest is the estimation of the parameter $\theta = (\beta,\sigma)$ by the maximum marginal likelihood(MML) which is given:

2 Marginal likelihood

The marginal likelihood of the above model is given by:

$$\begin{aligned} L(\theta;y) = P(Y = y) &= \int_{\mathbb{R}^n} P(Y = y \mid U = u)\ \varphi(u)du \\ &= \int_{\mathbb{R}^n} \prod_{i=1}^{n}\prod_{j=1,j\neq i}^{n} P(Y_{ij} = y_{ij} \mid u_i, u_j)\ \varphi(u)du \\ &= \int_{\mathbb{R}^n} \prod_{i=1}^{n}\prod_{j=1,j\neq i}^{n} \frac{\exp\left((z'_{ij}\beta + u_i - u_j)y_{ij}\right)}{1 + exp\left(z'_{ij}\beta + u_i - u_j\right)}\ \varphi(u)du \end{aligned}$$

where $\varphi()$ is the density distribution of the random effects U.

3 Data

In our case, we have 77 lizards with only 100 responses, so we have too many missing data. We have also four covariates $x_1,\ x_2,\ x_3$ and x_4.

We would like to estimate $(\beta_1, \beta_2, \beta_3, \beta_4, \sigma)$ by the marginal maximum likelihood (MML), using the *glmer* of the *lme4* R package.

Question

Could you please show me how to write the formula and to choose the different parameters of the *glmer* given below to estimate the five parameters $(\beta_1, \beta_2, \beta_3, \beta_4, \sigma)$.

glmer(formula, data, family, start=NULL,verbose=FALSE,nAGQ=1,doFit=TRUE, subset, weights, na.action, offset, contrasts=NULL,model=TRUE, control=list(),...)