Applied Time Series Analysis
SS 2014 – Week 10

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Forecasting Decomposed Series

The principle for forecasting time series that are decomposed into trend, seasonal effect and remainder is:

1) **Stationary Remainder**
   Is usually modelled with an ARMA(p,q), so we can generate a time series forecast with the methodology from before.

2) **Seasonal Effect**
   Is assumed as remaining "as is", or "as it was last" (in the case of evolving seasonal effect) and extrapolated.

3) **Trend**
   Is either extrapolated linearly, or sometimes even manually.

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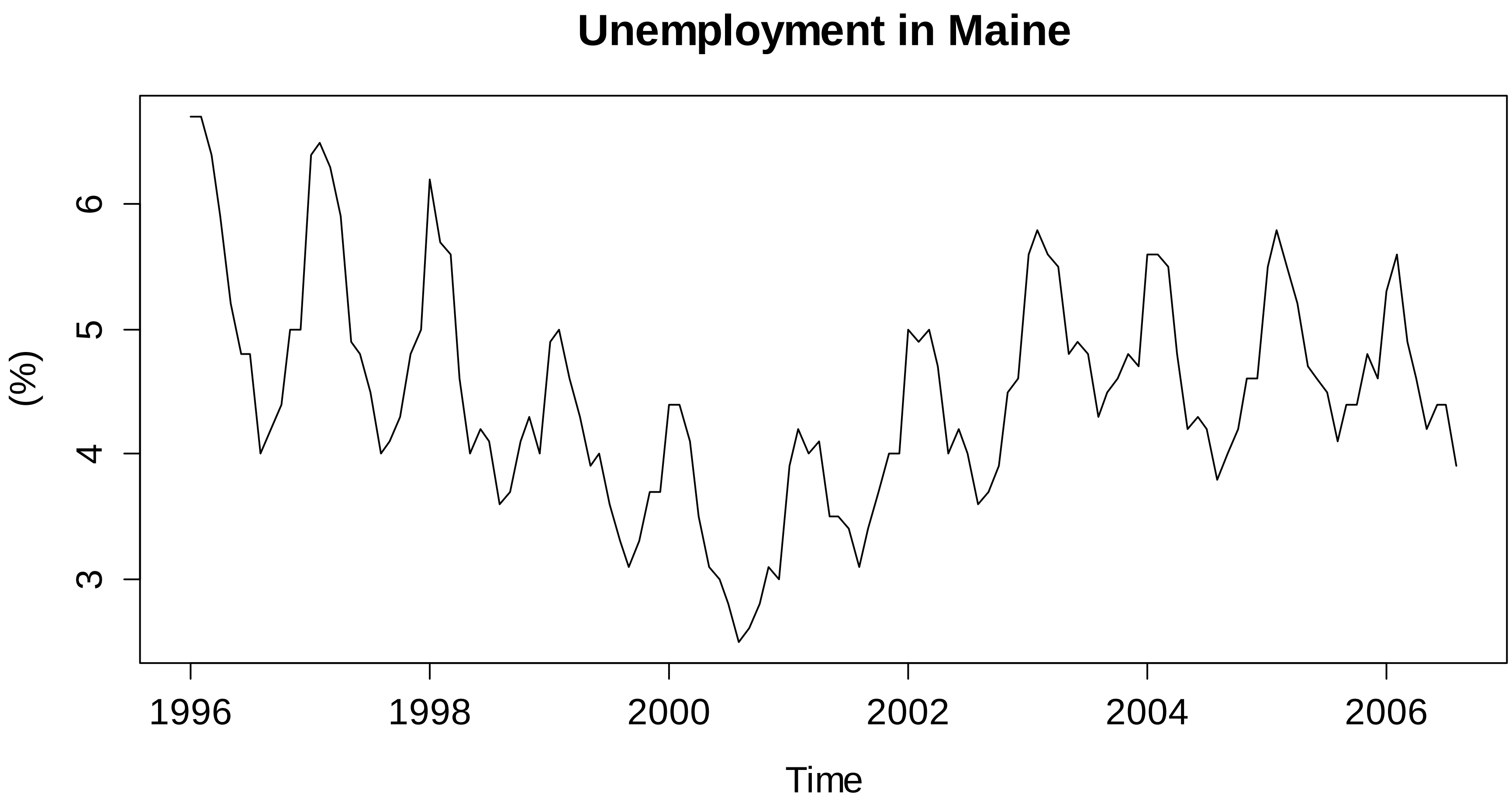
Forecasting Decomposed Series: Example



