

Lineare Regression: Tests

Statistik (Biol./Pharm./HST) – FS 2014



Ersatz: Cooper & Shuttle

- 12-Minuten Test nach Cooper (1968)
- 20m-Shuttle-Test nach Leger (1982)

Eur J Appl Physiol (1982) 49: 1–12

European Journal of
**Applied
Physiology**
and Occupational Physiology
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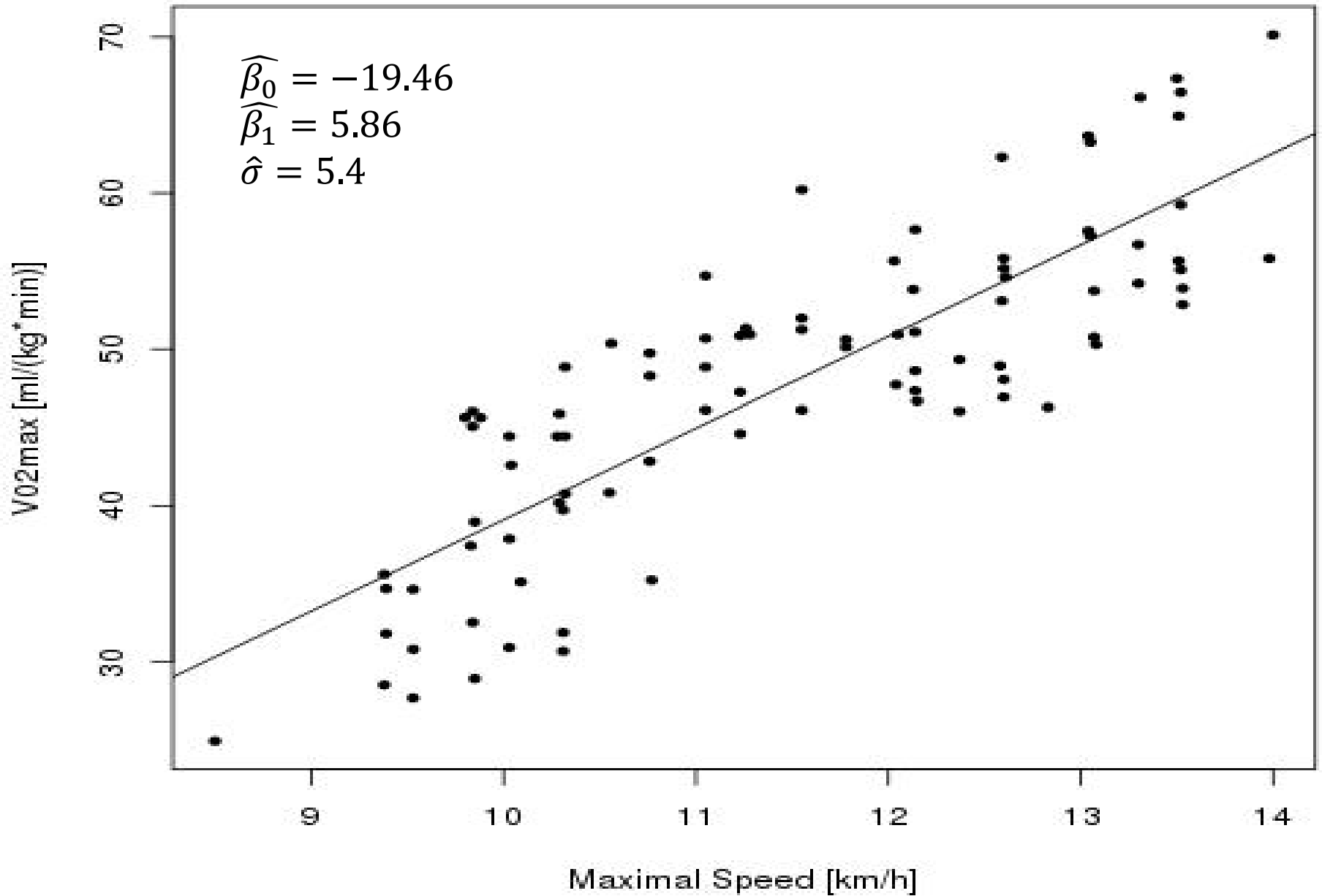
A Maximal Multistage 20-m Shuttle Run Test to Predict $\dot{V}O_2 \max^*$

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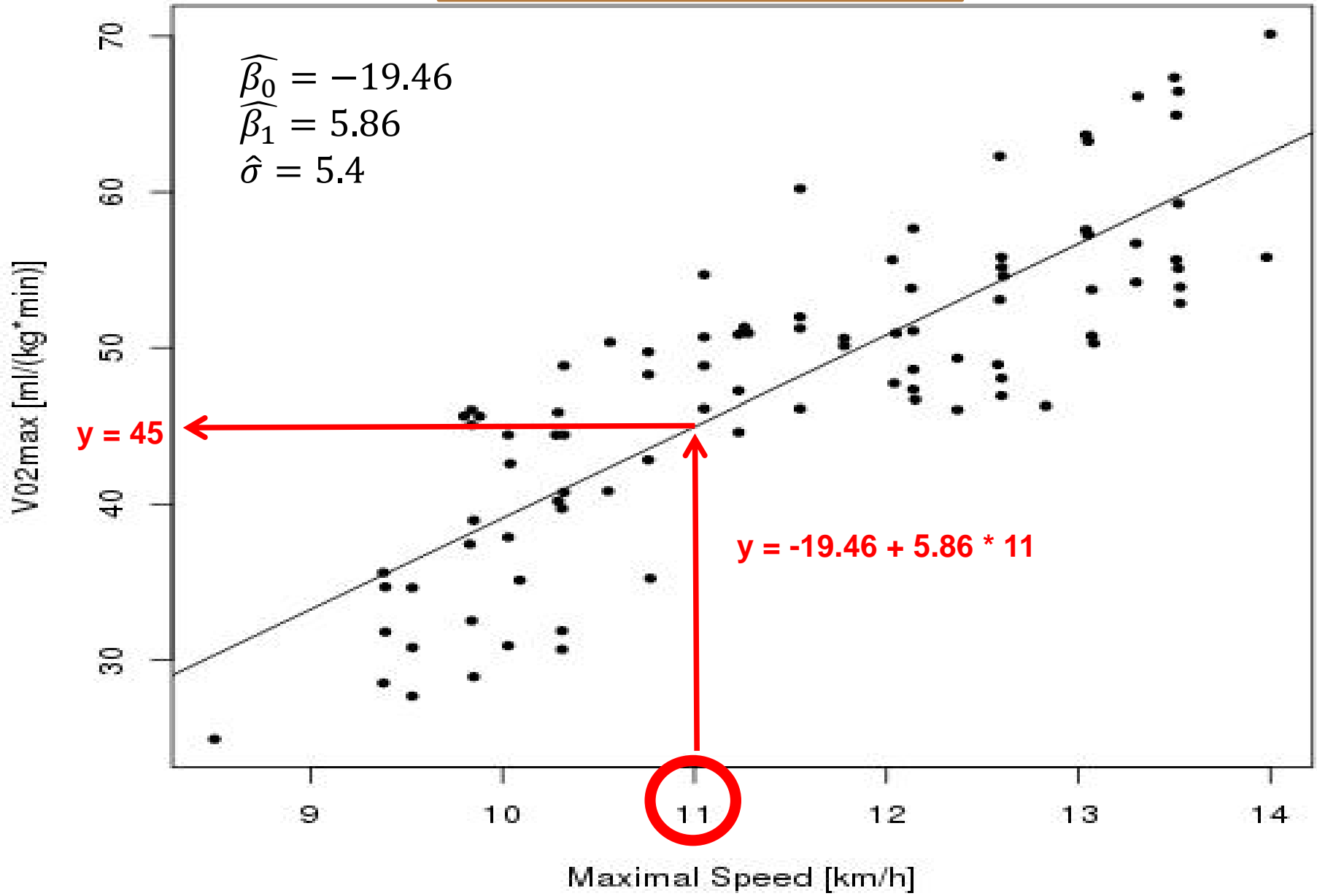
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Methode der kleinsten Quadrate



- Wie genau stimmen Parameter?
- Wie genau stimmt Vorhersage?



t-Test in der Linearen Regression: 1/2

1. Modell:

$$Y_i = \beta_0 + \beta_1 x_i + E_i, \quad E_1, \dots, E_n \text{ iid } \mathcal{N}(0, \sigma^2).$$

2. Nullhypothese: $H_0 : \beta_1 = 0$

Alternative: $H_A : \beta_1 \neq 0$ (Es wird hier üblicherweise ein zwei-seitiger Test durchgeführt)

3. Teststatistik:

$$T = \frac{\text{beobachtet} - \text{erwartet}}{\text{geschätzter Standardfehler}} = \frac{\hat{\beta}_1 - 0}{\widehat{\text{s.e.}}(\hat{\beta}_1)}.$$

Dabei ist der geschätzte Standardfehler

$$\widehat{\text{s.e.}}(\hat{\beta}_1) = \sqrt{\widehat{\text{Var}}(\hat{\beta}_1)} = \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^n (x_i - \bar{x}_n)^2}}.$$

Verteilung der Teststatistik unter H_0 : $T \sim t_{n-2}$

t-Test in der Linearen Regression: 2/2

4. **Signifikanzniveau:** α

5. **Verwerfungsbereich für die Teststatistik:**

$$K = \left(-\infty, -t_{n-2; 1-\frac{\alpha}{2}}\right] \cup \left[t_{n-2; 1-\frac{\alpha}{2}}, \infty\right)$$

6. **Testentscheid:** Überprüfe, ob der beobachtete Wert der Teststatistik im Verwerfungsbereich liegt.

