# QUANTILE CALCULATIONS IN R

Objective:
Showing how quantiles (esp. quartiles) are calculated in R.
R offers different functions to calculate quartiles, which can produce different output.

Examples:

```
> data <- c(1,12,14,3,96,111)

> summary(data)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   1.00    5.25   13.00   39.50   75.50  111.00

> quantile(data, c(0.25, 0.5, 0.75), type = 1)
25% 50% 75%
  3  12  96
```

Sources:
- http://stat.ethz.ch/R-manual/R-devel/library/grDevices/html/boxplot.stats.html
- http://stat.ethz.ch/R-manual/R-patched/library/stats/html/quantile.html
- http://en.wikipedia.org/wiki/Quantile
- https://stat.ethz.ch/pipermail/r-help/2012-January/300586.html

## 1.Defining test sets

| | Q1 | | | MEDIAN | | | Q3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **3** | | | **3** | | | **3** | | | **3** | |
| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 |

```
> data12 <- c(2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37)
> length(data12) / 4 ## Length of each quartile
[1] 3
```

| | Q1 | | | MEDIAN | | | Q3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **2.75** | | | **2.75** | | | **2.75** | | | **2.75** | |
| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 |

```
> data11 <- c(2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31)
> length(data11) / 4 ## Length of each quartile
[1] 2.75
```

| | Q1 | | | MEDIAN | | | Q3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **2.5** | | | **2.5** | | | **2.5** | | | **2.5** | |
| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 |

```
> data10 <- c(2, 3, 5, 7, 11, 13, 17, 19, 23, 29)
> length(data10) / 4 ## Length of each quartile
[1] 2.5
```

| | Q1 | | | MEDIAN | | | Q3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **2.25** | | | **2.25** | | | **2.25** | | | **2.25** | |
| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 |

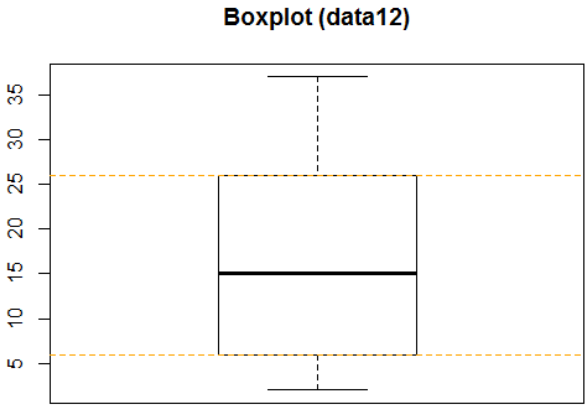
```
> data9 <- c(2, 3, 5, 7, 11, 13, 17, 19, 23)
> length(data9) / 4 ## Length of each quartile
[1] 2.25
```

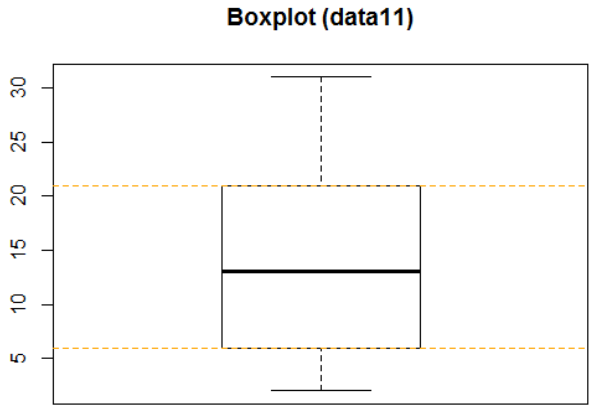
## Comparison of result sets for different functions

