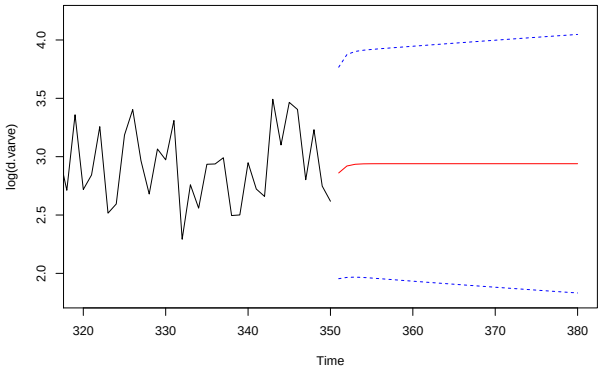
Solution to Series 6

1. **a)** The predicted value stabilizes after just a few points in time, and the prediction band grows linearly. Both these properties were to be expected from the theory (cf. lecture notes, 11.5.2).

R code:

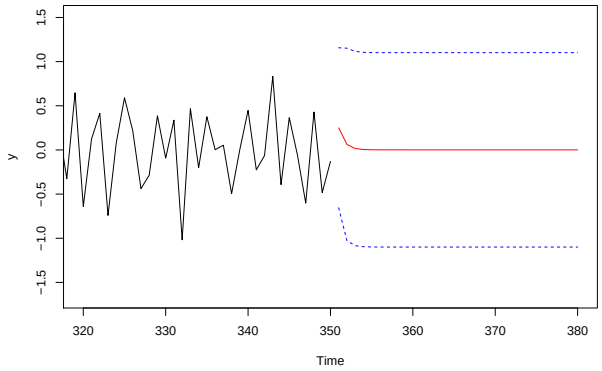
```
> ## Fitting the model
> d.varve <- ts(d.varve)
> r.varve <- arima(log(d.varve), order=c(1,1,1))
> r.pred <- predict(r.varve, n.ahead=30)                    ## Prediction
> plot(log(d.varve), xlim=c(320,380), ylim=c(1.8, 4.2))     ## Plotting
> lines(r.pred$pred, col=2)
> lines(r.pred$pred + 1.96 * r.pred$se, lty=2, col=4)
> lines(r.pred$pred - 1.96 * r.pred$se, lty=2, col=4)
```



b) Here the predicted change is also quite quick to stabilize at the mean of first-order differences. The prediction variance is constant after just a few time points, and is only marginally smaller than the variance of the process (cf lecture notes, 11.5.1).

R code:

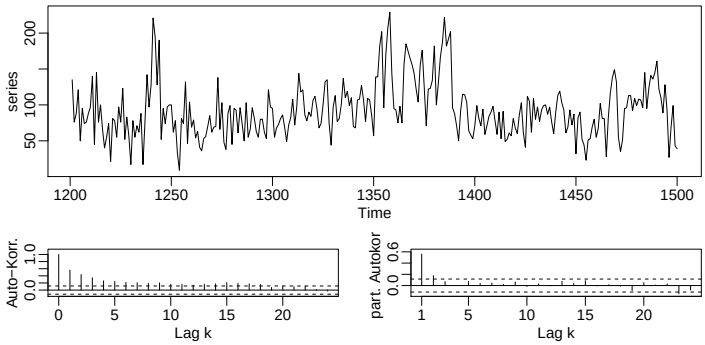
```
> y <-  diff(log(d.varve))
> ## Fitting the model
> r.varve.2 <- arima(y, order=c(1,0,1))
> r.pred.2 <- predict(r.varve.2, n.ahead=30)   ## Prediction
> plot(y, xlim=c(320,380))                     ## Plotting
> lines(r.pred.2$pred, col=2)
> lines(r.pred.2$pred + 1.96 * r.pred.2$se, lty=2, col=4)
> lines(r.pred.2$pred - 1.96 * r.pred.2$se, lty=2, col=4)
```



c) For Part **a)** the predicted values need only undergo an exp transformation. This means if we calculate `exp(r.pred$pred)`, we get the predicted values on the original scale. In Part **b)**, however, the formation of differences needs reversing (by taking the cumulative sum). If we apply `predict()` to an `arima` object, R does this for us. If we use the differences, we have to apply the cumulative sum to the predictet values first, then do the exp transformation and then add the last known entry of the timeseries: `exp(cumsum(r.pred.2$pred))+d.varve[350]`

2. **a)** Fit an ARIMA/SARIMA Model for all of these datasets. Use transformations if suitable. Compute a prediction and plot it along with the prediction band. Try different models and compare the outcome. Also analyse the residuals using `acf()/pacf()` and `qqnorm()`.
R-hint: `predict(....,n.ahead=...)`
You can either use the function `f.acf()`. Download (or copy) the file from here:
`http://stat.ethz.ch/WBL/Source-WBL-2/R/f.acf.R"` or draw acf and pacf plots. I offer only some suggestions instead of solutions. Other models might be just as well or even better.
Fir data:

```
> f.acf(d.foe)
```