Lineare Regression in R

Modell: $Y_i = \beta_0 + \beta_1 x_i + E_i$, $E_i \sim N(0, \sigma^2)$ i. i. d

Modell: $Y_i = -19.46 + 5.86x_i + E_i$, $E_i \sim N(0, 5.43^2)$ i. i. d

Standardfehler von $\widehat{\beta}_1 (= \widehat{\sigma}_{\widehat{\beta}_1})$

Approx. 95%-VI:

$$5.86 \pm 2 * 0.41$$

Exaktes 95%-VI:

$$5.86 \pm 1.99 * 0.41$$

$t_{89;0.975}$

```
> fitShuttle <- lm(vo2max ~ vmax, data = dat)
> summary(fitShuttle)
```

```
Call:
lm(formula = vo2max ~ vmax, data = dat)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-10.2230  -4.3976  -0.2016   4.7026  12.0348
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -19.4582     4.7239  -4.119  8.5e-05 ***
vmax         5.8566     0.4082  14.347 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 5.433 on 89 degrees of freedom
Multiple R-squared: 0.6981, Adjusted R-squared: 0.6948
F-statistic: 205.8 on 1 and 89 DF, p-value: < 2.2e-16
```

Beobachtete Teststatistik

im Test $H_0: \beta_1 = 0$ vs.
 $H_A: \beta_1 \neq 0$

P-Wert:

Angenommen $\beta_1 = 0$;
wie wa. ist Beobachtung
oder etwas extremere?

Freiheitsgrade: $n - (\text{Anz. } \beta\text{'s}) = 91 - 2 = 89$

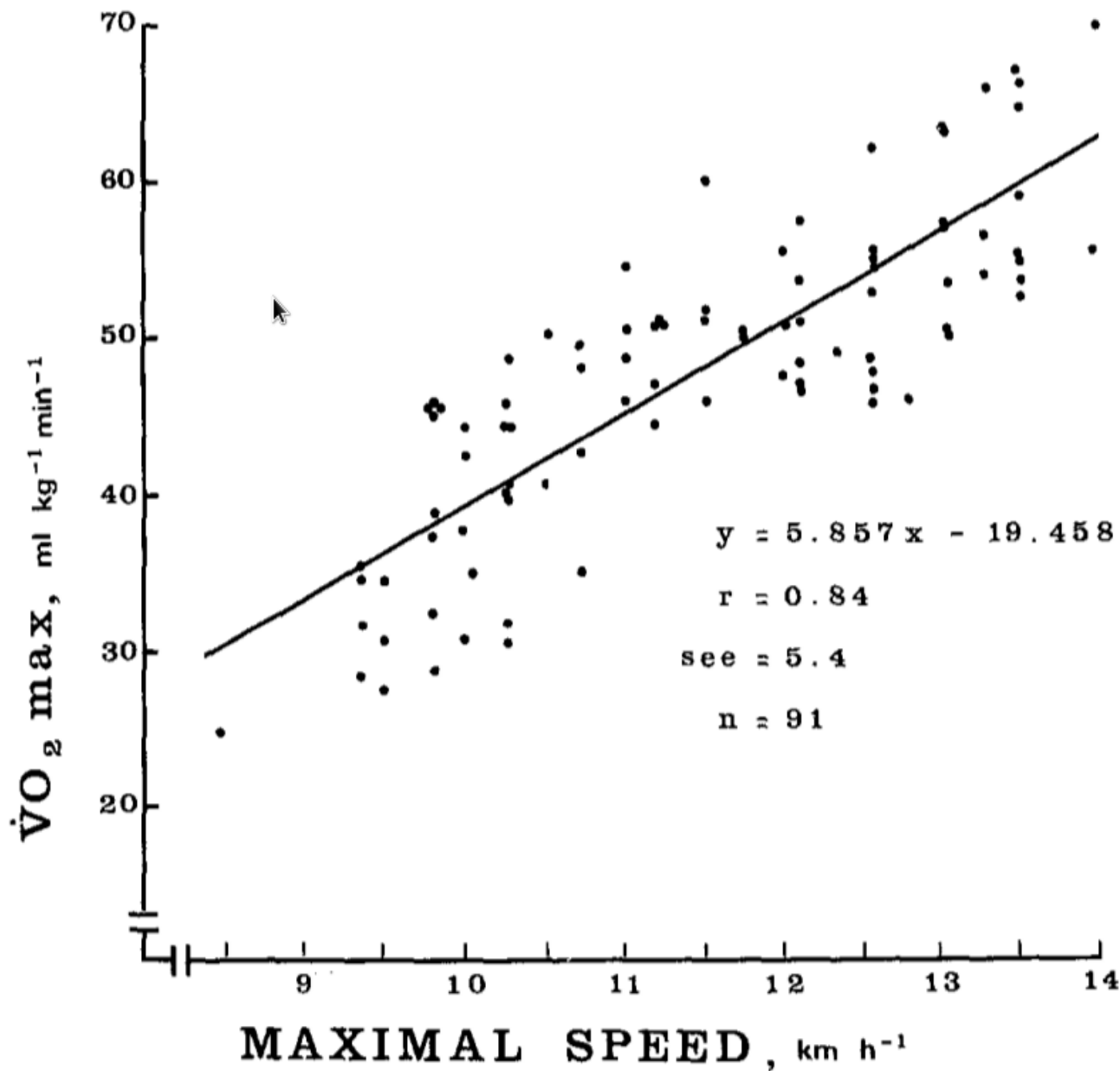


Fig. 2. $\dot{V}O_2$ max as a function of the maximal speed achieved in the 20-m shuttle run test for a total sample of 91 adult subjects. Each point in this figure represents maximal effort