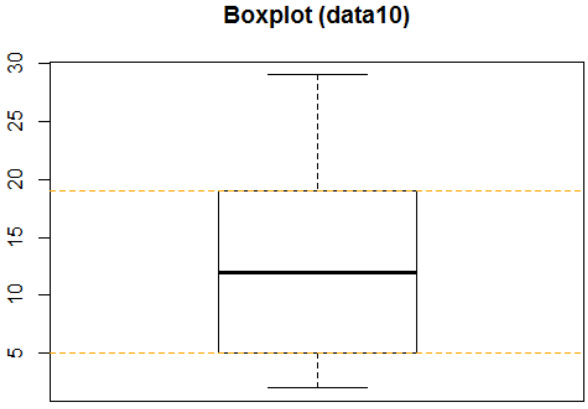
| DATA | FUNCTION | Q1 | MEDIAN | Q3 |
|---|---|---|---|---|
| data12 | quantile(data12, c(0.25, 0.5, 0.75), type = 1) | 5 | 13 | 23 |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 2) | 6 | 15 | 26 |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 3) | 5 | 13 | 23 |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 4) | 5 | 13 | 23 |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 5) | 6 | 15 | 26 |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 6) | 5.5 | 15 | 27.5 |
| | **quantile(data12, c(0.25, 0.5, 0.75), type = 7)** | **6.5** | **15** | **24.5** |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 8) | 5.833333 | 15 | 26.5 |
| | quantile(data12, c(0.25, 0.5, 0.75), type = 9) | 5.875 | 15 | 26.375 |
| | **summary(data12)** | **6.5** | **15** | **24.5** |
| | **boxplot(data12)** | **6** | **15** | **26** |
| data11 | quantile(data11, c(0.25, 0.5, 0.75), type = 1) | 5 | 13 | 23 |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 2) | 5 | 13 | 23 |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 3) | 5 | 13 | 19 |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 4) | 4.5 | 12 | 20 |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 5) | 5.5 | 13 | 22 |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 6) | 5 | 13 | 23 |
| | **quantile(data11, c(0.25, 0.5, 0.75), type = 7)** | **6** | **13** | **21** |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 8) | 5.333333 | 13 | 22.333333 |
| | quantile(data11, c(0.25, 0.5, 0.75), type = 9) | 5.375 | 13 | 22.25 |
| | **summary(data11)** | **6** | **13** | **21** |
| | **boxplot(data11)** | **6** | **13** | **21** |
| data10 | quantile(data10, c(0.25, 0.5, 0.75), type = 1) | 5 | 11 | 19 |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 2) | 5 | 12 | 19 |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 3) | 3 | 11 | 19 |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 4) | 4 | 11 | 18 |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 5) | 5 | 12 | 19 |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 6) | 4.5 | 12 | 20 |
| | **quantile(data10, c(0.25, 0.5, 0.75), type = 7)** | **5.5** | **12** | **18.5** |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 8) | 4.833333 | 12 | 19.333333 |
| | quantile(data10, c(0.25, 0.5, 0.75), type = 9) | 4.875 | 12 | 19.25 |
| | **summary(data10)** | **5.5** | **12** | **18.5** |
| | **boxplot(data10)** | **5** | **12** | **19** |
| data9 | quantile(data9, c(0.25, 0.5, 0.75), type = 1) | 5 | 11 | 17 |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 2) | 5 | 11 | 17 |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 3) | 3 | 7 | 17 |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 4) | 3.5 | 9 | 16 |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 5) | 4.5 | 11 | 17.5 |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 6) | 4 | 11 | 18 |
| | **quantile(data9, c(0.25, 0.5, 0.75), type = 7)** | **5** | **11** | **17** |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 8) | 4.333333 | 11 | 17.666667 |
| | quantile(data9, c(0.25, 0.5, 0.75), type = 9) | 4.375 | 11 | 17.625 |
| | **summary(data9)** | **5** | **11** | **17** |
| | **boxplot(data9)** | **5** | **11** | **17** |