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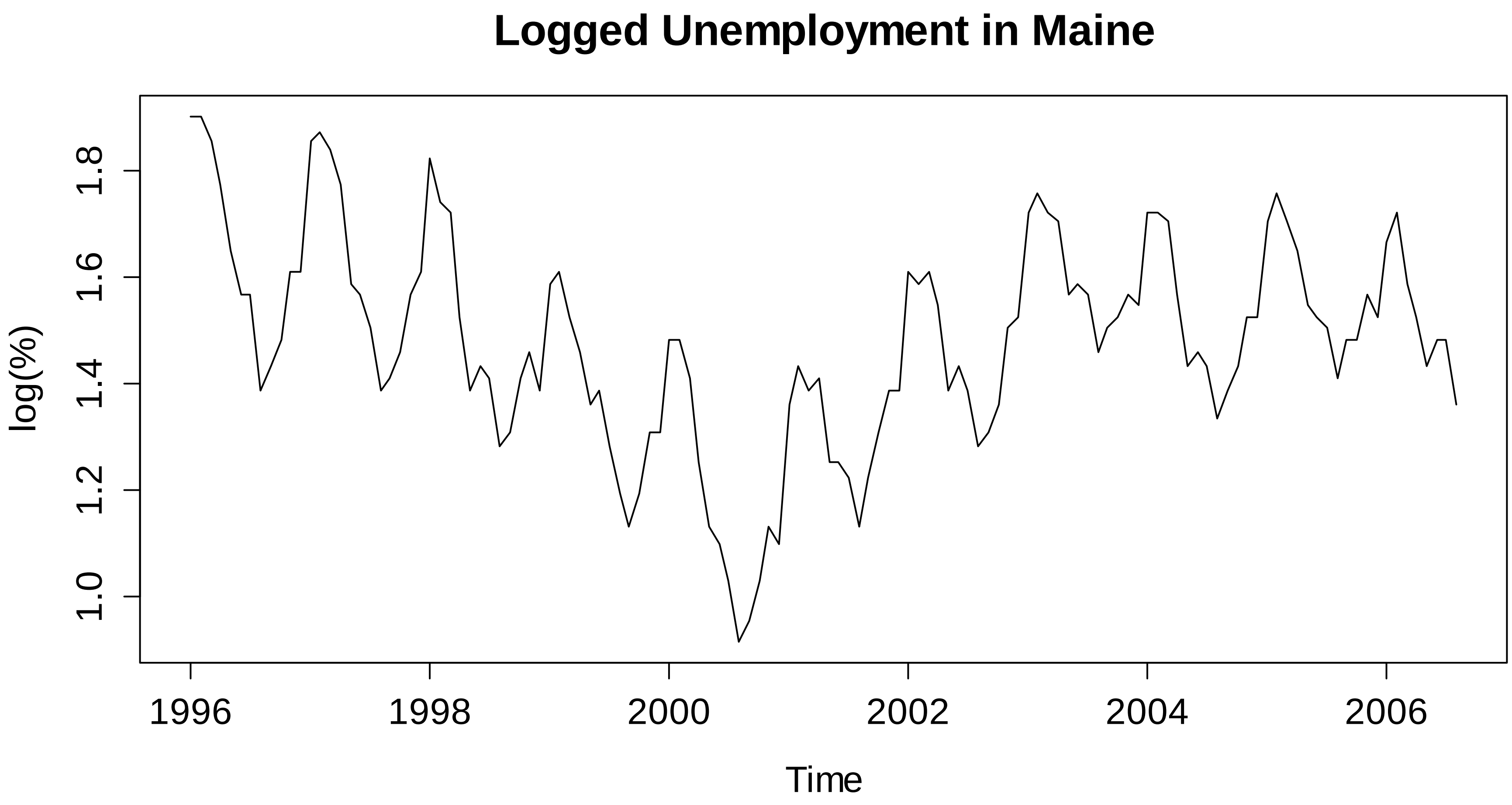
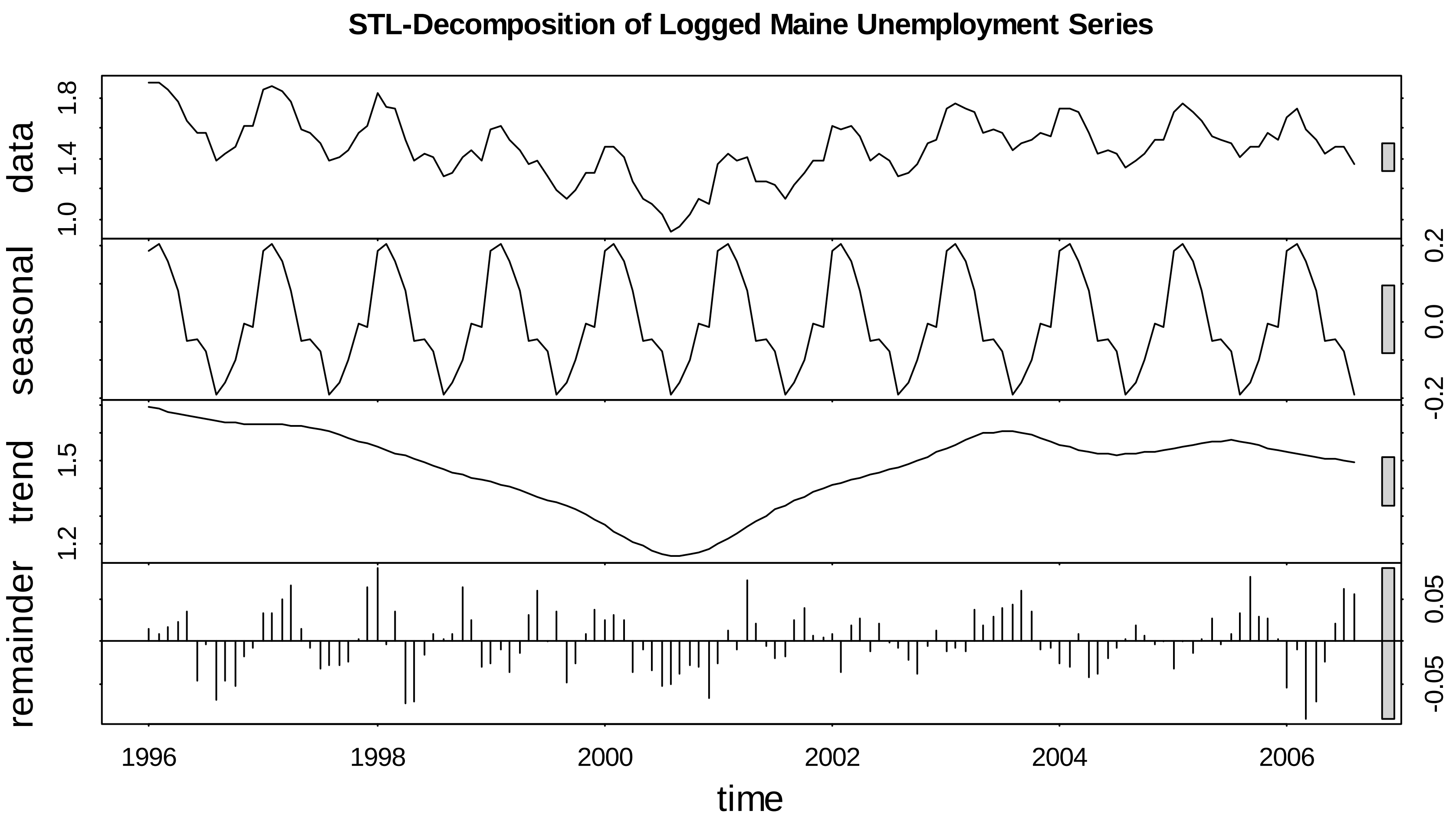
Forecasting Decomposed Series: Example



