Formally, for the purpose of the decomposition into common and specific shocks, the measurement equation of the state-space model is set up as follows[9]:

$$\underset{2\times 1}{\begin{bmatrix} s_t^1 \\ s_t^2 \end{bmatrix}} = \underset{2\times 3}{\begin{bmatrix} \alpha_{11} & 1 & 0 \\ \alpha_{21} & 0 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} Z_{0t} \\ Z_{1t} \\ Z_{2t} \end{bmatrix}}, \tag{4}$$

where the observable variables s_t^i are supply or demand shocks obtained from the structural VAR decomposition in the first step for country (region) i at time t. Z_{0t} is the unobservable common component while Z_{it} are the unobservable country (region)-specific components for region 1 or 2 at time t.

The unobservable components to be recovered follow a transition equation as:

$$\underset{3\times 1}{\begin{bmatrix} Z_{0t} \\ Z_{1t} \\ Z_{2t} \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} \varpi_{0t} \\ \varpi_{1t} \\ \varpi_{2t} \end{bmatrix}}, \tag{5}$$

where the ϖ_{0t} and ϖ_{it} are assumed to have no serial correlation with variance–covariances matrix specified as follows:

$$E(\varpi\varpi') = \begin{bmatrix} 1 & 0 & 0 \\ 0 & P_{11} & 0 \\ 0 & 0 & P_{22} \end{bmatrix}, \tag{6}$$

where for purpose of identification the common and specific shocks are assumed to be orthogonal, the variance of common shocks is set to unity and the variances of province-specific shocks are P_{ii}.

The state-space model outlined above allows us to decompose structural shocks (supply or demand) facing province i into a component that is common to all provinces (Z_{0t}) and one that is province-specific (Z_{it}), with coefficients α_{it} measuring how strongly structural shocks facing province i are related to the common component. The state-space model is estimated using the Kalman filter.[10]