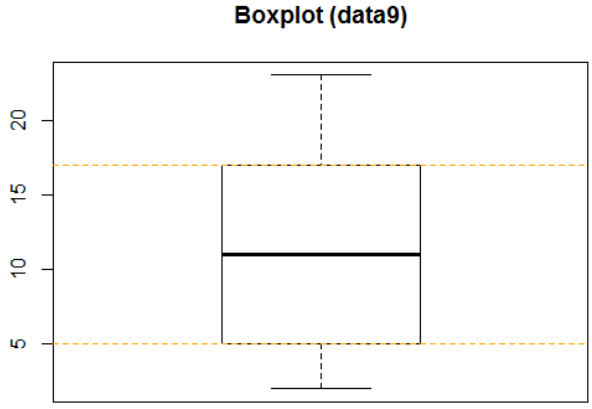
## Boxplots

```
boxplot(data12, main="Boxplot (data12)")
abline(h= 6, col="orange", lty=2, )
abline(h=26, col="orange", lty=2)

boxplot(data11, main="Boxplot (data11)")
abline(h= 6, col="orange", lty=2, )
abline(h=21, col="orange", lty=2)

boxplot(data10, main="Boxplot (data10)")
abline(h= 5, col="orange", lty=2, )
abline(h=19, col="orange", lty=2)

boxplot(data9, main="Boxplot (data9)")
abline(h= 5, col="orange", lty=2, )
abline(h=17, col="orange", lty=2)
```




## Conclusions so far...

The function summary() seems to use the same algorithm for calculating Q1, median and Q3 as does the function quantiles() with type set to 7.

Sometimes, the function quantiles() generates the same results with different types set.

The function boxplot() does not seem to use one of the 9 types that the function quantiles() uses to calculate Q1, median and Q3.

## Box Plot Statistics

```
> ?boxplot.stats
```

*The two ‘hinges’ are versions of the first and third quartile, i.e., close to quantile(x, c(1,3)/4). The hinges equal the quartiles for odd n (where n <- length(x)) and differ for even n. Whereas the quartiles only equal observations for n %% 4 == 1 (n = 1 mod 4), the hinges do so additionally for n %% 4 == 2 (n = 2 mod 4), and are in the middle of two observations otherwise.*

Based on this information, the two 'hinges' in boxplot(data9) should be equal to the calculation of the first and third quartile using function quantile(data9, c(1,3)/4)).
The same should apply to boxplot(data11) and quantile(data11, c(1,3)/4).
This is confirmed by the data in the table on page 2.

In addition, quartiles only equal observations for n %% 4 == 1 and n %% 4 == 2, and are in the middle of two observations otherwise.

Based on this insights and the defined data sets data9, data10, data11 and data12 a custom function can be defined to calculate the first and third quantile according to 'boxplot logic'.

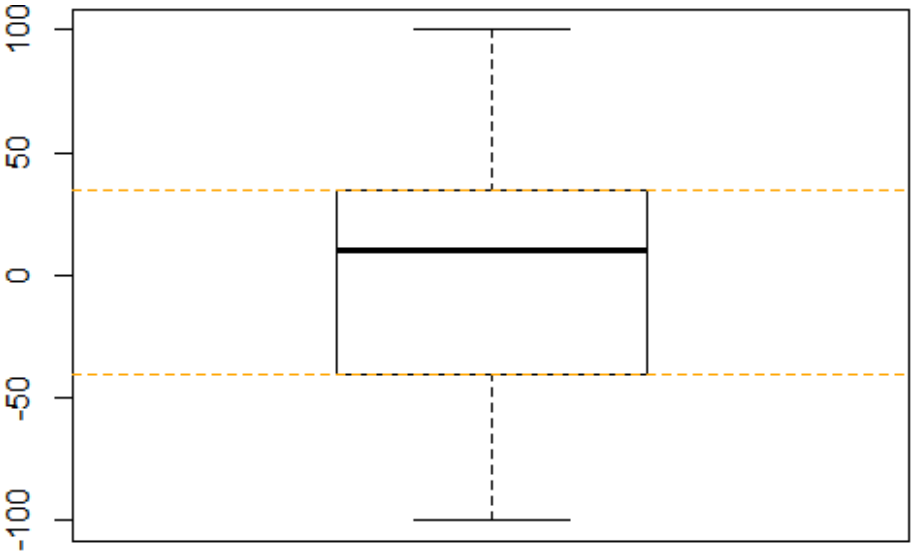
```
BoxplotQuartiles <- function (v) {
                       v  <- sort(v)
                       n  <- length(v)
                       ql <- n/4   ## Length of a quartile
                       p1 <- ql*1  ## Position Q1 in vector
                       p3 <- ql*3  ## Position Q3 in vector
                       f1 <- p1%%1 ## Fractional part of p1
                       f3 <- p3%%1 ## Fractional part of p3
                       if (f1 == 0.25)
                          Q1 <- v[p1 + 0.75]
                       else if (f1 == 0.50)
                          Q1 <- v[p1 + 0.50]
                       else if (f1 == 0.75)
                          Q1 <- (v[p1 + 0.25] + v[p1 + 0.25 + 1.00]) / 2
                       else if (f1 == 0.00)
                          Q1 <- (v[p1 + 0.00] + v[p1 + 0.00 + 1.00]) / 2
                       else
                          Q1 <- 'Error in calculation Q1'
                       if (f3 == 0.25)
                          Q3 <- (v[p3 - 0.25] + v[p3 - 0.25 + 1.00]) / 2
                       else if (f3 == 0.50)
                          Q3 <- v[p3 + 0.50]
                       else if (f3 == 0.75)
                          Q3 <- v[p3 + 0.25]
                       else if (f3 == 0.00)
                          Q3 <- (v[p3 - 0.00] + v[p3 - 0.00 + 1.00]) / 2
                       else
                          Q3 <- 'Error in calculation Q3'
                       return(c(Q1, Q3))
}
```