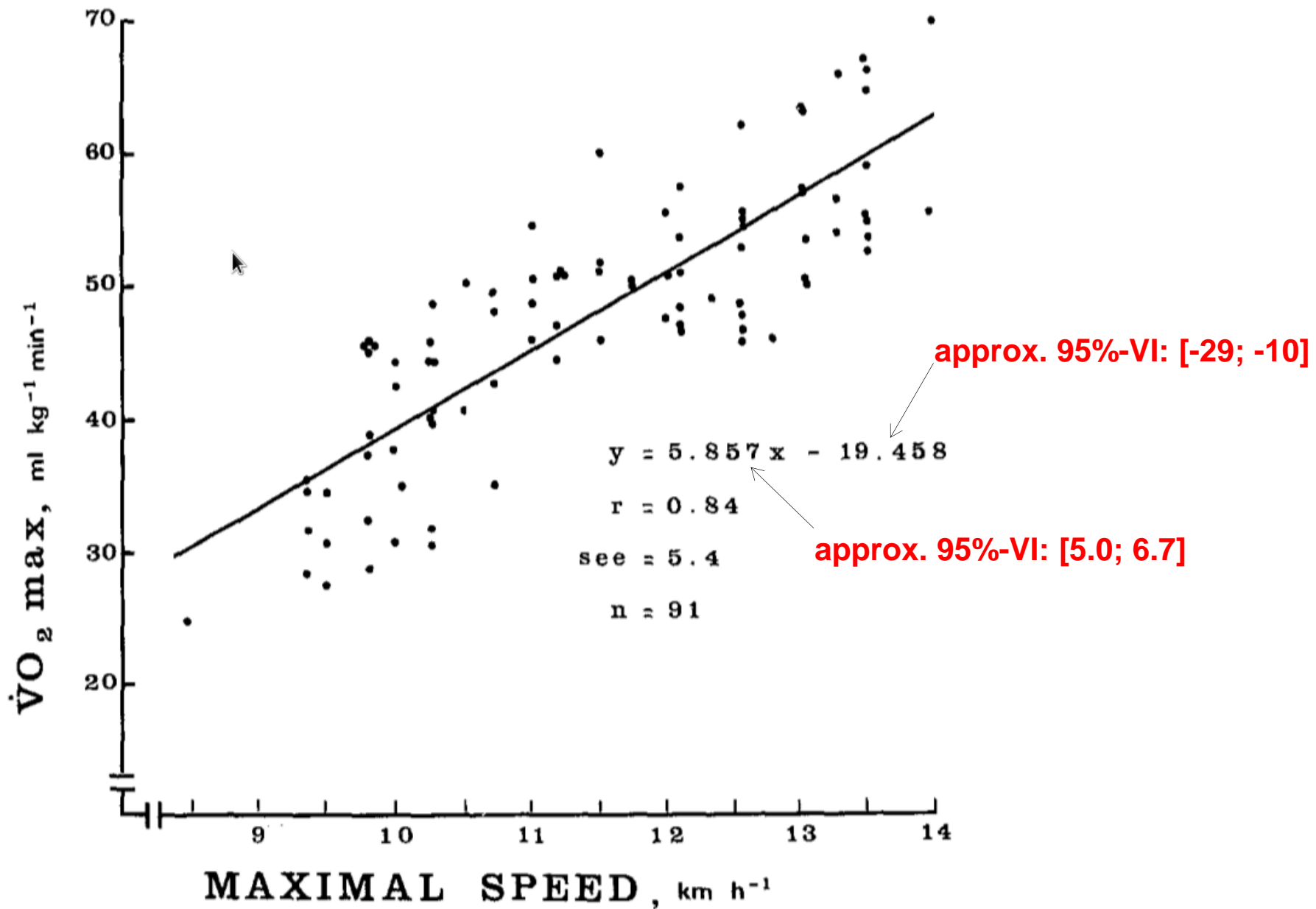


Fig. 2. $\dot{V}O_2$ max as a function of the maximal speed achieved in the 20-m shuttle run test for a total sample of 91 adult subjects. Each point in this figure represents maximal effort

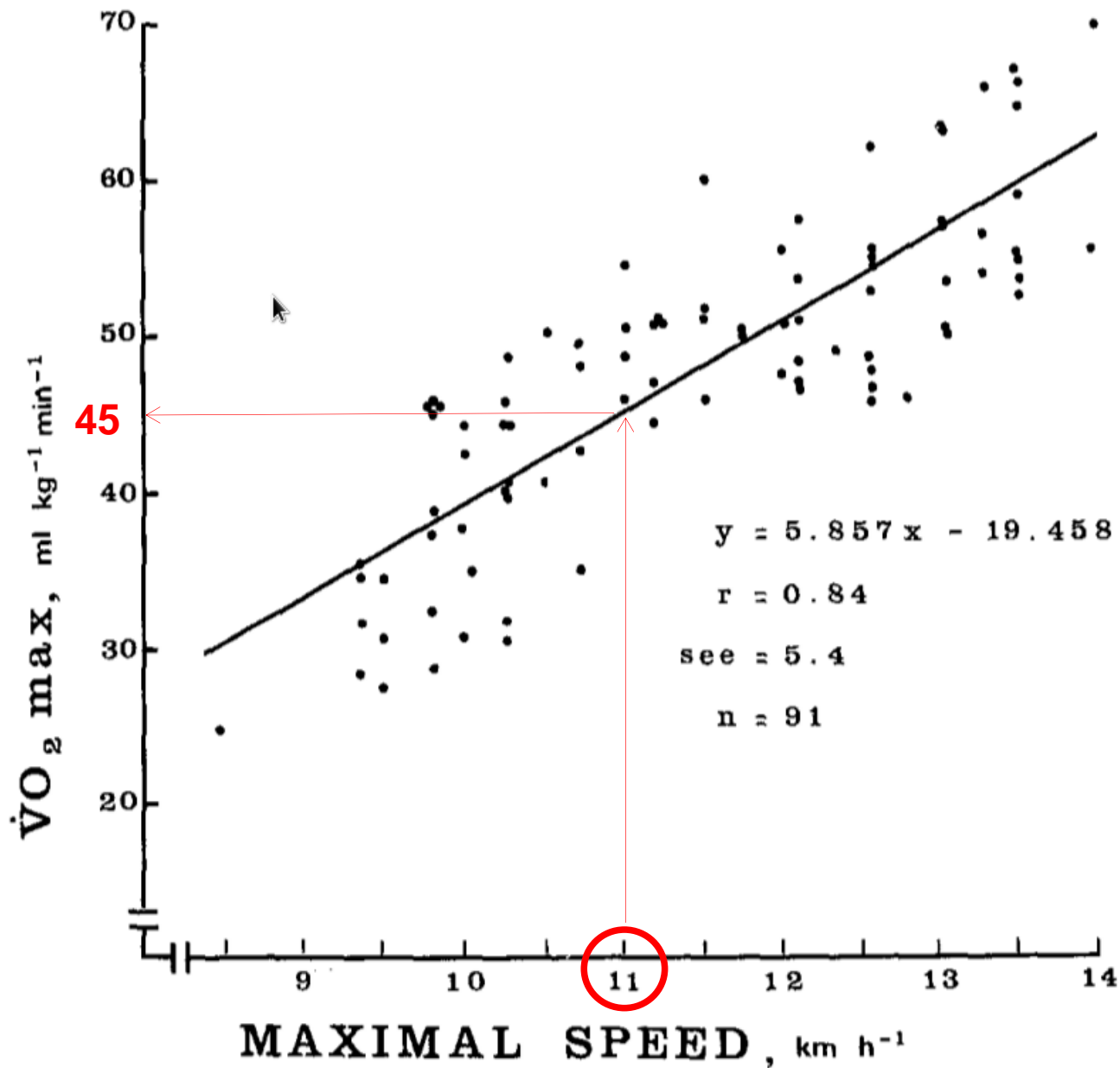


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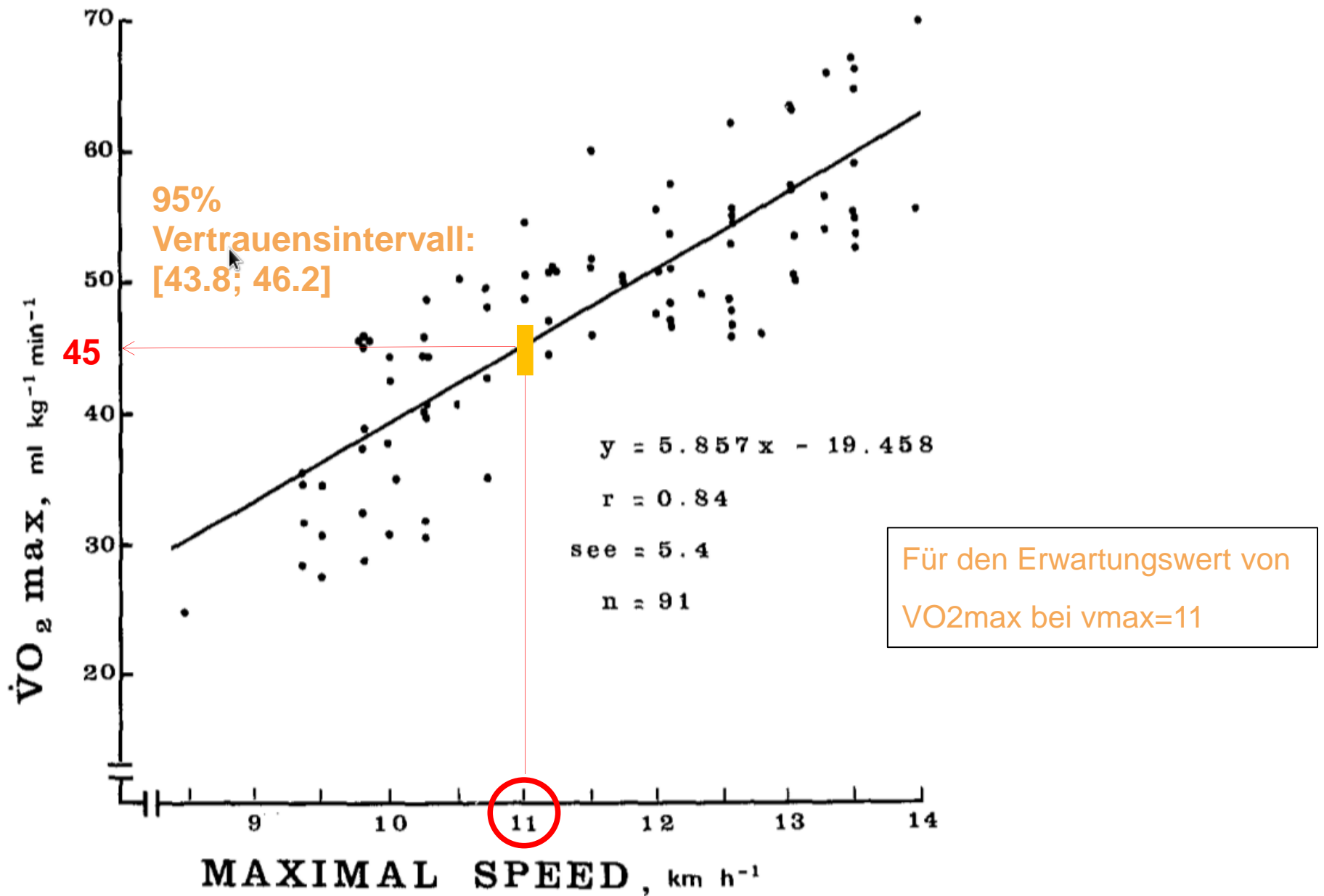


