Series 4

1. Since simulations can be of use in model validation, we would like to use this exercise to simulate several time series by means of an ARMA model:

(i) AR(2) model with coefficients $\alpha_1 = 0.9$ and $\alpha_2 = -0.5$.

(ii) MA(3) model with coefficients $\beta_1 = 0.8$, $\beta_2 = -0.5$ and $\beta_3 = -0.4$.

(iii) ARMA(1, 2) model with coefficients $\alpha_1 = -0.75$, $\beta_1 = -1$ and $\beta_2 = 0.25$.

For all models, the error E_t follows the standard normal distribution $\mathcal{N}(0, 1)$.

a) How should the autocorrelations behave based on the theory?

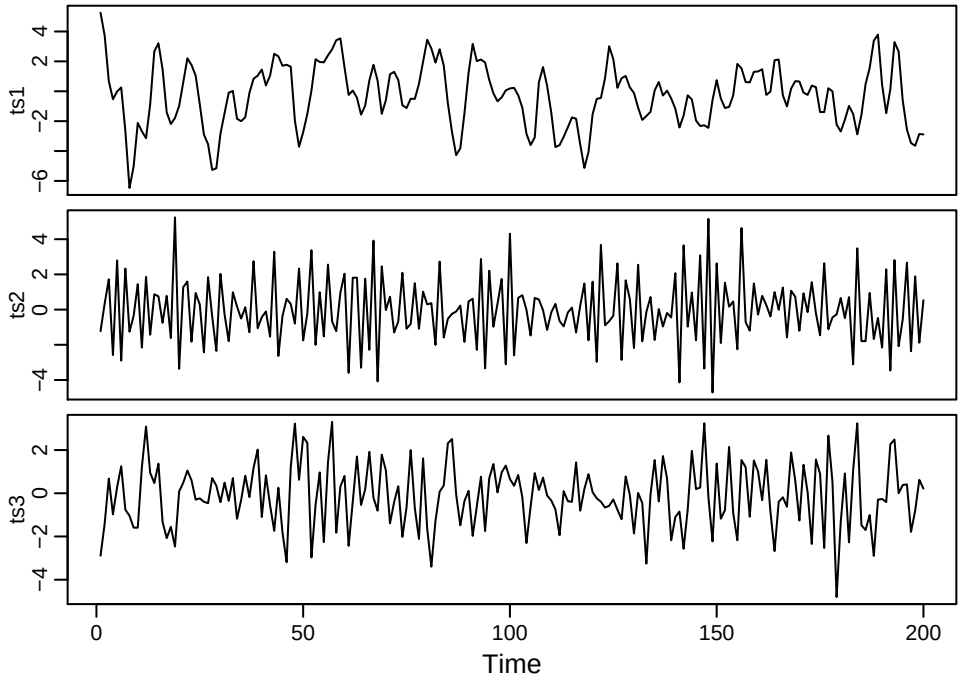
b) Use the function `ARMAacf()` to compute the *theoretical* autocorrelations and partial autocorrelations, and plot them up to lag 30.
R hints: Use the arguments `ar` and `ma` of `ARMAacf()` to specify the parameters of the models. With the argument `pacf`, you can specify whether partial or "normal" autocorrelations should be calculated.

c) Simulate all three models (i) to (iii). Take several different lengths for the time series: $n = 200$, $n = 500$ and $n = 1000$. Repeat these simulations several times to develop some intuition on what is "chance" and what is "structure". You don't have to print out all these plots, just have a look at them.
For each model, make a plot of one simulation for $n = 200$ and the corresponding correlograms. Compare the empirical autocorrelations to the theoretical ones from Part b).
R hints:
You can use the procedure `arima.sim()` to simulate the time series. The length of the simulated series you can choose by setting the argument `n`, and the model by setting the parameter `model` (to a list!).

```
> ar.sim <- arima.sim(n = ..., model = list(ar = c(0.9, -0.5)))
```

2. For each of the following three time series, find a suitable ARMA(p, q) model and estimate its parameters. To read the data, call:

```
> data <- read.table("http://stat.ethz.ch/Teaching/Datasets/ARMAsim.dat",
   header = TRUE)
> ts1 <- ts(data[, "ts1"])
> ts2 <- ts(data[, "ts2"])
> ts3 <- ts(data[, "ts3"])
```


ts1
ts2
ts3
Time
0
50
100
150
200