

Series 7

1. During their yearly spring melt, glaciers deposit layers of sand and mud. These annual sediments, known as *varves*, can be reconstructed in New England for the whole time between the beginning (about 12'600 years ago) till the end (6'000 years ago) of glacial retreat. From these varves, approximations of paleoclimatic parameters can in turn be computed, such as temperature (a warmer year yields more sediment).

In the dataset `varve.dat`, you will find 350 annual sediment diameters (contained in lines 201 through 550) starting at 11'660 years ago. After loading these data, first construct a time series object from them:

```
> t.url <- "http://stat.ethz.ch/Teaching/Datasets/WBL/varve.dat"
> d.varve <- ts(scan(t.url))[201:550]
```

Comment: The procedure `scan()` is a more general data loading function than `read.table()`. We use it here to avoid putting the data into a data frame. Do not worry about the exact choice of procedure for reading data here: simply believe us when we say that `scan()` does what we need, or read the help file.

- a) Draw a time series plot. If need be, transform the original data. Is this a stationary time series? Give reasons for your conclusion.

R Hint: `diff()`

- b) Choose a suitable model that fits the data. Which model have you chosen? Check the convergence of the algorithm and write down the model with its estimated coefficients.

R Hints:

```
> r.varve <- arima(log(d.varve), order=c(..., ..., ...))
> str(r.varve)
> plot(resid(r.varve), ...)
> acf(...)
> pacf(...)
```

- c) Does your model fit? Analyze the residuals and comment on the results.

R Hints:

```
> t.resid <- resid(...)
> qqnorm(...)
> qqline(...)
```

Source: R. Shumway, D. Stoffer, Time Series Analysis and its Applications, 2000.

2. The dataset `GARCH.dat` contains a simulated timeseries of length 500. Load the data. You will also need the R-package `tseries`.

```
t.x <- read.table("http://stat.ethz.ch/Teaching/Datasets/WBL/GARCH.dat")
t.x <- ts(t.x[,1])
```

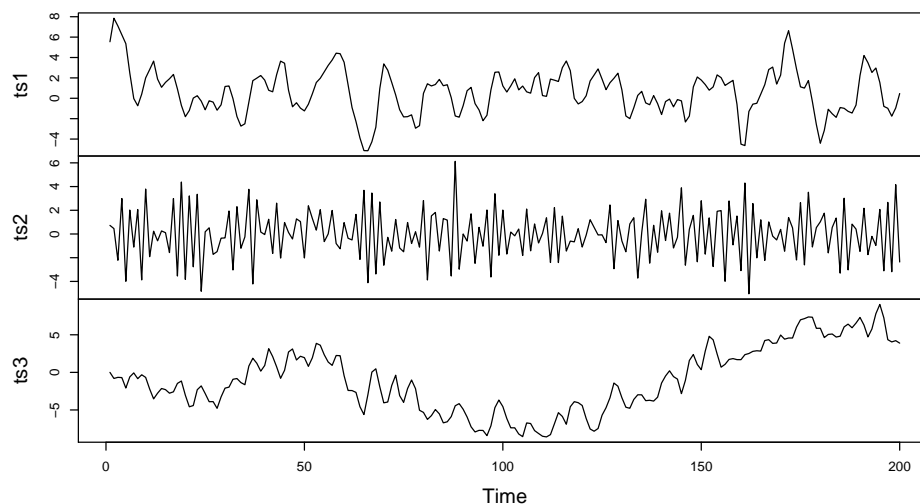
- a) Look at the timeseries plot. Is it stationary?
- b) Fit a GARCH(p,q)-model. Find the order (p,q) with minimal AIC. Choose p and q from 0 to 2. Both cannot be zero at the same time.

R Hint: `garch(...)`

- c) Estimate the coefficients for your model from b).

3. For each of the following three time series, find a suitable ARIMA(p,d,q) model and estimate its parameters. To read the data, call:

```
> t.url <- "http://stat.ethz.ch/Teaching/Datasets/WBL/ARIMAsim2.dat"
> t.d <- ts(read.table(t.url, header=T))
```



4. Since simulations can be of use in model validation, we would like to use this exercise to simulate several time series by means of an ARMA model.

- AR(2) model with coefficients $\alpha_1 = 0.9$ and $\alpha_2 = -0.5$.
- MA(3) model with coefficients $\beta_1 = 0.8$, $\beta_2 = -0.5$ and $\beta_3 = -0.4$.
- ARMA(1,2) model with coefficients $\alpha_1 = -0.75$, $\beta_1 = -1$ and $\beta_2 = 0.25$.

For all models, the error E_t follows the standard normal distribution $N(0,1)$.

- First think how the autocorrelations should behave based on the theory.
- Use the procedure `ARMAacf()` to compute the *theoretical* autocorrelations.

R Hints:

```
## Theoretical autocorrelations
> plot(0:30, ARMAacf(ar=c(0.9,-0.5), lag.max=30), type="h",
      ylab="ACF")
## Theoretical partial autocorrelations
> plot(1:30, ARMAacf(ar=c(0.9,-0.5), lag.max=30, pacf=T), type="h",
      ylab="PACF")

> plot(0:30, ARMAacf(ma=c(..., ..., ...), ...), ...)
> ...
```

- Now simulate all three models a) - c). Take several different lengths for the time series: $n = 200, 500$ and 1000 . Repeat these simulations several times to develop some intuition on what is “chance” and what is “structure”.

R Hints:

You can use the procedure `arma.sim()` to simulate the time series. The length of the simulated series you can choose by setting the argument `n`, and the model by setting the parameter `model` (to a list!).

```
> r.sim1 <- arma.sim(n=..., model=list(ar=c(0.9,-0.5)))
```

- Regard the time series plot and the correlograms with the ordinary and partial autocorrelations.

Exercise hour: Monday, April 07.