Fig. 2. $\dot{V}O_2 \text{ max}$ as a function of the maximal speed achieved in the 20-m shuttle run test for a total sample of 91 adult subjects. Each point in this figure represents maximal effort

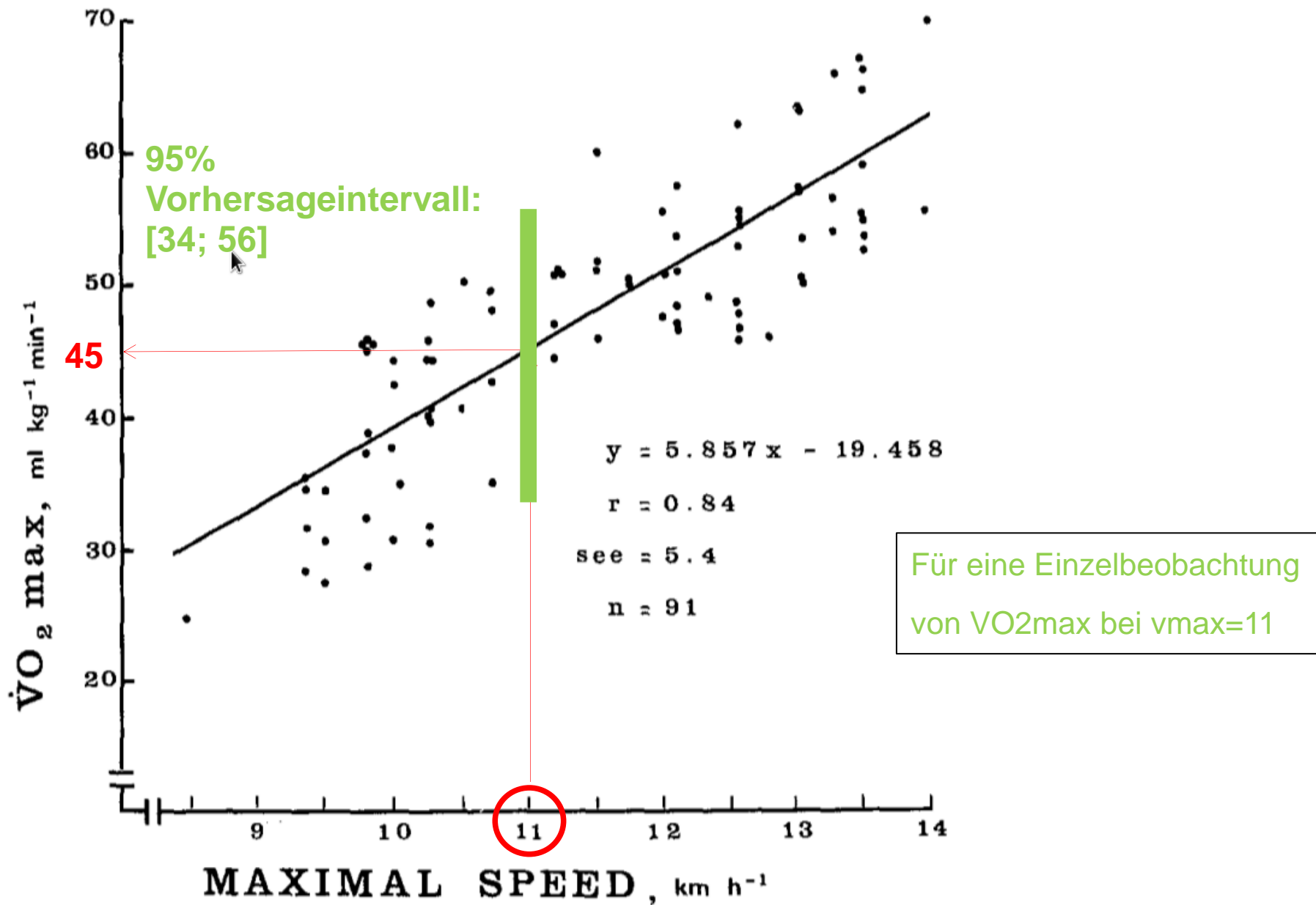
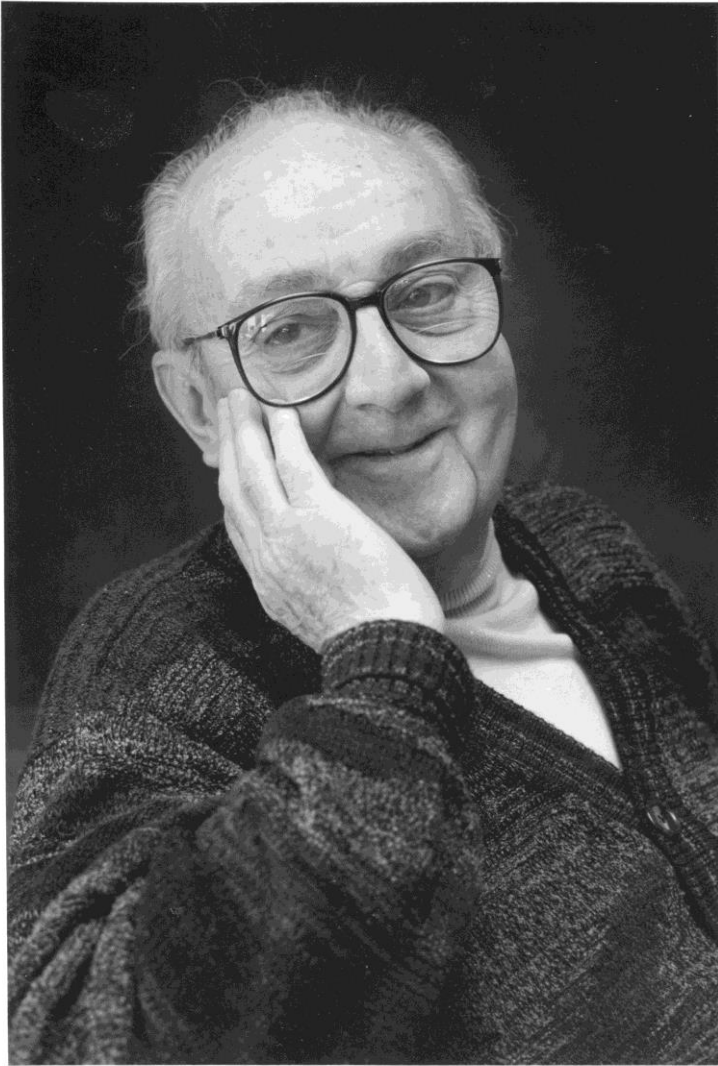


Fig. 2. $\dot{V}O_2 \text{ max}$ as a function of the maximal speed achieved in the 20-m shuttle run test for a total sample of 91 adult subjects. Each point in this figure represents maximal effort

George E.P. Box



“Essentially,
all models are
wrong,
but some are
useful.“

Residuenanalyse: Wie gut stimmt das Modell ?

