

Negative Binomial

$i$  = rows

$j$  = columns

$D_{ij}$  = cumulative claims

$C_{ij}$  = cumulative claims

In R this is named *loss*:

	1	2	3	4	5	6	7	8	9	10
1	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
2	106	4285	5396	10666	13782	15599	15496	16169	16704	
3	3410	8992	13873	16141	18735	22214	22863	23466		
4	5655	11555	15766	21266	23425	26083	27067			
5	1092	9565	15836	22169	25955	26180				
6	1513	6445	11702	12935	15852					
7	557	4020	10946	12314						
8	1351	6947	13112							
9	3133	5395								
10	2063									

So  $D_{11} = 5012$ ,  $D_{12} = 8269 \dots$

In R this is called *All*:

	1	2	3	4	5	6	7	8	9	10
1	0	8.51959	9.020269	9.29716	9.376278	9.51333	9.691593	9.798627	9.831347	6.481577
2	0	4.663439	8.362876	8.593413	9.274816	9.531119	9.654962	9.648337	9.690851	
3	0	8.134468	9.104091	9.5377	9.689118	9.838149	10.00848	10.03728		
4	0	8.640295	9.354874	9.665611	9.964865	10.06156	10.16904			
5	0	6.995766	9.165866	9.670041	10.00645	10.16412				
6	0	7.32185	8.77106	9.367515	9.467692					
7	0	6.322565	8.299037	9.300729						
8	0	7.2086	8.846065							
9	0	8.049746								
10	0									

So  $8.51959 = \ln(5012)$  represents  $\log(D_{i,j-1})$  where  $i = 1, j = 2$

(using  $C_{ij}$  or  $D_{ij}$  makes no difference)

Now,

$$E[C_{ij}] = m_{ij} = (\lambda_j - 1)D_{i,j-1}$$

$$\eta_{ij} = \log(m_{ij}) = \log(\lambda_j - 1) + \log(D_{i,j-1})$$

$$\log(\lambda_j - 1) = c + \alpha_{j-1} \quad \text{with: } \alpha_1 = 0, j \geq 2$$

$$\log(m_{ij}) = c + \alpha_{j-1} + \log(D_{i,j-1})$$

For  $1 \leq i \leq 10$  and  $j = 1$

$$\log(m_{11}) = c$$

$$\log(m_{21}) = c$$

⋮

$$\log(m_{10,1}) = c$$

Since  $\alpha_{j-1} = \alpha_0$  does not exists.

Then for  $1 \leq i \leq 9$  and  $j = 2$

$$\log(m_{12}) = c + \alpha_1 + \log(D_{1,1})$$

$$\log(m_{22}) = c + \alpha_1 + \log(D_{2,1})$$

⋮

$$\log(m_{92}) = c + \alpha_1 + \log(D_{9,1})$$

But  $\alpha_1 = 0$

Then for  $1 \leq i \leq 8$  and  $j = 3$

$$\log(m_{13}) = c + \alpha_2 + \log(D_{1,2})$$

⋮

$$\log(m_{83}) = c + \alpha_2 + \log(D_{8,2})$$

And so on until  $i = 1, j = 10$

$$\log(m_{1,10}) = c + \alpha_9 + \log(D_{1,9})$$

In R:

```
> loss #the first table above
```

```
> claims <- as.vector(loss)
```

```

> n.origin <- nrow(loss)

> n.dev <- ncol(loss)-1 #since  $\alpha_1 \dots \alpha_9$ 

> dev <- factor(col <- rep(0:n.dev, each=n.origin))

> All #the second table above

> cum <- as.vector(All)

> mack <- data.frame(claims=claims,dev=dev,cum=cum)

> library(MASS)

> model.nb1 <- glm.nb(claims ~ dev + offset(cum), data = mack)

> summary(model.nb1)

```

The parameters:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	7.7787	0.1865	41.72	<2e-16 ***
dev1	-5.6752	0.2709	-20.95	<2e-16 ***
dev2	-7.2505	0.2796	-25.93	<2e-16 ***
dev3	-7.5053	0.2905	-25.84	<2e-16 ***
dev4	-7.6107	0.3044	-25.00	<2e-16 ***
dev5	-7.6592	0.3228	-23.72	<2e-16 ***
dev6	-7.7363	0.3487	-22.19	<2e-16 ***
dev7	-7.7449	0.3880	-19.96	<2e-16 ***
dev8	-10.6810	0.4571	-23.37	<2e-16 ***
dev9	-8.8398	0.6217	-14.22	<2e-16 ***

But they are not correct.

dev1 which represent  $\alpha_1$  should be zero and the result should be

	Parameter estimates
Constant	0.6928
$\alpha_2$	-1.1652
$\alpha_3$	-1.9989
$\alpha_4$	-2.4550
$\alpha_5$	-2.8698
$\alpha_6$	-3.8645
$\alpha_7$	-4.0961
$\alpha_8$	-4.7711
$\alpha_9$	-5.3796

I think that the problem is when I'm specifying the data.frame.

```
> mack
   claims dev      cum
1     5012  0  0.000000
2     106   0  0.000000
3    3410   0  0.000000
4    5655   0  0.000000
5   1092   0  0.000000
6   1513   0  0.000000
7    557   0  0.000000
8   1351   0  0.000000
9   3133   0  0.000000
10  2063   0  0.000000
11  8269   1  8.519590
12  4285   1  4.663439
13  8992   1  8.134468
14 11555   1  8.640295
15  9565   1  6.995766
16  6445   1  7.321850
17  4020   1  6.322565
18  6947   1  7.208600
19  5395   1  8.049746
20    NA   1      NA
21 10907   2  9.020269
22  5396   2  8.362876
23 13873   2  9.104091
24 15766   2  9.354874
25 15836   2  9.165866
26 11702   2  8.771060
27 10946   2  8.299037
28 13112   2  8.846065 ...and so on.
```

I need to tell R that under the column named *dev*, the first ten zeros, should not take them as a label of a parameter. Since as I specified before, the first ten claims, does not depend on a parameter but only on the intercept.

How can I do it please?