Forecasting Decomposed Series: Example



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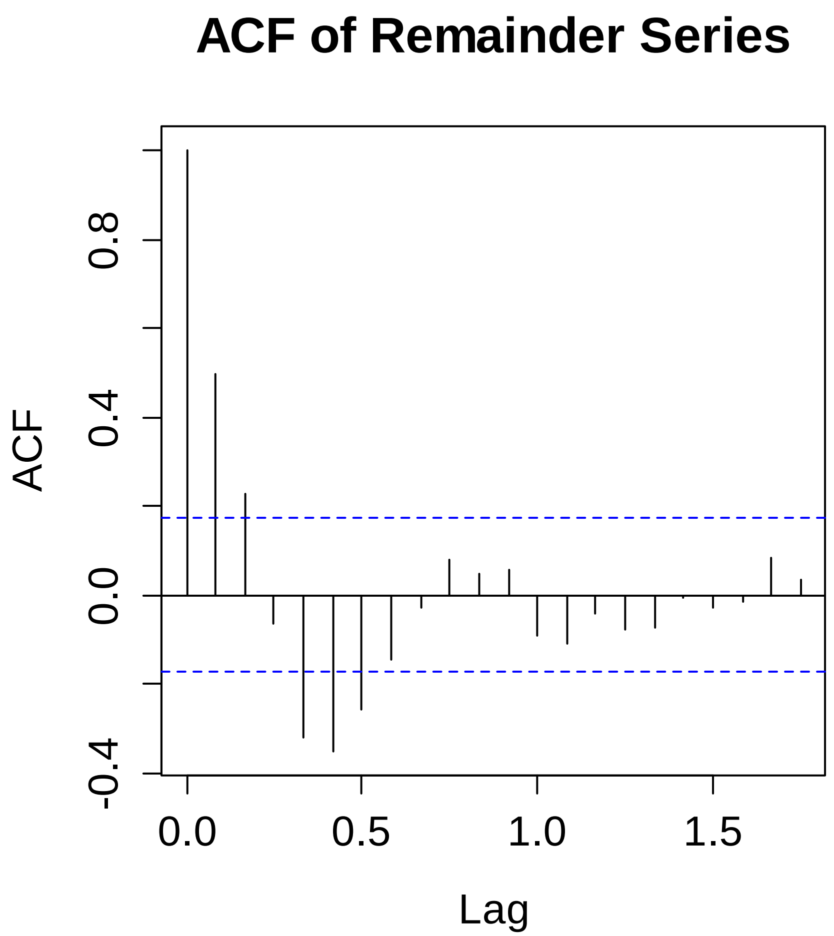
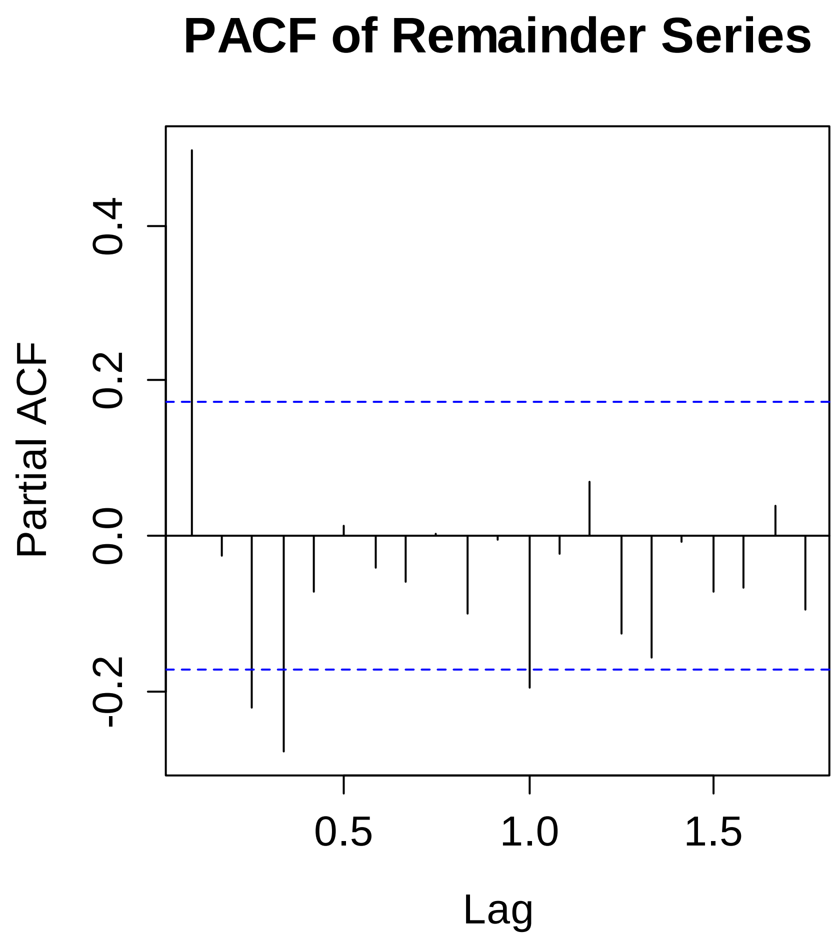
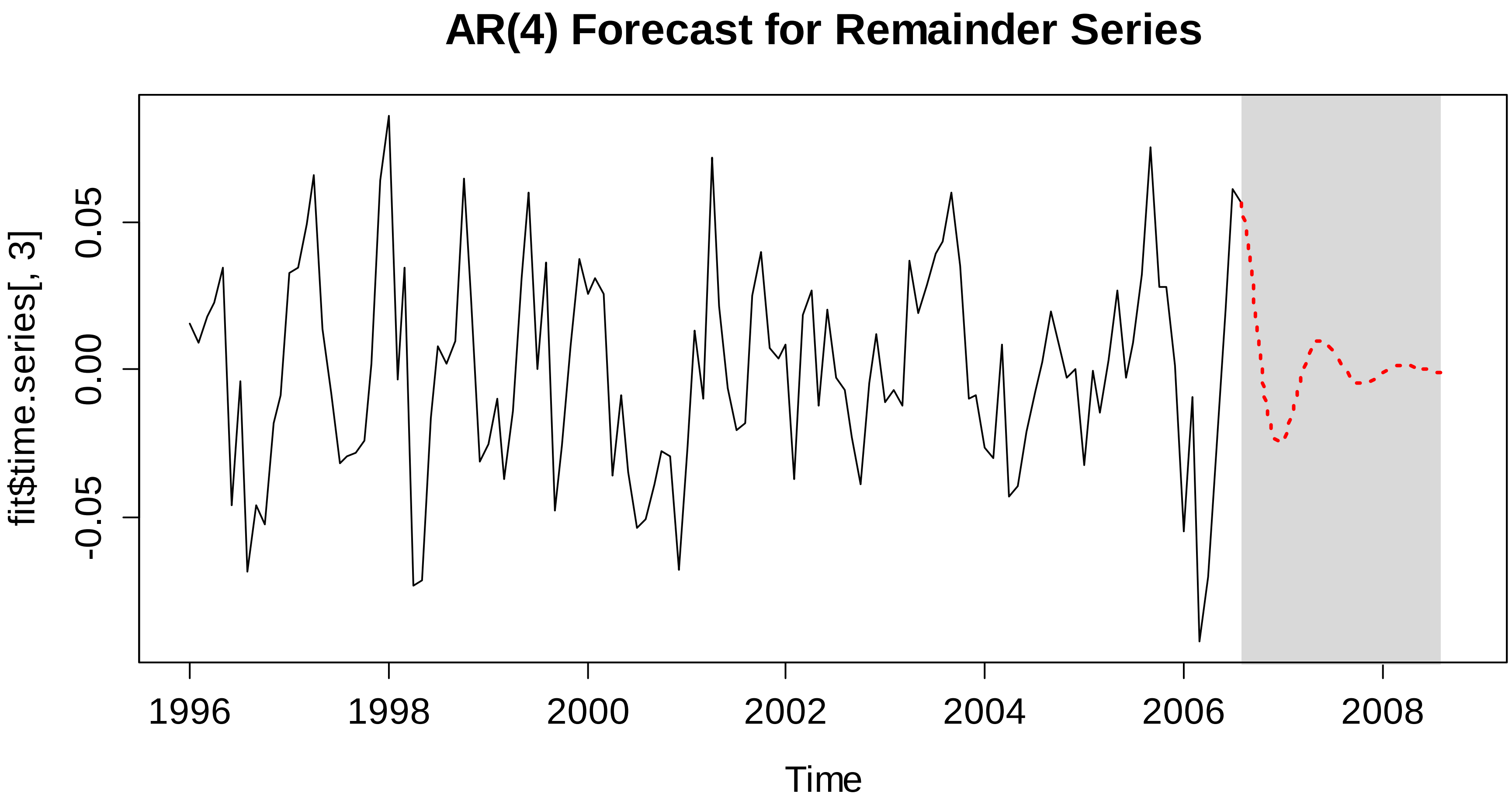
Forecasting Decomposed Series: Example