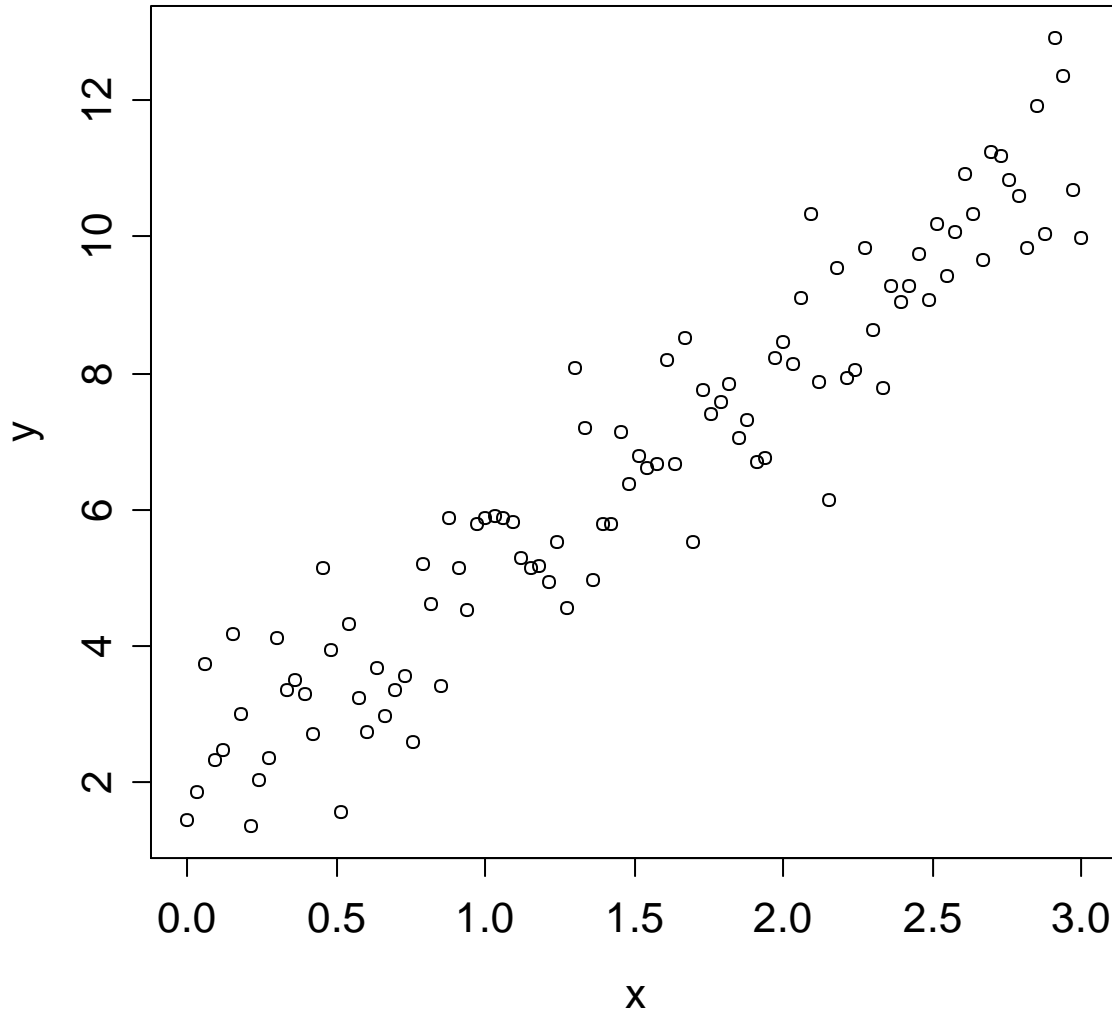
$$\underline{Y_i} = \beta_0 + \beta_1 x_i + \varepsilon_i ; \varepsilon_i \sim \underline{N(0, \sigma^2)} \quad iid$$

- Form des funktionellen Zusammenhangs
- Varianz der Fehler ist konstant
- Fehler sind normalverteilt

Einfache Regression:
Streudiagramm
Multiple Regression:
Tukey-Anscombe Plot

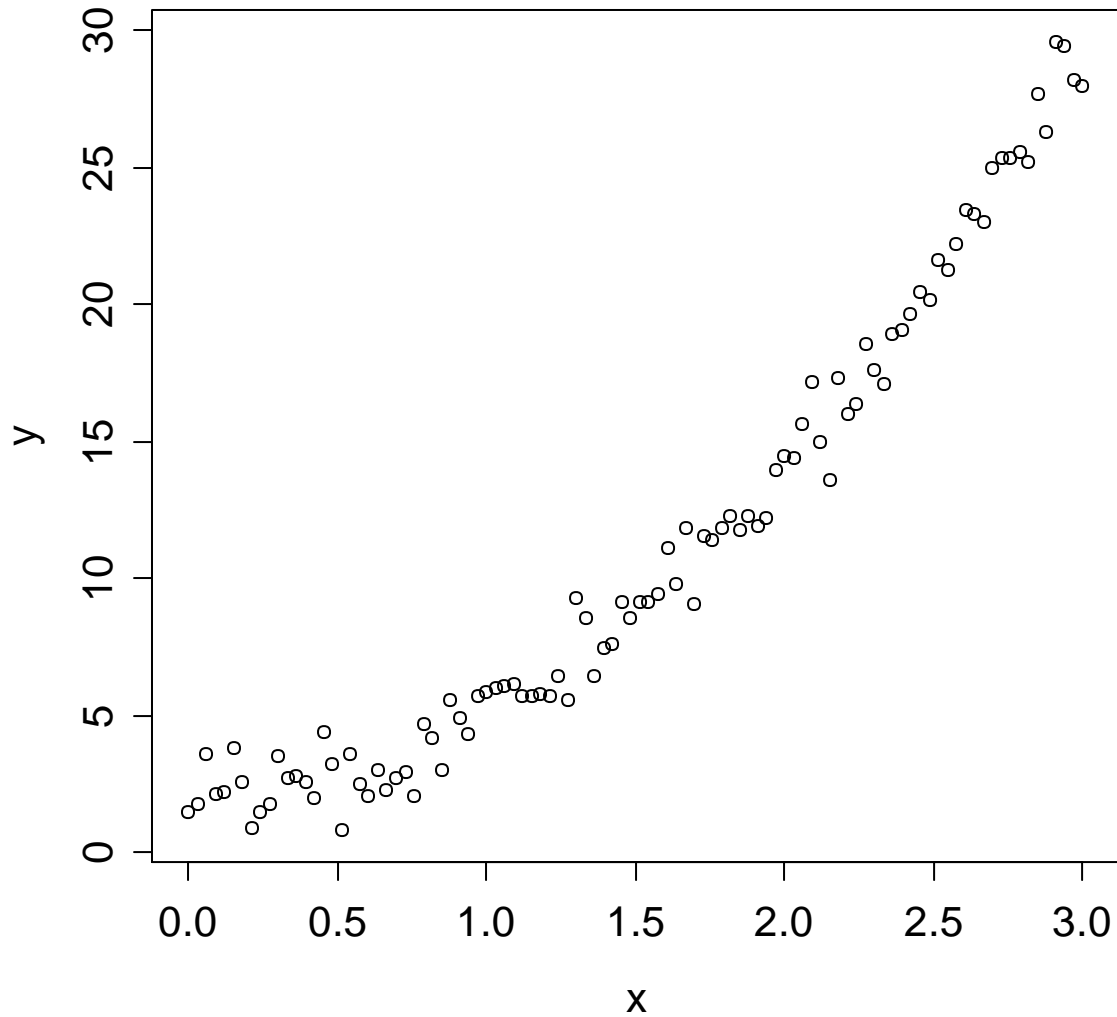
QQ-Plot der
Residuen

Streudiagramm bei einfacher linearer Regression



OK

Streudiagramm bei einfacher linearer Regression

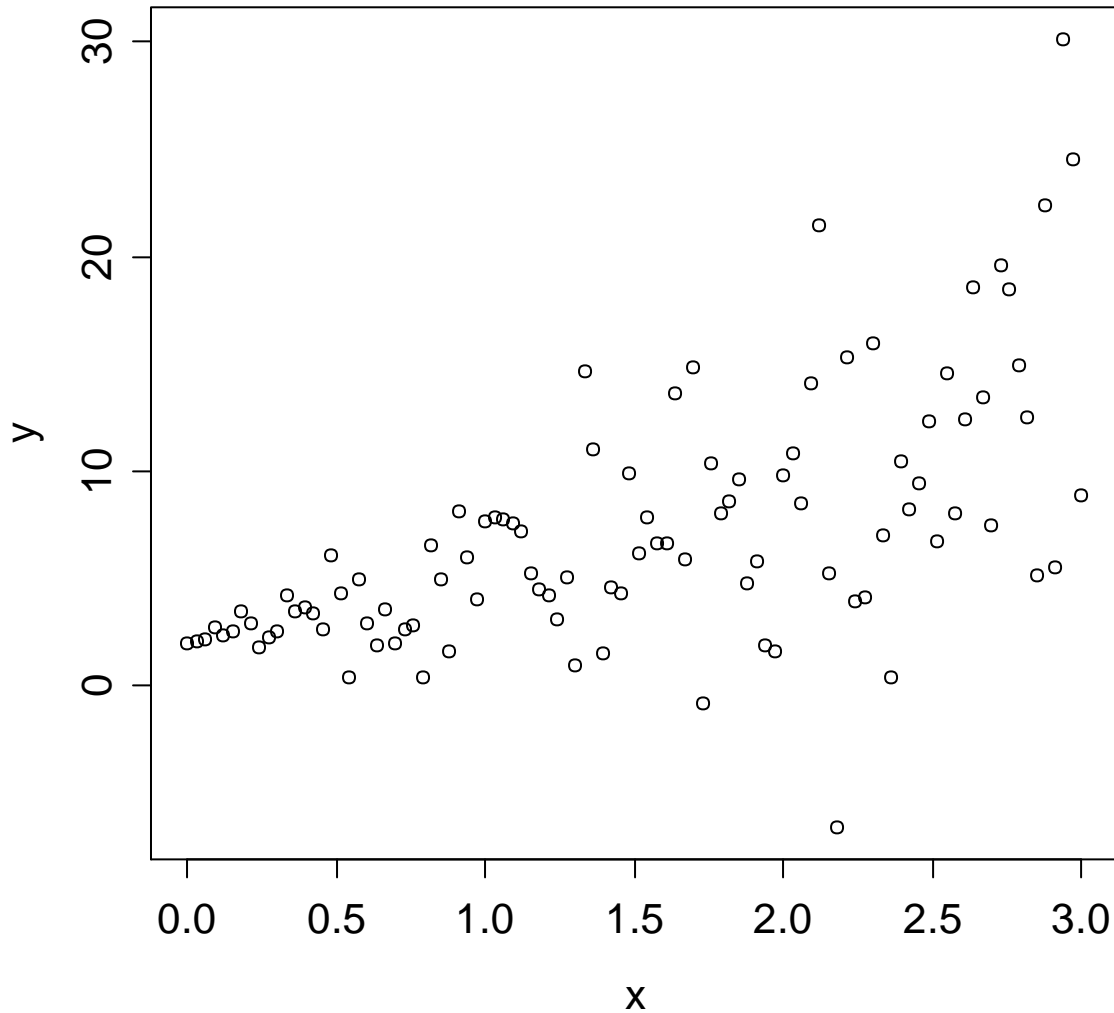


Systematischer Fehler

Krümmung:

