The Limited Dependent Variable (LDV) (1999) Model

This model was developed by Lesmond (1999) and only needs daily security returns to estimate costs of liquidity. The model recognises that investors will only trade if they expect a positive return. If transaction costs of trading are higher than expected returns, trading will not take place, assuming all investors are equally informed. This results in zero returns being observed.

The LDV (1999) model is developed as follows:

- In the absence of all transaction costs (external or internal), the return should be:

$$R^*_{j,t} = \beta_j \cdot Rm_t + \varepsilon_{j,t}$$

where $R^*_{j,t}$ is required return, β_j, Rm_t and $\varepsilon_{j,t}$ are the beta coefficient to market-wide and new firm-specific information, respectively. Also j is a particular firm and t is time.

- Because of illiquidity costs however, Mendelson (1986) showed that actual returns $R_{j,t}$ require the required returns to be adjusted for liquidity premium as follows:

$$R_{j,t} = R^*_{j,t} - \alpha_{i,j}$$

where $R_{j,t}$ is the measured return, $\alpha_{2,j}$ is the effective buy-side cost and $\alpha_{1,j}$ is the effective sell-side cost for firm j at time t (Lesmond, 1999).

- The general methodology for limited dependent variable models is detailed in Maddal (1983). The effect of liquidity on equity returns is then generally modelled by combining the required return and the liquidity constraint. This gives the following returns:

$$R_{j,t} = R^*_{j,t} - \alpha_{1,j} \quad \text{if} \quad R^*_{j,t} < \alpha_{1,j} \quad \text{and} \quad \alpha_{1,j} < 0,$$

$$R_{j,t} = 0 \quad \text{if} \quad \alpha_{1,j} \leq R^*_{j,t} \leq \alpha_{2,j}$$

$$R_{j,t} = R^*_{j,t} - \alpha_{2,j} \quad \text{if} \quad R^*_{j,t} > \alpha_{2,j} \quad \text{and} \quad \alpha_{2,j} > 0.$$

The estimates, α_2-α_1, provide liquidity threshold for informed investors and are termed the LDV model. If the measured return, $R_{j,t}$=0, it indicates that neither the buy nor sell side threshold can be exceeded (Lesmond, 1999).

- Finally, the resulting likelihood estimator is:

$$L\left(\alpha_{1,j}, \alpha_{2,j}, \beta_j, \sigma_j \middle| R_{j,t}, Rm_t\right) =$$

$$\prod_1 \frac{1}{\sigma_j} n\left[\frac{R_{j,t} + \alpha_{1,j} - \beta_j \cdot Rm_t}{\sigma_j}\right]$$

$$\times \prod_0 \left[N\left(\frac{\alpha_{2,j} - \beta_j \cdot Rm_t}{\sigma_j}\right) - N\left(\frac{\alpha_{1,j} - \beta_j \cdot Rm_t}{\sigma_j}\right)\right]$$

$$\times \prod_2 \frac{1}{\sigma_j} n\left[\frac{R_{j,t} + \alpha_{2,j} - \beta_j \cdot Rm_t}{\sigma_j}\right]$$

For liquidity purposes, the $\alpha_{i,j}$ estimates are the focus.

n() and N() are the probability and cumulative normal distribution respectively.

The numerical subscripts (0, 1, and 2) under each likelihood operator represent different regions as follows:

- 1 is the non-zero measured return region where the market return is negative.
- 0 is the zero measured return regions.
- 2 is the non-zero measured return where the market return is positive.