Forecasting Decomposed Series: Example



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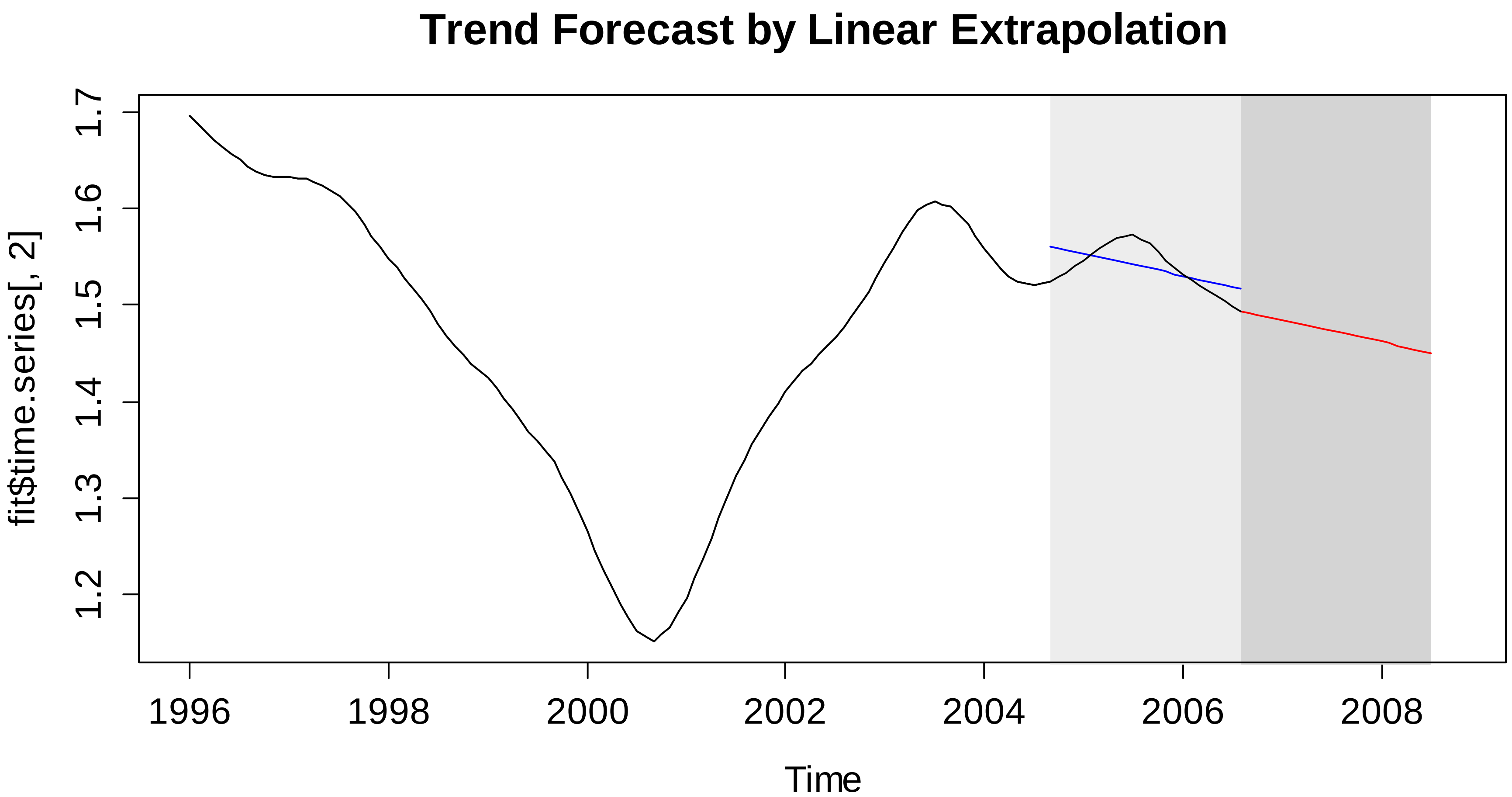
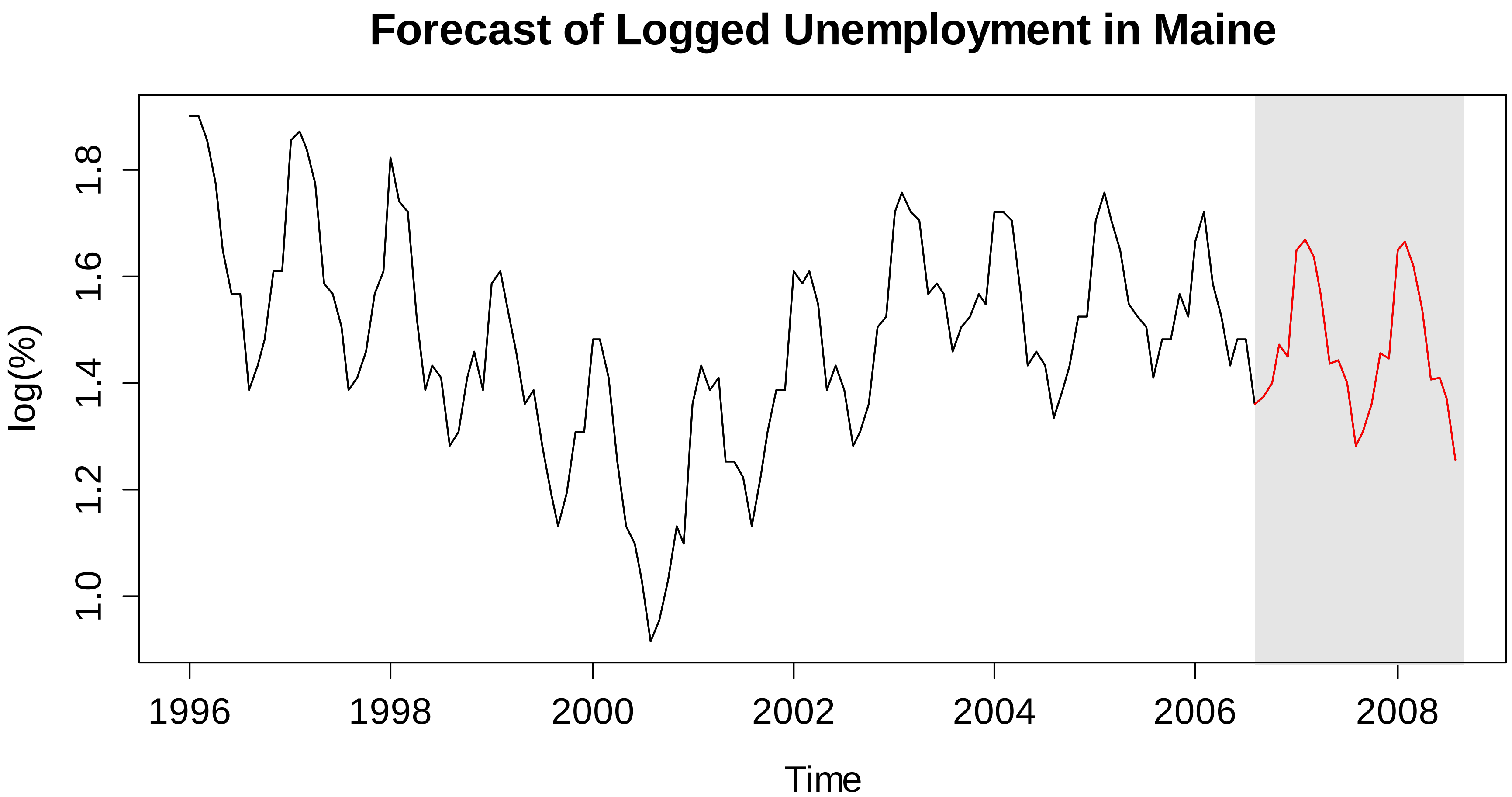
Forecasting Decomposed Series: Example



Forecasting Decomposed Series: Example



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Simple Exponential Smoothing

This is a quick approach for estimating the current level of a time series, as well as for forecasting future values. It works for any stationary time series without a trend and season.

The **simple, intuitive idea behind** is:

$$\hat{X}_{n+1,1:n} = \sum_{i=0}^{n-1} w_i x_{n-i} \quad \text{where } w_0 \geq w_1 \geq w_2 \geq \ldots \geq 0 \text{ and } \sum_{i=0}^{n-1} w_i = 1$$

The weights are often chosen to be exponentially decaying, two examples with different parameters are on the next slide. However, there is also a deeper mathematical notion of ExpSmo.

