

Reference to “Practical Regression and Anova using R”, Jullian Faraway,
 July 2002 The goodness of fit is measured by coefficient of determination.

$$R^2 = 1 - \frac{\sum(\widehat{y}_i - y_i)^2}{\sum(y_i - \bar{y})^2}$$

For Model 1:

$$R_{y_1}^2 = 1 - \frac{\sum(\widehat{y}_{1i} - y_i)^2}{\sum(y_i - \bar{y})^2}$$

For Model 2:

$$R_{y_2}^2 = 1 - \frac{\sum(\widehat{y}_{2i} - y_i)^2}{\sum(y_i - \bar{y})^2}$$

$$R_{\frac{y_2+y_1}{2}}^2 = 1 - \frac{\sum(\frac{\widehat{y}_{1i} + \widehat{y}_{2i}}{2} - y_i)^2}{\sum(y_i - \bar{y})^2} = 1 - \frac{A}{\sum(y_i - \bar{y})^2}$$

$$\frac{R_{y_1}^2 + R_{y_2}^2}{2} = 1 - \frac{\frac{1}{2} \sum(\widehat{y}_{1i} - y_i)^2 + \sum(\widehat{y}_{2i} - y_i)^2}{\sum(y_i - \bar{y})^2} = 1 - \frac{B}{\sum(y_i - \bar{y})^2}$$

Where,

$$A = \sum\left(\frac{\widehat{y}_{1i} + \widehat{y}_{2i}}{2} - y_i\right)^2 = \frac{1}{4} \sum(2\widehat{y}_{1i}\widehat{y}_{2i} + \widehat{y}_{1i}^2 + \widehat{y}_{2i}^2 - 4(\widehat{y}_{1i} + \widehat{y}_{2i})y_i + 4y_i^2) = \frac{1}{4} \sum(a + c)$$

$$B = \frac{1}{2}(\sum(\widehat{y}_{1i} - y_i)^2 + \sum(\widehat{y}_{2i} - y_i)^2) = \frac{1}{4} \sum(\widehat{y}_{1i}^2 + \widehat{y}_{2i}^2 + \widehat{y}_{1i}^2 + \widehat{y}_{2i}^2 - 4(\widehat{y}_{1i} + \widehat{y}_{2i})y_i + 4y_i^2) = \frac{1}{4} \sum(b + c)$$

$$a = 2\widehat{y}_{1i}\widehat{y}_{2i}, \quad b = \widehat{y}_{1i}^2 + \widehat{y}_{2i}^2$$

$b \geq a$ is always hold

So

$B \geq A$ is always hold, therefore

$$R_{\frac{y_2+y_1}{2}}^2 \geq \frac{R_{y_1}^2 + R_{y_2}^2}{2}$$