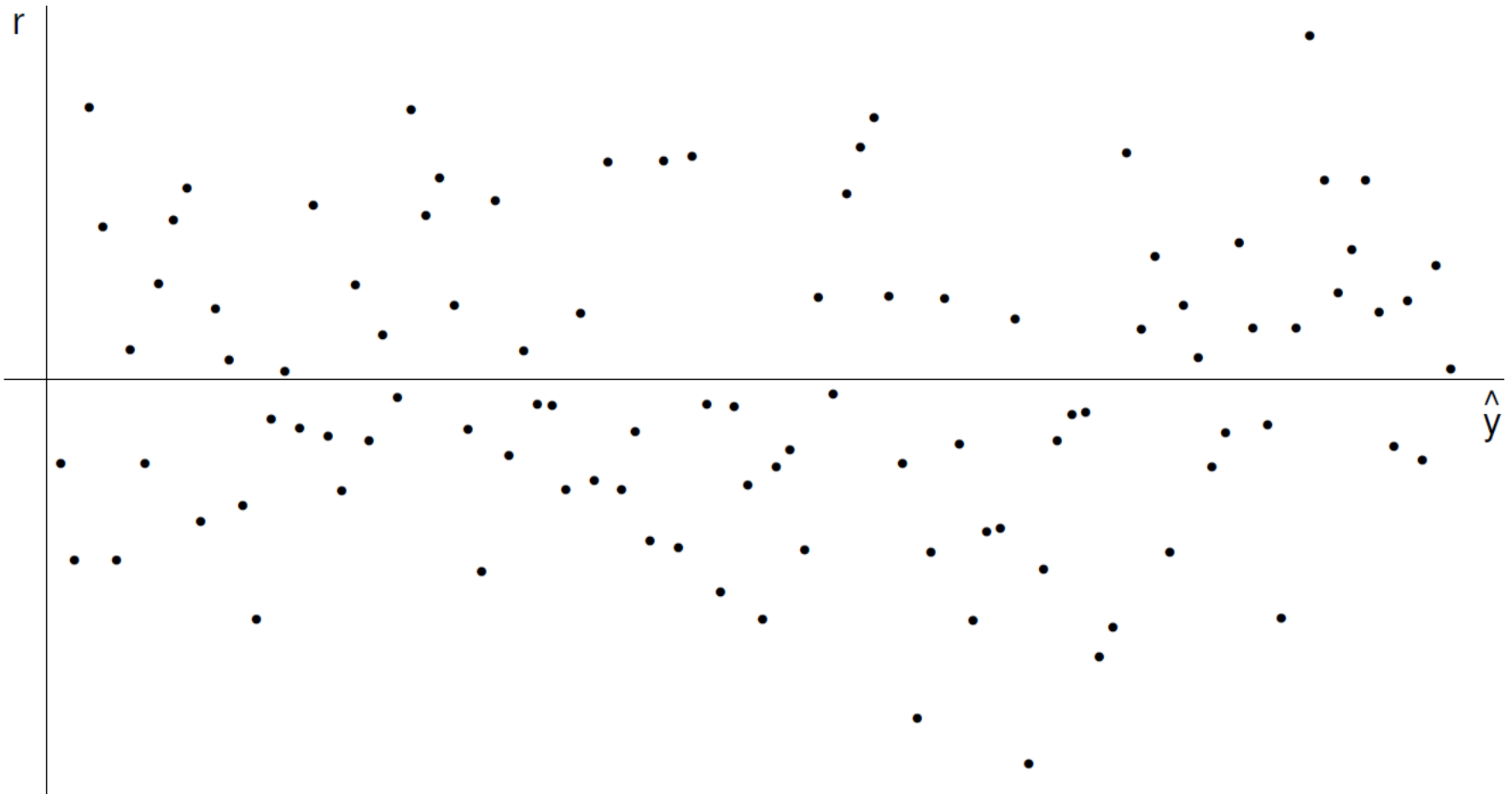
$$y = b_0 + b_1x + b_2x^2$$

Streudiagramm bei einfacher linearer Regression

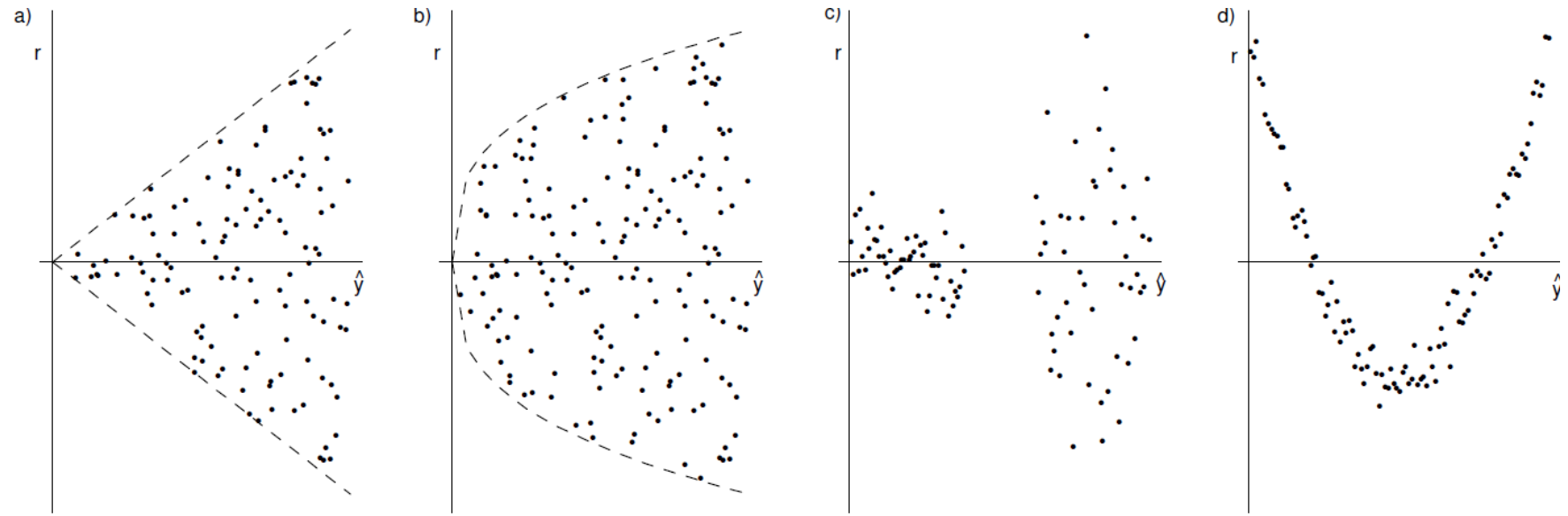


Fehlervarianz
nicht konstant

Beispiel für guten Tukey-Anscombe Plot



Beispiele für schlechte Tukey-Anscombe Plots



Fehlervarianz nicht konstant

Systematischer Fehler

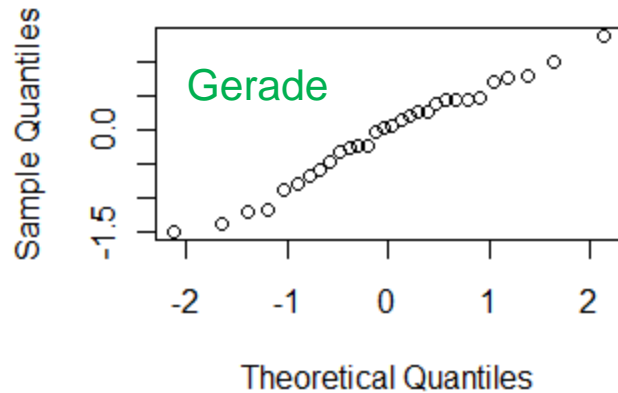
Residuenanalyse: QQ-Plot

Gerade = "gut"