→ **See the blackboard for the derivation...**

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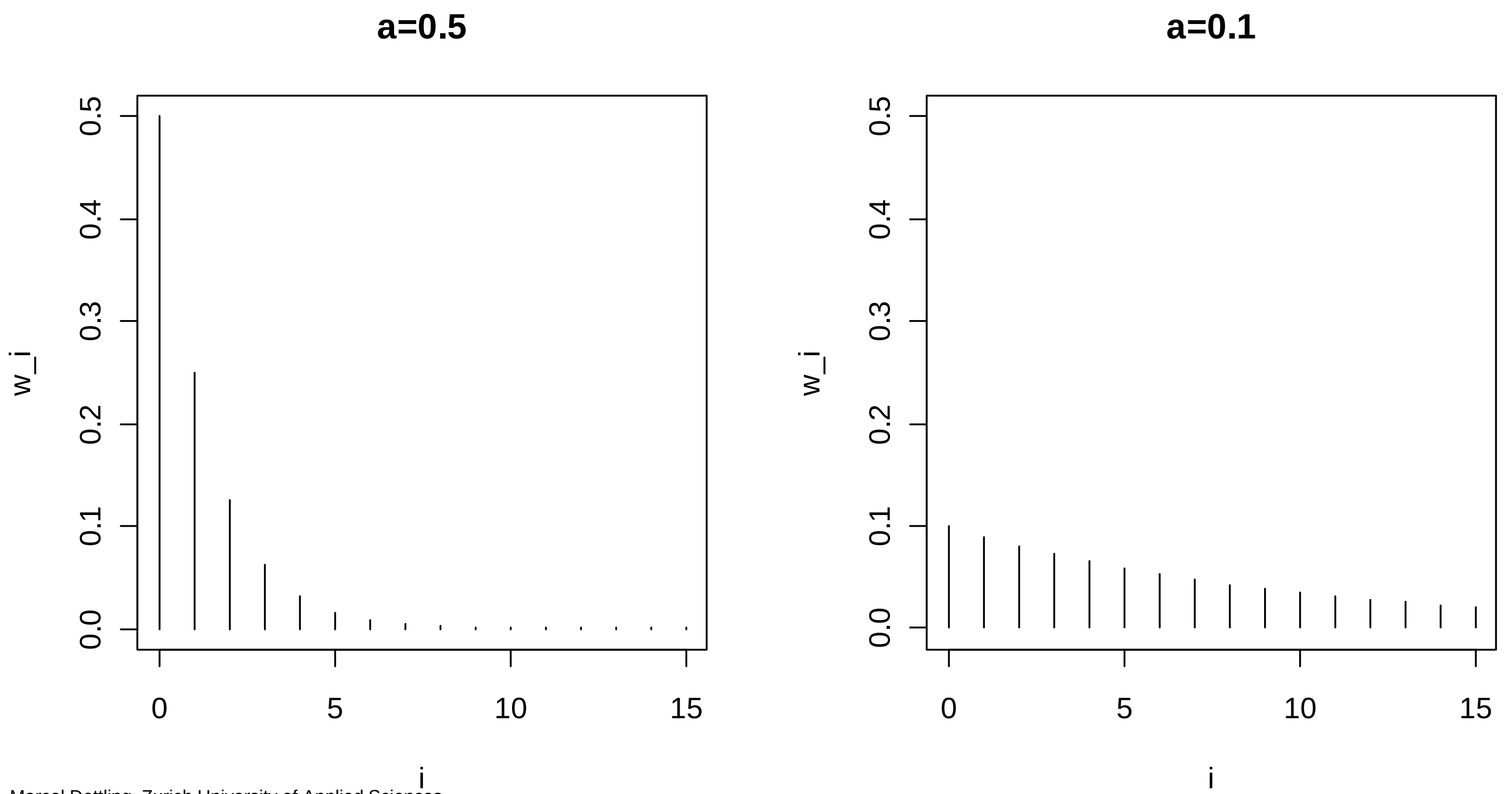
Choice of Weights

An usual choice are exponentially decaying weights:

$$w_i = \alpha(1-\alpha)^i \quad \text{where } \alpha \in (0,1)$$



Simple Exponential Smoothing: Summary

What is it?

- A method for estimating and forecasting the conditional mean

Basic notion: $X_t = \mu_t + E_t$

- μ_t is the conditional expectation, which we try to estimate from the data. The estimate a_t is called level of the series.
- E_t is a completely random innovation term.

Estimation of the level: two notions exist…

- Weighted updating: $a_t = \alpha x_t + (1-\alpha) a_{t-1}$
- Exponential smoothing: $a_t = \sum_{i=0}^{\infty} \alpha(1-\alpha)^i x_{t-i}$

Forecasting with Exponential Smoothing

The forecast, for any horizon $k > 0$ is:

$$\hat{X}_{n+k,1:n} = a_n$$

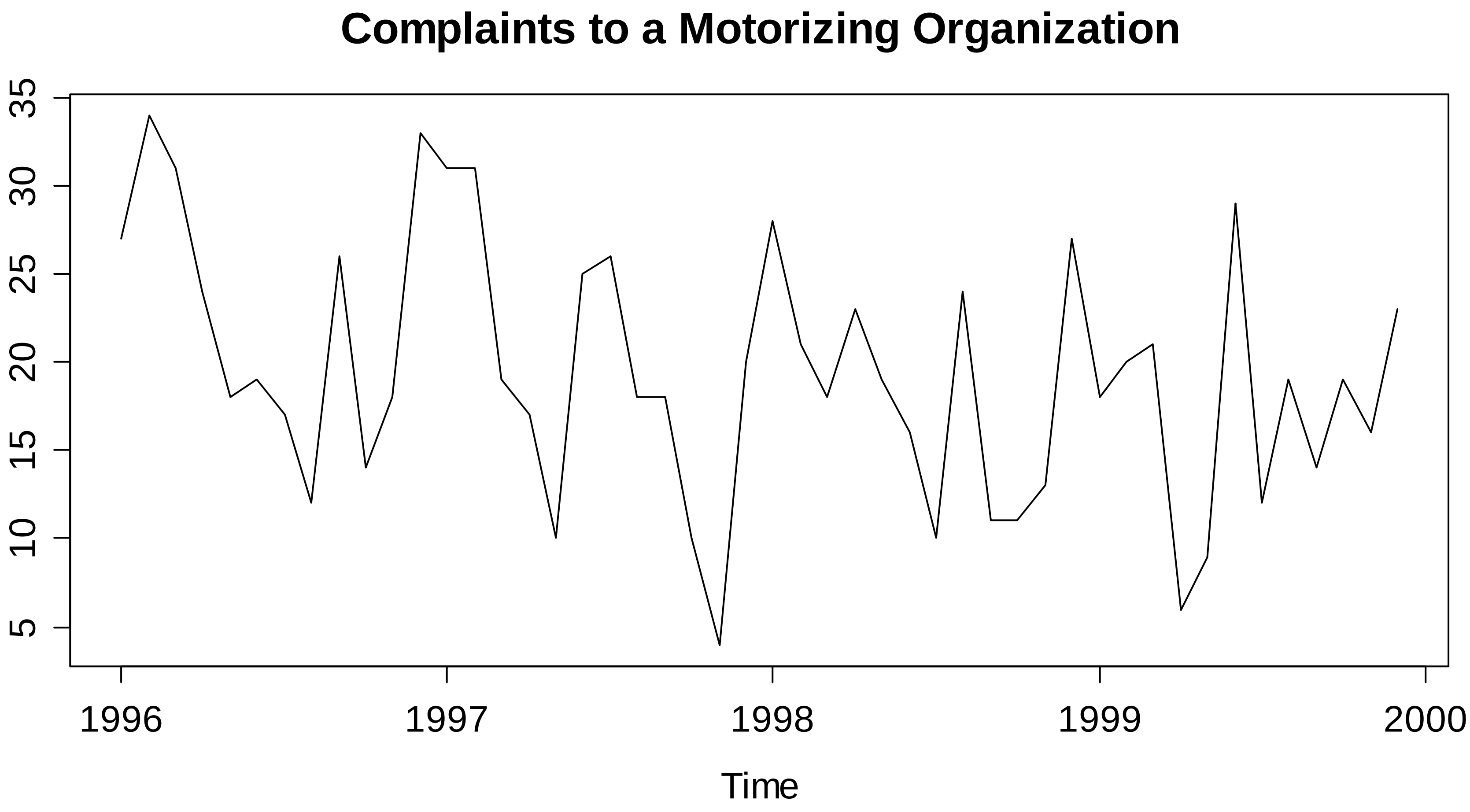
Hence, the forecast is given by the current level, and it is constant for all horizons k . However, it does depend on the choice of the smoothing parameter α . In R, a data-adaptive solution is available by minimizing SS1PE:

