Concerns with Singular Value Decomposition in R Software

Using singular value decomposition, any second-order tensor is given as

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}, \tag{1}$$

where $\mathbf{U}$ and $\mathbf{V}$ are the orthogonal tensors, and $\boldsymbol{\Sigma}$ is the diagonal matrix (Eigenvalue matrix). For a symmetric matrix, the orthogonal tensors are the same, i.e., $\mathbf{U}$=$\mathbf{V}$. There is an issue with the singular value decomposition function (svd) in R-software which is illustrated using the following example.

Example:
The symmetric matrix $\mathbf{A}$ is assumed as

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix}. \tag{2}$$

Upon finding the singular value decomposition of matrix $\mathbf{A}$ using svd($\mathbf{A}$) in R software, one arrives at

$$\begin{aligned} \mathbf{U} &= \begin{bmatrix} -0.7071068 & -0.7071068 \\ -0.7071068 & 0.7071068 \end{bmatrix} \\ \boldsymbol{\Sigma} &= \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \\ \mathbf{V} &= \begin{bmatrix} -0.7071068 & 0.7071068 \\ -0.7071068 & -0.7071068 \end{bmatrix} \end{aligned} \tag{3}$$

The tensors $\mathbf{U}$ and $\mathbf{V}$ should be the same for a symmetric matrix. However, the tensors (Eq.(3)) for the chosen symmetric matrix given in Eq.(2) are different, and the components of the diagonal matrix are incorrect. As a result of these issues, a difference is observed not only in the exponential of a matrix but also in finding the even power of a matrix. Comparison of the square of the chosen matrix '$\mathbf{A}$' using the direct method and singular value decomposition is as follows

$$\mathbf{A}^2 = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix} \quad \text{Direct} \quad \text{Method} \tag{4}$$

$$\mathbf{A}^2 = \begin{bmatrix} 8 & 17 \\ 17 & 8 \end{bmatrix} \quad \text{SVD} \tag{5}$$

Observations: Observing the above equations, it is clear that there is a huge difference in the square of matrix obtained using direct method and SVD.