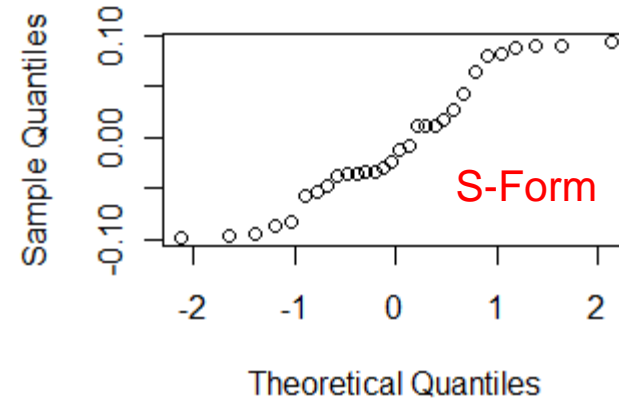
Krümmung = "schlecht"

Normal Q-Q Plot

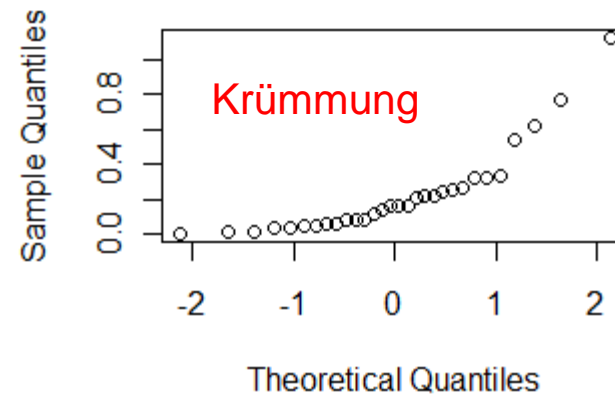
OK



Normal Q-Q Plot



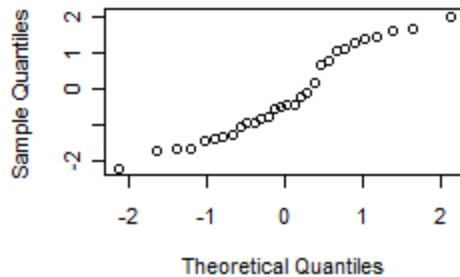
Normal Q-Q Plot



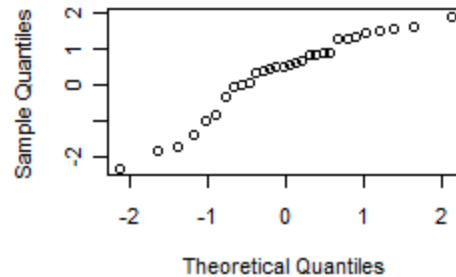
QQ-Plots: Streuung von “guten” QQ-Plots

$(n = 30, R_i \sim N(0, 1))$

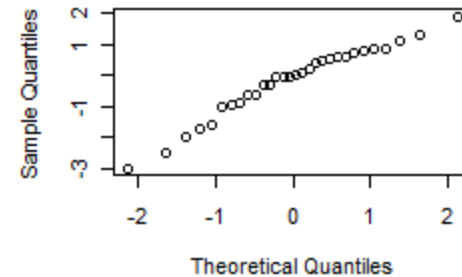
Normal Q-Q Plot



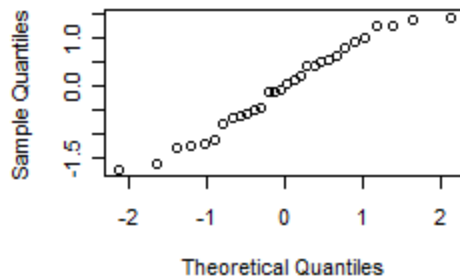
Normal Q-Q Plot



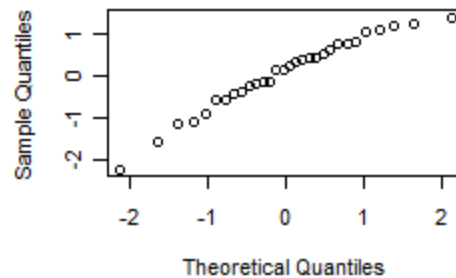
Normal Q-Q Plot



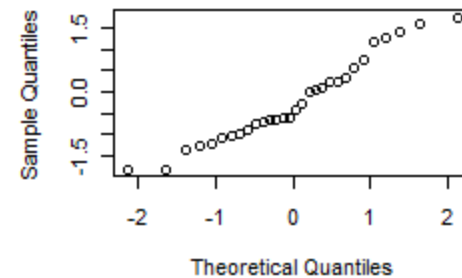
Normal Q-Q Plot



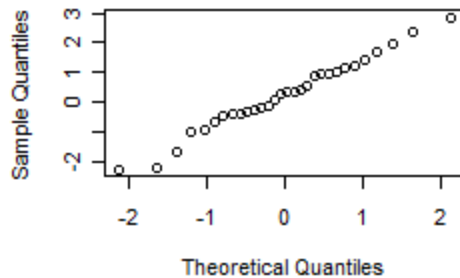
Normal Q-Q Plot



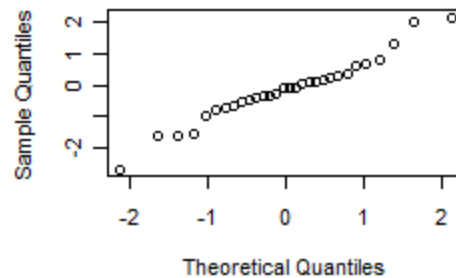
Normal Q-Q Plot



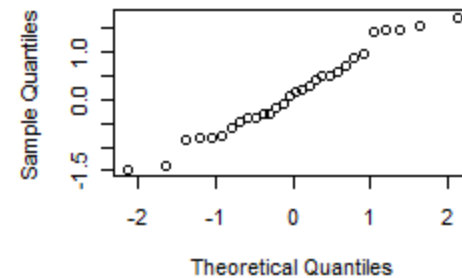
Normal Q-Q Plot



Normal Q-Q Plot



Normal Q-Q Plot



Falls Residuenplots schlecht

- Oft helfen Transformationen von x oder y
- Achtung: Vorsicht beim Interpretieren der neuen Parameter
- Bsp: $\log(y)$ statt y

Vorher: $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

Wenn x durch $x+1$ ersetzt wird, ändert sich Y im Mittel zu $Y + \beta_1$

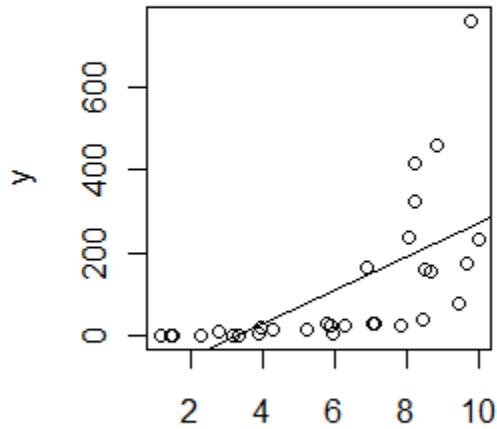
Nachher:

$$\log(Y_i) = \beta_0 + \beta_1 x_i + \varepsilon_i \leftrightarrow Y_i = \exp(\beta_0 + \beta_1 x_i + \varepsilon_i)$$

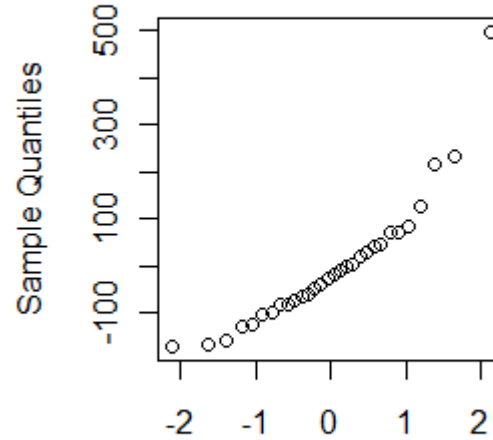
Wenn x durch $x+1$ ersetzt wird, ändert sich Y "im Mittel" zu $Y * \exp(\beta_1)$