1-step-prediction-error: $e_t = x_t - \hat{X}_{t;1:(t-1)} = x_t - a_{t-1}$

$$\hat{\alpha} = \arg\min_{\alpha} \sum_{i=2}^{n} e_t^2$$

The solution needs to be found with numerical optimization.

Exponential Smoothing: Example



Exponential Smoothing: Example

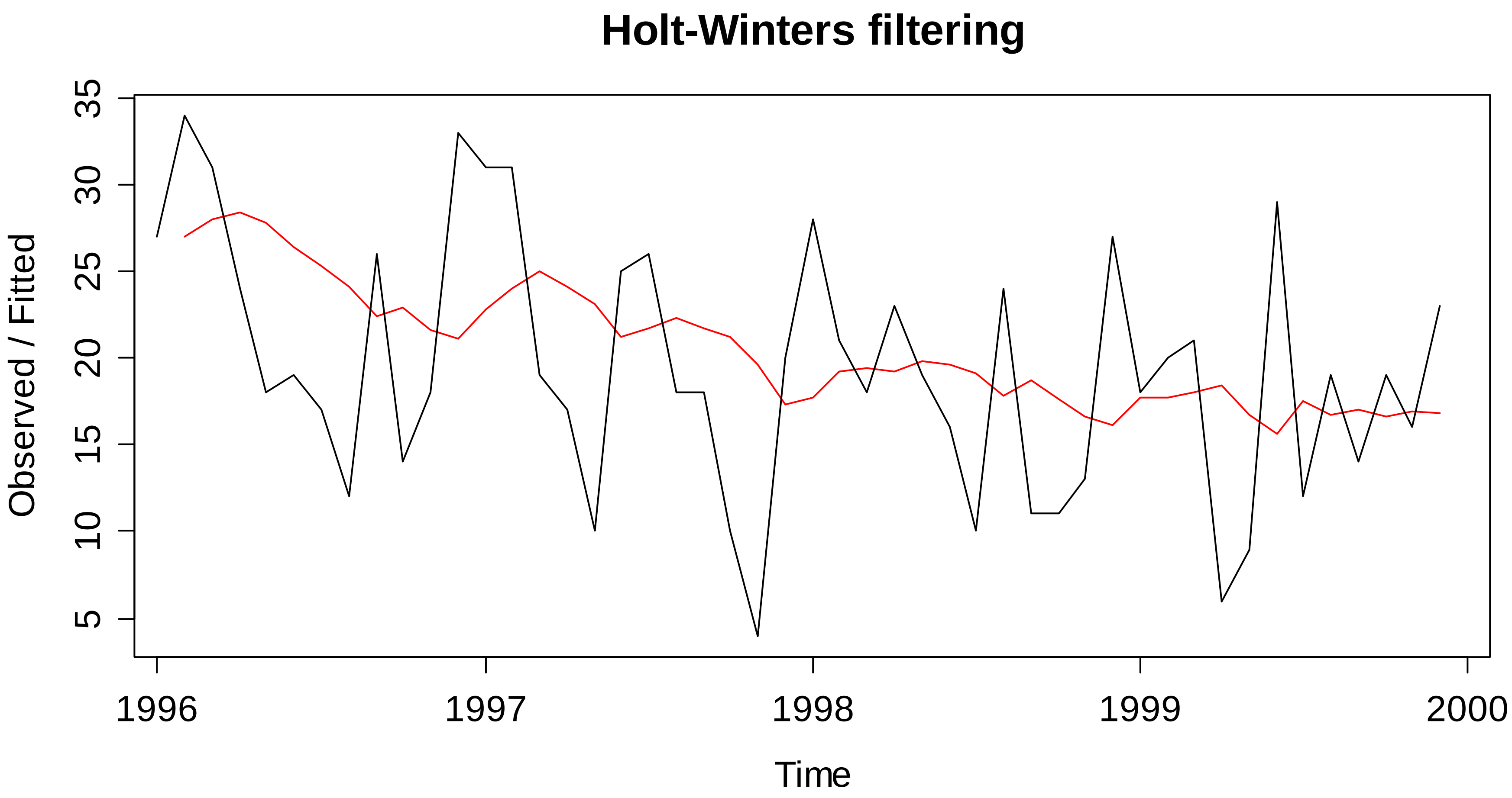
```
> fit  <- HoltWinters(cmpl, beta=F, gamma=F)

Holt-Winters exponential smoothing without trend
and without seasonal component.

Smoothing parameters:
 alpha:  0.1429622
 beta :  FALSE
 gamma:  FALSE

Coefficients:
      [,1]
a 17.70343
```

Exponential Smoothing: Example



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Holt-Winters Method

Purpose:

- is for time series with deterministic trend and/or seasonality
- is still a heuristic, model-free approach
- again based on weighted averaging

Is based on these 3 formulae:

$$\begin{aligned} a_t &= \alpha(x_t - s_{t-p}) + (1-\alpha)(a_{t-1} + b_{t-1}) \\ b_t &= \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1} \\ s_t &= \gamma(x_t - a_t) \quad + (1-\gamma)s_{t-p} \end{aligned}$$

