# Stat*4360 Winter 2009: Final Examination

**Made available by 11:59pm, Sunday April 12, 2009 NO EXCEPTIONS**

The purpose of this final examination is to bring together all of the concepts learned this semester. Please begin your R session with `library(MASS)`.

You will each receive a custom made data set *via* your email. Upload your data into your local drives where in the following, $X_t$, will be in reference to the first column of data and $Y_t$ is in reference to the second column of data.

The data is something I generated individually from the following model:

$$Y_t = \sum_{s=m_1}^{m_2} \xi_s X_{t-s} + N_t, \ t = 1, \ldots, 1000 \ .$$

A description of the terms and parameters go as follows:

- The $\{X_t\}$ is from a stationary $AR(p_X)$ process with autoregressive parameters

  $$\alpha_{X,1}, \ldots, \alpha_{X,p_X},$$

  variance (innovation) $\sigma_X^2 > 0$, and the autoregressive order $p_X > 0$.

- The $\{N_t\}$ is from a stationary $ARMA(p_N, q_N)$ process with autoregressive parameters and moving average parameters

  $$\alpha_{N,1}, \ldots, \alpha_{N,p_N}, \quad \beta_{N,1}, \ldots, \beta_{N,q_N},$$

  variance $\sigma_N^2 > 0$, and autoregressive and moving average orders $p_N, q_N \geq 0$.

- The order of the filter satisfies $m_2 > m_1 > 0$ as well as coefficients of the filter being

  $$\xi_{m_1}, \ldots, \xi_{m_2}.$$

Thus there are: 5 order parameters, $p_X, p_N, q_N, m_1, m_2$; 2 variance (innovative) parameters $\sigma_X^2, \sigma_N^2$; the filter parameters $\xi_{m_1}, \ldots, \xi_{m_2}$; the autoregressive parameters for $X_t$, $\alpha_{X,1}, \ldots, \alpha_{X,p_X}$, the autoregressive parameters for $N_t$, $\alpha_{N,1}, \ldots, \alpha_{N,p_N}$, and the moving average parameters for $N_t$, $\beta_{N,1}, \ldots, \beta_{N,q_N}$.

The objective of this examination is to fit all of the above parameters. Here are some hints on how to proceed.

**Step 1:** Identify and fit $\{X_t\}$.

**Step 2:** Use the method of pre-whitening to fit the parameters $m_2 > m_1 > 0$, followed by the parameters $\xi_{m_1}, \ldots, \xi_{m_2}$.

**Step 3:** Next get an estimate for the noise process $\{N_t\}$, after which you identify and fit an appropriate $ARMA(p_N, q_N)$ model.

**Step 4:** Transfer everything back to the original form with the estimated parameters and run a final `armia` fit using the `xreg` option.

**Step 5:** Reflect back upon your work and ask whether this is the "best" model you can fit, *ie*, are all of the assumptions satisfied, can one get the same performance with less parameters.

After you have done your analysis, please write your report as formally as possible (meaning correct grammar and spelling) providing details as to what you believe the true model is, how and why you arrived at your conclusions. You can include graphs to back your conclusions, however, your report should be no longer than 10 pages inclusive.

**Submission:** Please use electronic submission and 20% of this examination will be graded on presentation.