## Test function with the defined data sets

```
> BoxplotQuartiles(data12)
[1]  6 26

> BoxplotQuartiles(data11)
[1]  6 21

> BoxplotQuartiles(data10)
[1]  5 19

> BoxplotQuartiles(data9)
[1]  5 17
```

## Test function with another odd data set

```
> data25 <- c(-100,99,99,100,96,10,-80,85,4,30,30,30,-40,-50,0,0,6,17,19,35,10,71,-99,-98,-97)

> BoxplotQuartiles(data25)
[1] -40  35

> boxplot(data25)
> abline(h=-40, col="orange", lty=2)
> abline(h= 35, col="orange", lty=2)
```

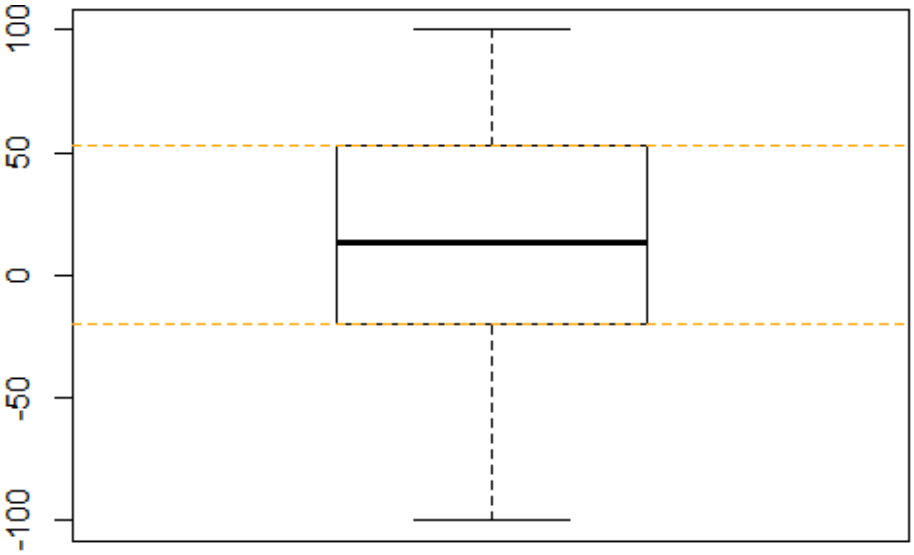



Expectation:
Qantile and summary functions give equal results for the first and third quartile compared to the custom function BoxplotQuartiles() because of the odd data set.

```
> quantile(data25, c(1,3)/4, type=7)
25% 75%
-40  35

> summary(data25)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-100.00  -40.00   10.00    7.08   35.00  100.00
```