→ **See the blackboard for the derivation...**

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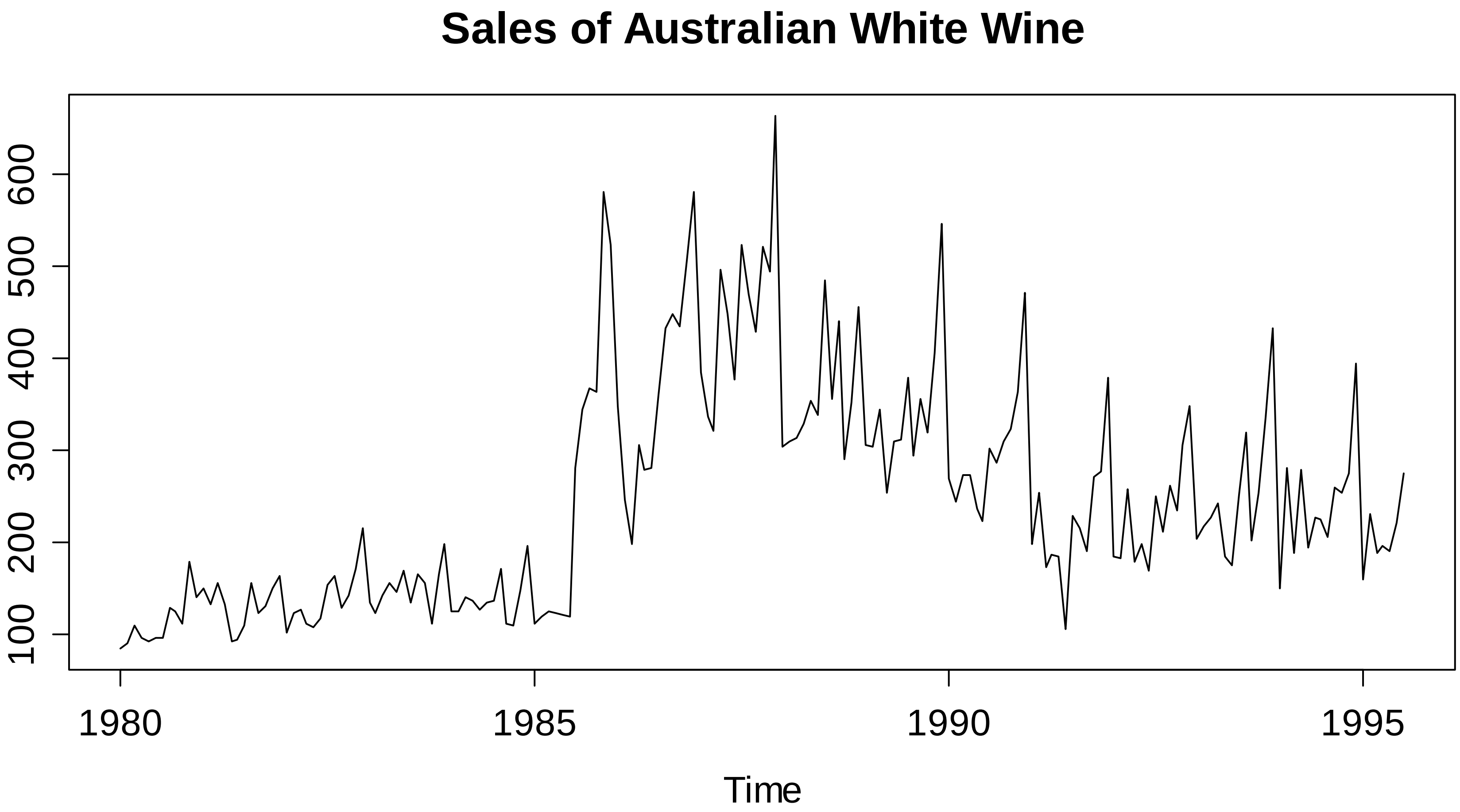
Holt-Winters: Example



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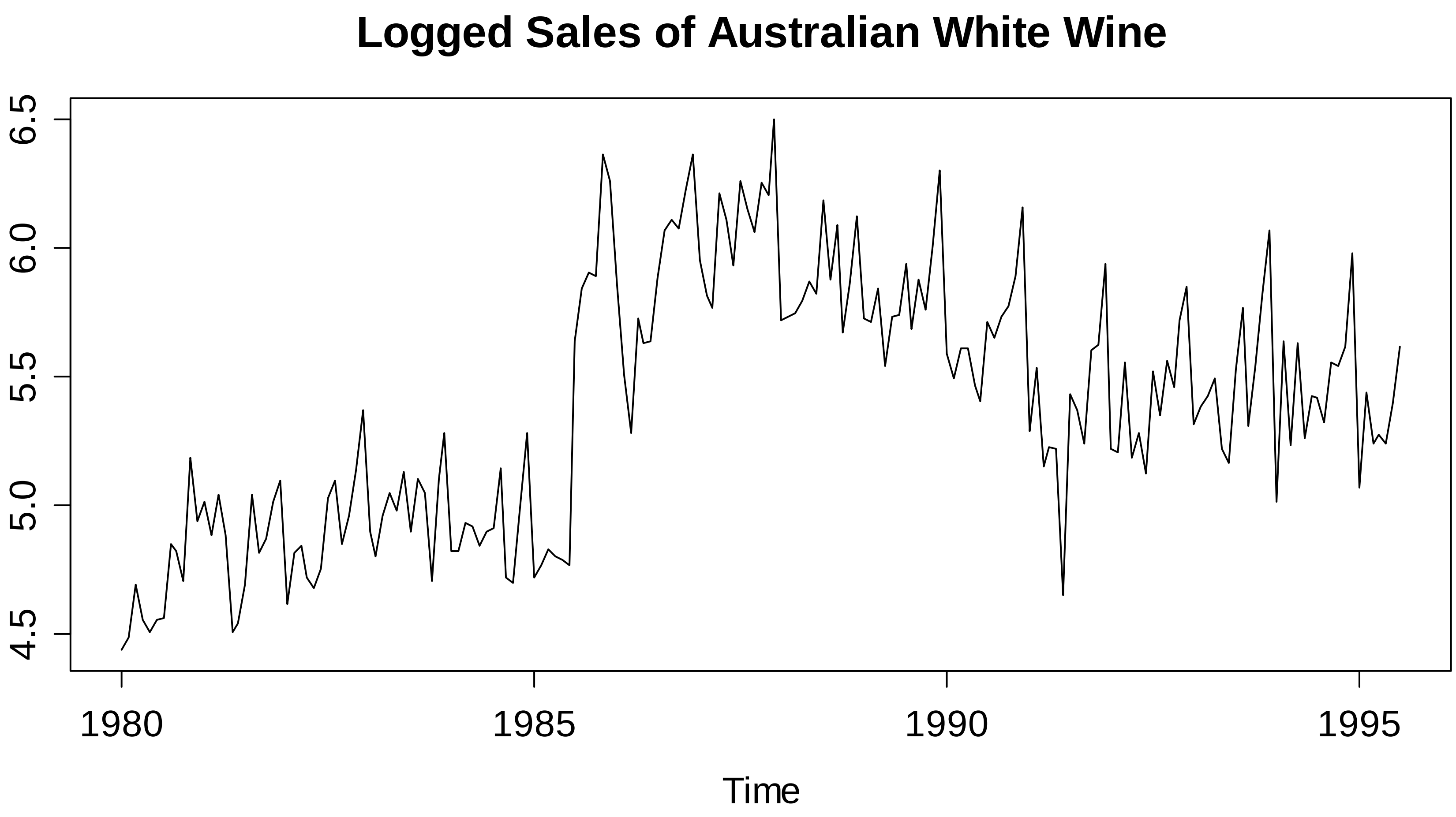
Holt-Winters: Example



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Holt-Winters: R-Code and Output

```
> HoltWinters(x = log(aww))

Holt-Winters exponential smoothing with trend and
additive seasonal component.

Smoothing parameters:
 alpha:  0.4148028; beta :  0; gamma:  0.4741967

Coefficients:
a     5.62591329; b    0.01148402
s1  -0.01230437; s2   0.01344762; s3   0.06000025
s4   0.20894897; s5   0.45515787; s6  -0.37315236
s7  -0.09709593; s8  -0.25718994; s9  -0.17107682
s10 -0.29304652; s11 -0.26986816; s12 -0.01984965
```