Bsp: Ohne Log-Transformation

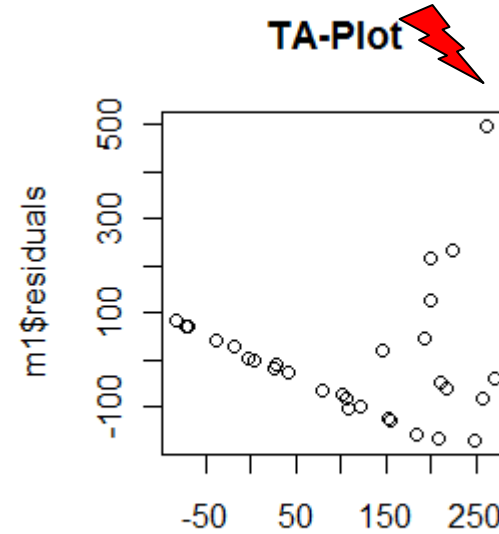
Streudiagramm



Normal Q-Q Plot

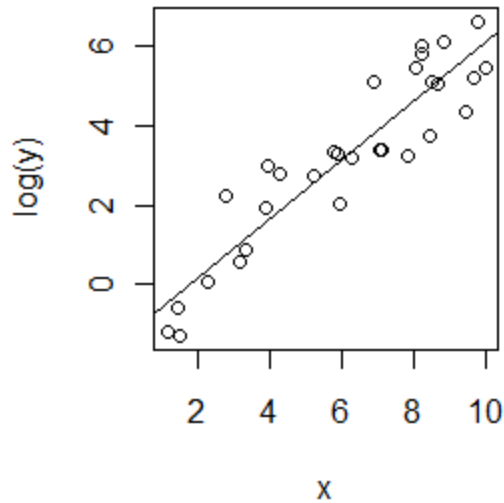


TA-Plot

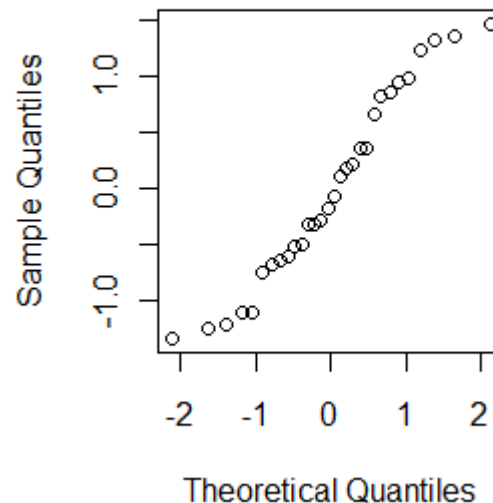


y

Streudiagramm (log(y))

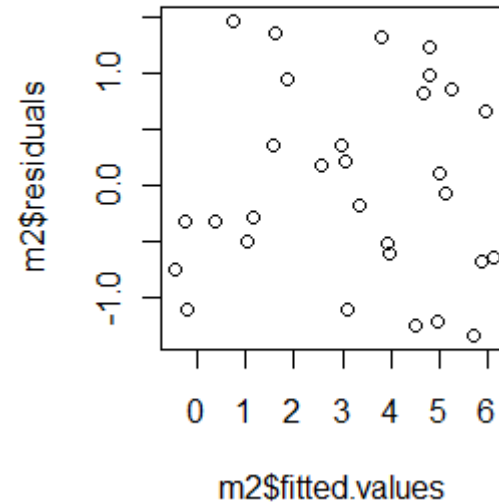


Normal Q-Q Plot



TA-Plot

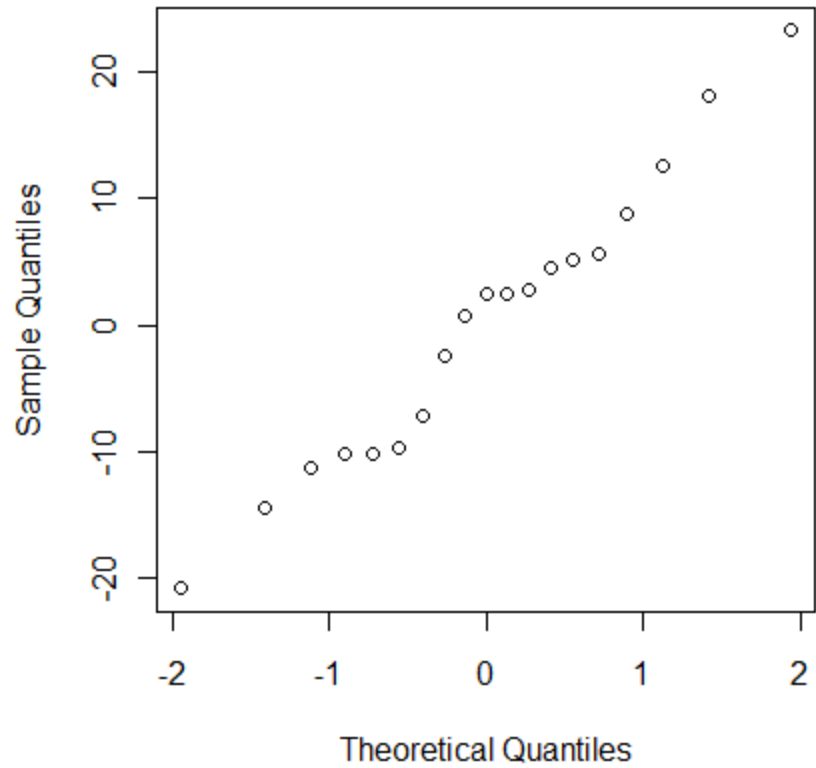
OK



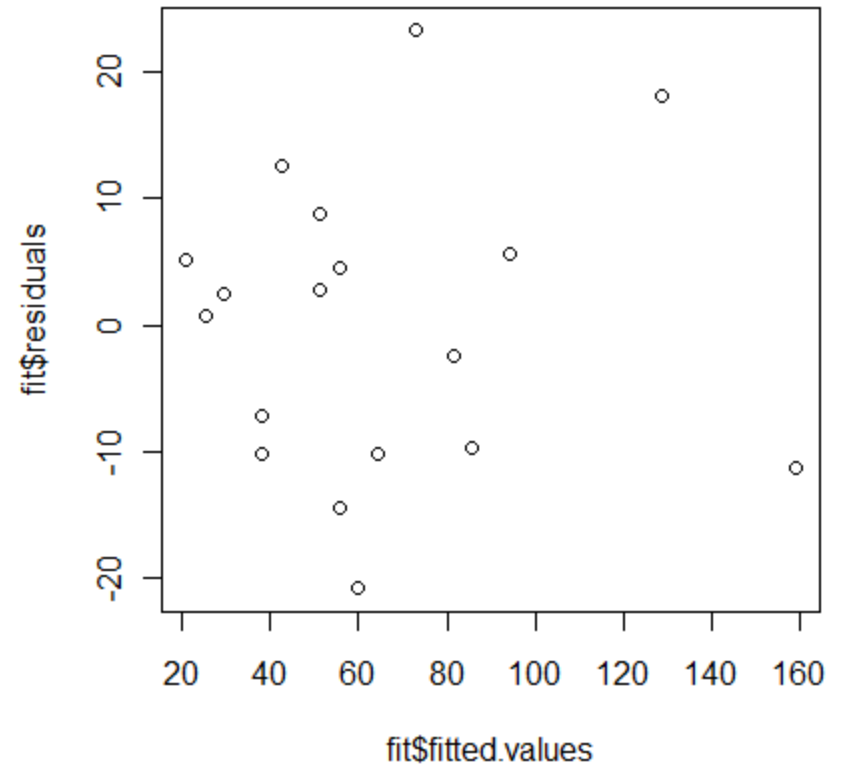
log(y)

Residuenanalyse: Supermarkt

Normal Q-Q Plot OK

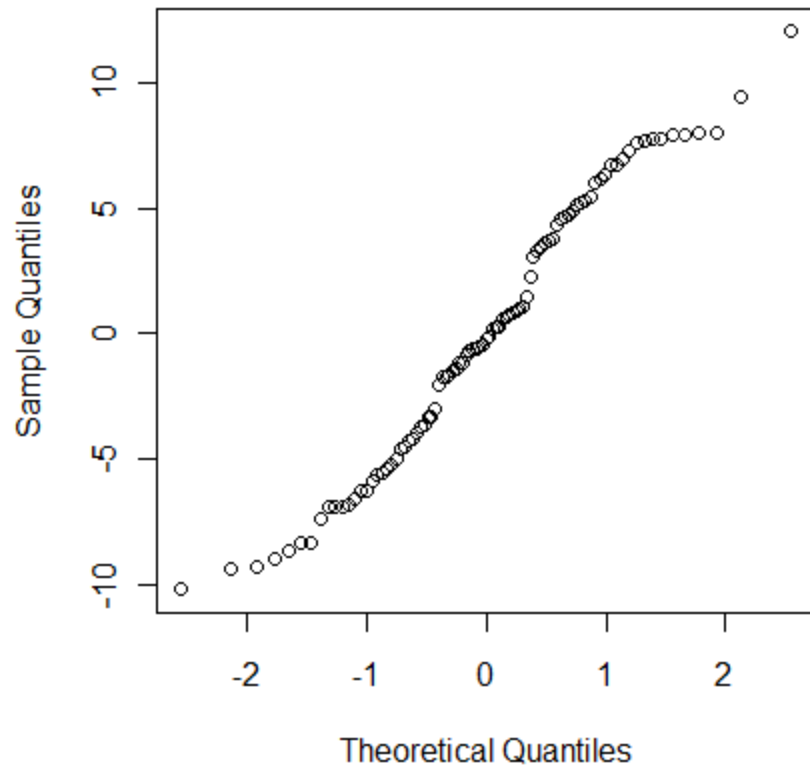


TA-Plot OK



Residuenanalyse: Beep-Test

Normal Q-Q Plot OK



TA-Plot OK

