

If Y follows a Weibull distribution, the loglikelihood to be maximized is:

$$\begin{aligned}
 l(\boldsymbol{\theta}, \boldsymbol{\omega}) = & \sum_{i=1}^n \left[\epsilon_i \delta_{1_i} \ln \left(\sum_{j=1}^m \frac{\alpha_{ij}}{\sigma} \exp \left(\frac{\ln(u_i + z_{r_i} - s_j) - \mu - \beta \ln(s_j)}{\sigma} \right) \right. \right. \\
 & \left. \left. - e^{\frac{\ln(u_i + z_{r_i} - s_j) - \mu - \beta \ln(s_j)}{\sigma}} \right) \omega_j \right) \\
 & + \epsilon_i \delta_{2_i} \ln \left(\sum_{j=1}^m \alpha_{ij} \exp \left(- e^{\frac{\ln(u_i + z_{r_i} - s_j) - \mu - \beta \ln(s_j)}{\sigma}} \right) \omega_j \right) \\
 & + \epsilon_i (1 - \delta_{1_i})(1 - \delta_{2_i}) \ln \left(\sum_{j=1}^m \alpha_{ij} \left[1 - \exp \left(- e^{\frac{\ln(u_i + z_{r_i} - s_j) - \mu - \beta \ln(s_j)}{\sigma}} \right) \right] \omega_j \right) \\
 & \left. + (1 - \epsilon_i) \ln \left(\sum_{j=1}^m \alpha_{ij} \omega_j \right) \right],
 \end{aligned}$$

where $n = 361$ and $m = 204$ (months).

Maximization is restricted by the known constraints on $\boldsymbol{\omega}$ and $\sigma > 0$.