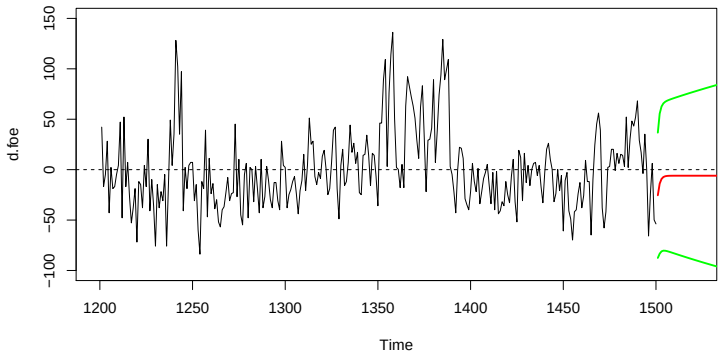


The acf-plot is decaying very slowly. There might be a trend.

```
> f.acf(diff(d.foe, 1))
```

The differenced data looks better, could either be MA(1) or ARMA(1,1). If you check the residuals, ARMA(1,1) yields better results and MA(1) still shows some dependence. So we continue with an ARIMA(1,1,1)-model. There is no season since we have yearly data.

```
> d.foe <- d.foe-mean(d.foe)
> fit <- arima(d.foe, order=c(1,1,1))
> fit

Call:
arima(x = d.foe, order = c(1, 1, 1))

Coefficients:
        ar1     ma1
      0.406  -0.907
s.e.  0.075   0.041

sigma^2 estimated as 1005:  log likelihood = -1458,  aic = 2922
> t.pr <- predict(fit, n.ahead=50)
> t.u <- t.pr$pred+1.96*t.pr$se
> t.l <- t.pr$pred-1.96*t.pr$se
> plot(d.foe, xlim=c(1200, 1520), ylim=c(-100, 150))
> abline(h=0, lty=2)
> lines(t.pr$pred, col="red", lwd=2)
> lines(t.u, col="green", lwd=2)
> lines(t.l, col="green", lwd=2)
```



Airline data: Use the log-transform, we have seasonal data and a trend.

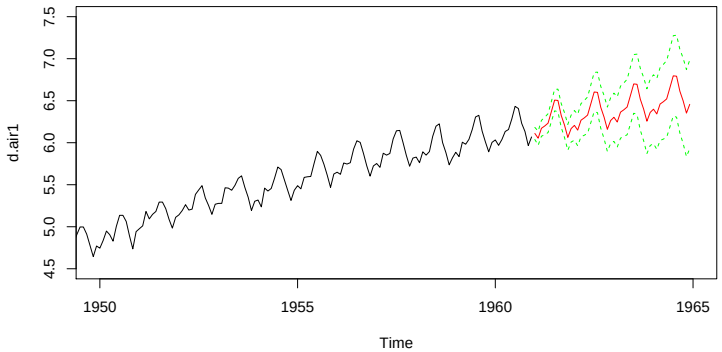
```
> d.air1 <- log(AirPassengers)
> d.air2 <- diff(d.air1, lag=12)
> d.air3 <- diff(d.air2, 1)
> f.acf(d.air3)
```

A $SARIMA(0,1,1) * (0,1,1)^{12}$ could be an appropriate model.

```
> s1.air <- arima(log(AirPassengers),order=c(0,1,1),seasonal=c(0,1,1))
> s1.air

Call:
arima(x = log(AirPassengers), order = c(0, 1, 1), seasonal = c(0, 1, 1))

Coefficients:
          ma1     sma1
      -0.402  -0.557
s.e.   0.090   0.073

sigma^2 estimated as 0.00135:  log likelihood = 245,  aic = -483
> f.acf(s1.air$residuals)
```

The residuals look ok.

```
> t.pr <- predict(s1.air, n.ahead=48)
> plot(d.air1, xlim=c(1950,1965), ylim=c(4.5,7.5))
> t.u <- t.pr$pred+1.96*t.pr$se
> t.l <- t.pr$pred-1.96*t.pr$se
> lines(t.pr$pred, col="red")
> lines(t.u, col="green", lty=2)
> lines(t.l, col="green", lty=2)
```



The AT$T bonds- data could be modelled using an ARIMA(0,1,1).

b) Now fit a polynomial model with a season if necessary. Again compute the prediction and plot it. Check the residuals.