## Test function with another even data set

```
> data24 <- c(-100,99,99,100,96,10,-80,85,4,30,30,30,-40,-50,0,0,6,17,19,35,10,71,-99,-98)

> BoxplotQuartiles(data24)
[1] -20  53

> boxplot(data24)
> abline(h=-20, col="orange", lty=2)
> abline(h= 53, col="orange", lty=2)
```




Expectation:
Qantile and summary functions give other results for the first and third quartile compared to the custom function BoxplotQuartiles() because of the even data set.

```
> quantile(data24, c(1,3)/4, type=7)
25% 75%
-10  44

> summary(data24)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-100.00  -10.00   13.50   11.42   44.00  100.00
```

## Beautiful solutions

Peter Dalgaard (https://stat.ethz.ch/pipermail/r-help/2012-January/300586.html) suggested two beautiful solutions.

The first one is the function fivenum().

```
> fivenum(data12, na.rm = TRUE)
[1]  2  6 15 26 37

> fivenum(data11, na.rm = TRUE)
[1]  2  6 13 21 31

> fivenum(data10, na.rm = TRUE)
[1]  2  5 12 19 29

> fivenum(data9, na.rm = TRUE)
[1]  2  5 11 17 23
```

The second one exists of two custom functions, build on the idea that the 'hinges' are the medians of the bottom and top halves of the sorted observations with the middle observation counting in both groups if n is odd.

```
Q1 <- function (x) {
         median(sort(x)[1:floor((length(x)+1)/2)])
      }

Q3 <- function (x) {
        median(sort(x)[ceiling((length(x)+1)/2):length(x)])
      }
```

Q1()

```
> Q1(data12)
[1] 6

> Q1(data11)
[1] 6

> Q1(data10)
[1] 5

> Q1(data9)
[1] 5
```

Q3()

```
> Q3(data12)
[1] 26

> Q3(data11)
[1] 21

> Q3(data10)
[1] 19

> Q3(data9)
[1] 17
```

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## Final tests...

```
> test_odd <- floor(rnorm(121)*100)

> BoxplotQuartiles(test_odd)
[1] -73  49

> fivenum(test_odd)
[1] -299  -73   -1   49  256

> Q1(test_odd)
[1] -73

> Q3(test_odd)
[1] 49
```

```
> test_even <- floor(rnorm(122)*100)

> BoxplotQuartiles(test_even)
[1] -75  88

> fivenum(test_even)
[1] -269.0  -75.0    2.5   88.0  252.0

> Q1(test_even)
[1] -75

> Q3(test_even)
[1] 88
```

## Custom quantile functions per type

```
QuantileType1 <- function (v, p) {
                                  v = sort(v)
                                  m = 0
                                  n = length(v)
                                  j = floor((n * p) + m)
                                  g = (n * p) + m - j
                                  y = ifelse (g == 0, 0, 1)
                                  ((1 - y) * v[j]) + (y * v[j+1])
                                }


QuantileType2 <- function (v, p) {
                                  v = sort(v)
                                  m = 0
                                  n = length(v)
                                  j = floor((n * p) + m)
                                  g = (n * p) + m - j
                                  y = ifelse (g == 0, 0.5, 1)
                                  ((1 - y) * v[j]) + (y * v[j+1])
                                }


QuantileType3 <- function (v, p) {
                                  v = sort(v)
                                  m = -0.5
                                  n = length(v)
                                  j = floor((n * p) + m)
                                  g = (n * p) + m - j
                                  y = ifelse(trunc(j/2)*2==j, ifelse(g==0, 0, 1), 1)
                                  ((1 - y) * v[j]) + (y * v[j+1])
                                }


QuantileType7 <- function (v, p) {
                                  v = sort(v)
                                  h = ((length(v)-1)*p)+1
                                  v[floor(h)]+((h-floor(h))*(v[floor(h)+1]- v[floor(h)]))
                                }
```

Example:

```
> data12 <- c(2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37)
> QuantileType1(data12, 0.25)
[1] 5
```

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