Theory

Random variable R has Student-t distribtuion truncated to [α, β] when its density function is expressed as follows:

$$g(x) = \sigma^{-1} f_v\left(\frac{x-\mu}{\sigma}\right)\left(F_v\left(\frac{\beta-\mu}{\sigma}\right) - F_v\left(\frac{\alpha-\mu}{\sigma}\right)\right)^{-1}, \alpha < x < \beta \qquad (1)$$

where:

f_v - density function of standard Student-t distribution

F_v - cumulative distribution function of standard Student-t distribution

v - number of degrees of freedom

Expected value of random variable R could be expressed as follows:

$$E(R) = \int_\alpha^\beta x g(x) dx \quad \text{for } v > 1 \qquad (2)$$

or equivalently:

$$E(R) = \mu + \sigma \frac{\Gamma\left(\frac{v-1}{2}\right)\left((v+a^2)^{-\left(\frac{v-1}{2}\right)} - (v+b^2)^{-\left(\frac{v-1}{2}\right)}\right) v^{v/2}}{2(F_v(b) - F_v(a))\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)} \quad \text{for } v > 1 \qquad (3)$$

where:

Γ(arg) - gamma function

a = (α - μ) / σ

b = (α - μ) / σ

Γ(arg) and F_v(arg) are available in R ('gamma' and 'pt' respectively)

Model

$$\mu = \begin{pmatrix} 55 \\ 40 \\ 50 \\ 35 \\ 45 \\ 30 \end{pmatrix} \qquad \Sigma = \begin{pmatrix} 1 & 1 & 0 & 2 & -1 & -1 \\ 1 & 16 & -6 & -6 & -2 & 12 \\ 0 & -6 & 4 & 2 & -2 & -5 \\ 2 & -6 & 2 & 25 & 0 & -17 \\ -1 & -2 & -2 & 0 & 9 & -5 \\ -1 & -12 & -5 & -17 & -5 & 36 \end{pmatrix}$$

In R language:

```
> library(tmvtnorm)
> meann = c(55, 40, 50, 35, 45, 30)
> covv = matrix(c(  1, 1, 0, 2, -1, -1,
+                   1, 16, -6, -6, -2, 12,
+                   0, -6, 4, 2, -2, -5,
+                   2, -6, 2, 25, 0, -17,
+                  -1, -2, -2, 0, 9, -5,
+                  -1, 12, -5, -17, -5, 36), 6, 6)
> df = 4
> lower = c(20, 20, 20, 20, 20, 20)
> upper = c(60, 60, 60, 60, 60, 60)
> X1 <- rtmvt(n=100000, meann, covv, df, lower, upper)
```

Results:

```
> sum(X1[,1]) / 100000
[1] 54.98258
> sum(X1[,2]) / 100000
[1] 40.36153
> sum(X1[,3]) / 100000
[1] 49.83571
> sum(X1[,4]) / 100000
[1] 34.69571
> sum(X1[,5]) / 100000
[1] 44.81081
> sum(X1[,6]) / 100000
[1] 31.10834
```

Results using equation (3):

$$\begin{pmatrix} 54.97 \\ 40 \\ 49.95 \\ 35.31 \\ 44.94 \\ 31.32 \end{pmatrix}$$