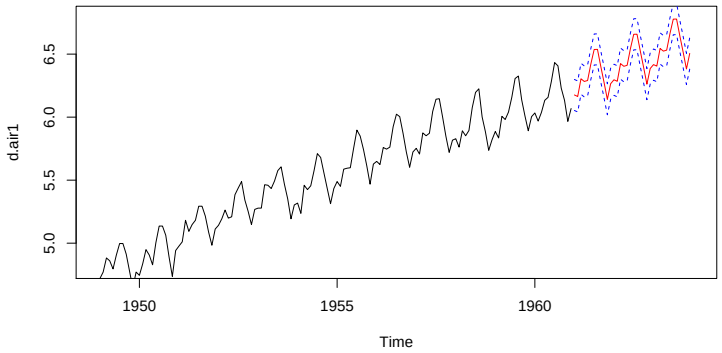
A possible solution is shown for the airline data only. The trend looks linear, but I added a quadratic term, after looking at a model using only a linear trend in time.

```
> plot(d.air1)
> time <- 1:length(d.air1)
> time <- time/12+1949
```

```
> Month <- factor(rep(month.name, length(d.air1)/12, levels=month.name))
> fit <- lm(d.air1~Month+time+I(time^2))
> fit <- lm(d.air1~Month+log(time))
> summary(fit)
> ts.fit <- ts(fitted(fit), start=1949, frequency=12)
> lines(ts.fit, col="red", lty=2)
```

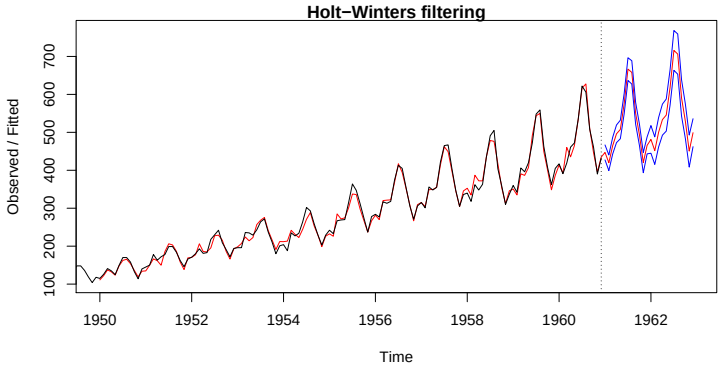
Prediction:

```
> t.pr <- predict(fit, newdata=data.frame(time=seq(1961,1964-1/12, by=1/12),
                        Month=rep(month.name, 3)), interval="prediction")
> plot(d.air1, xlim=c(1949,1964), ylim=c(4.8,6.8))
> t.pr <- ts(t.pr, start=1961, frequency=12)
> lines(t.pr[,1], col="red")
> lines(t.pr[,2], col=4, lty=2)
> lines(t.pr[,3], col=4, lty=2)
```



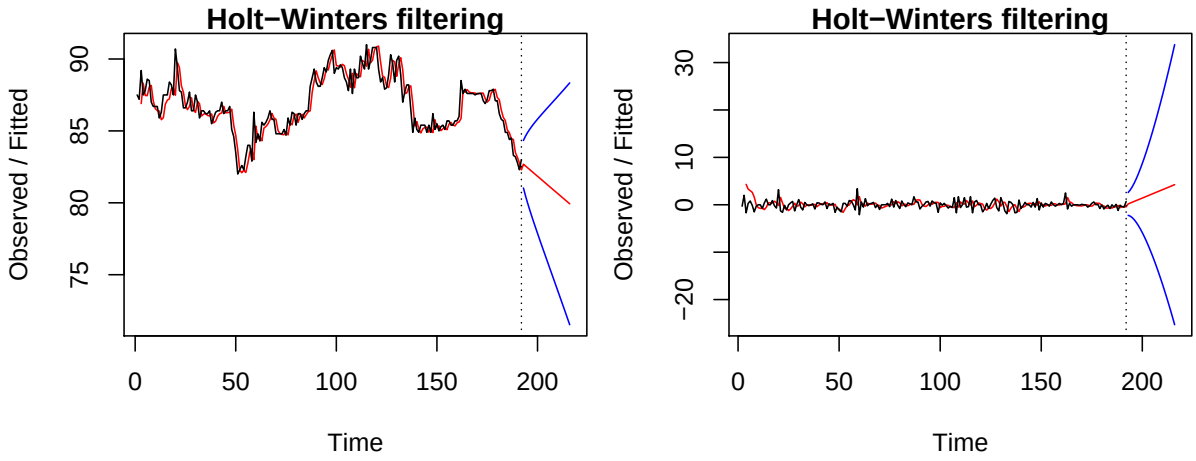
c) Now use exponential smoothing. Again make predictions, plot the predicted timeseries, including confidence bounds and check the residuals.
R-hint: `?HoltWinters`

```
> fit <- HoltWinters(d.air, seasonal="multiplicative")
> t.pr <- predict(fit, 24, prediction.interval=T)
> plot(fit, t.pr)
```



AT $ T bonds:

```
> fit <- HoltWinters(d.att, gamma=FALSE)
> t.pr <- predict(fit, 24, prediction.interval=T)
> fit2 <- HoltWinters(diff(d.att,1), gamma=FALSE)
> t.pr2 <- predict(fit2, 24, prediction.interval=T)
> par(mfrow=c(1,2))
> plot(fit, t.pr)
> plot(fit2,t.pr2)
```



This method does not seem to yield meaningful predictions in the case of the AT$T- data. You can try to play around with the α and β values in the `HoltWinters`-function, but it will probably not get better, since the data contains hardly any structure or information.