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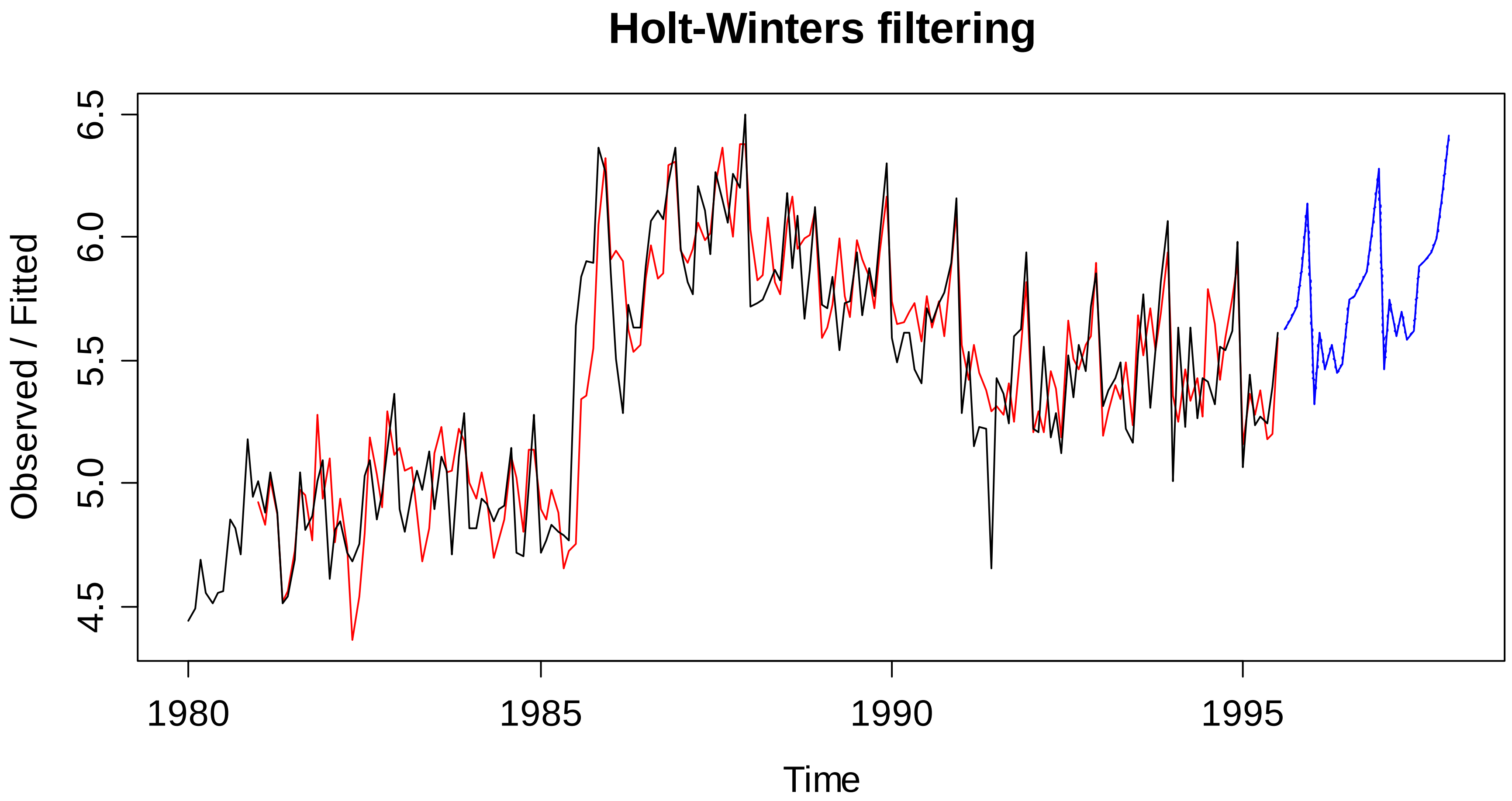
Holt-Winters: Fitted Values & Predictions



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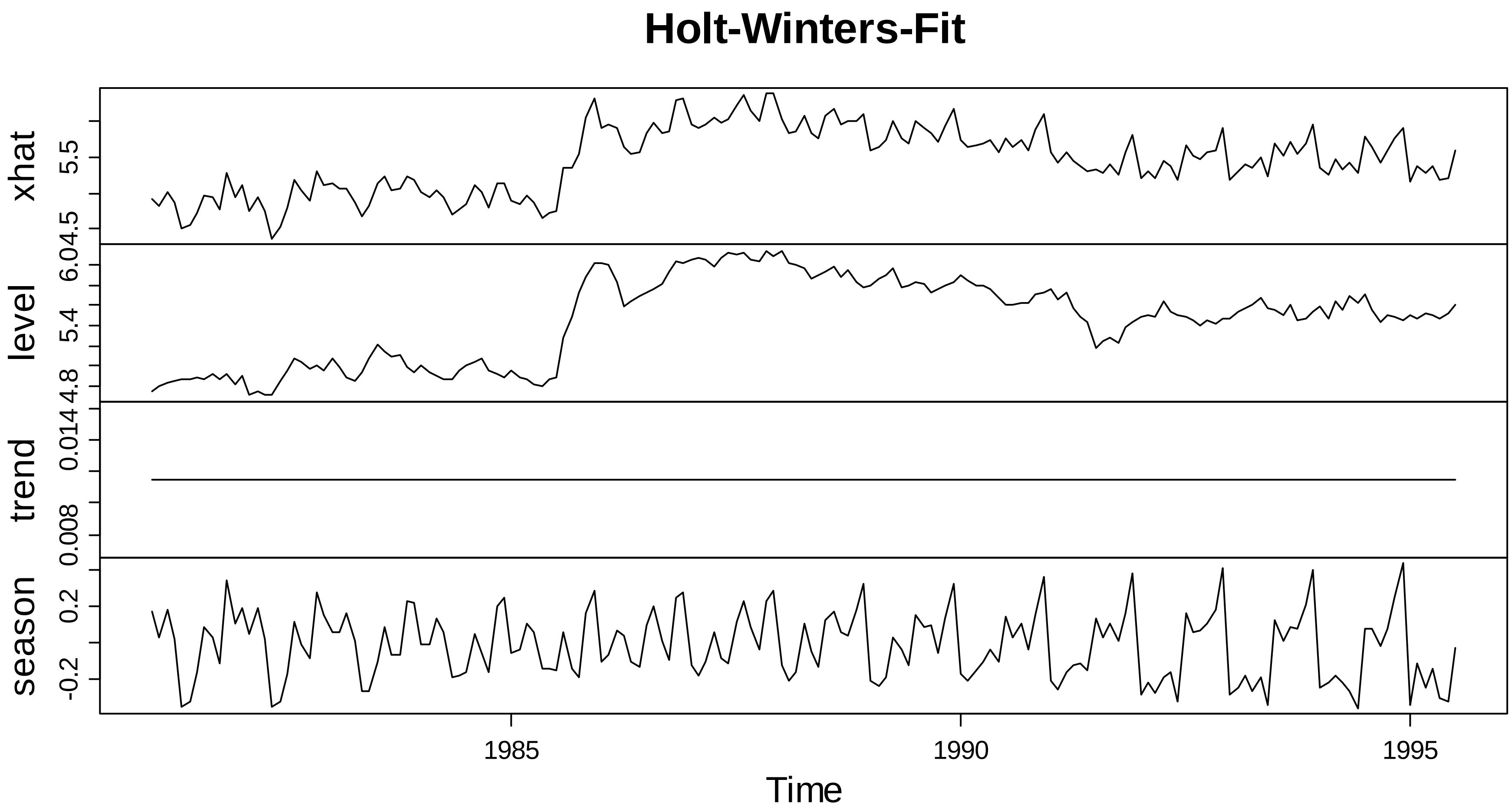
Holt-Winters: In-Sample Analysis



Holt-Winters: Predictions on Original Scale