Statistic Seminar: 6th talk

ETHZ FS2010

Prediction of New Observations

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Papers: Welham (2004), Yiang (2007)

Content

- Introduction
- Prediction of Mixed Effects
- Prediction of Future Observation
- Principles of Prediction
  - Prediction Process
  - Fixed and Random Terms
- Example: Split-Plot Design

Linear mixed model

$$\boxed{y = X\beta + Zb + \varepsilon = W\gamma + \varepsilon}$$

$y \cong$ $n \times 1$ vector of observations

$X \cong$ $n \times p$ matrix associating observations with the appropriate combination of fixed effects

$\beta \cong$ $p \times 1$ vector of fixed effects

$Z \cong$ $n \times q$ matrix associating observations with the app. comb. of random effects

$b \cong$ $q \times 1$ vector of random effects

$\varepsilon \cong$ $n \times 1$ vector of residual errors

$W, \gamma \cong$ combined design matrix resp. vector of effects

Linear mixed model

$$\boxed{y = X\beta + Zb + \varepsilon = W\tau + \varepsilon} \quad b \perp \epsilon$$

Assumption

$$\left(Y \middle| B = b\right) \sim N\left(\mathbf{X}\beta + \mathbf{Z}b, \sigma_1^2 I_n\right)$$

$$B \sim N\left(0, \mathbf{\Sigma}_\theta\right)$$

where $\mathbf{\Sigma}_\theta = \mathrm{cov}(B)$ q×q, symmetric

$$= \sigma^2 \mathbf{\Lambda}_\theta \mathbf{\Lambda}'_\theta$$

Use: $B = \mathbf{\Lambda}_\theta U$

$$U \sim N\left(0, \sigma_1^2 I_q\right) \longrightarrow \boxed{\begin{array}{c}\left(Y \middle| U = u\right) \sim N\left(\mathbf{X}\beta + \mathbf{Z}\mathbf{\Lambda}_\theta u, \sigma_1^2 I_n\right) \\ U \sim N\left(0, \sigma_1^2 I_q\right)\end{array}}$$

Linear mixed model

Compute $\hat{u}$ and $\hat{\beta}_\theta$

$$\begin{bmatrix} \hat{u} \\ \hat{\beta}_\theta \end{bmatrix} = \arg\min_{u,\beta} \left\| \begin{bmatrix} y \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{Z}\boldsymbol{\Lambda}_\theta\mathbf{X} & \mathbf{X} \\ I & 0 \end{bmatrix} \begin{bmatrix} u \\ \beta \end{bmatrix} \right\|^2$$

$$\underbrace{\begin{bmatrix} \boldsymbol{\Lambda}_\theta\mathbf{Z}'\mathbf{Z}\boldsymbol{\Lambda}_\theta + I_q & \boldsymbol{\Lambda}'_\theta\mathbf{Z}'\mathbf{X} \\ \mathbf{X}'\mathbf{Y}\boldsymbol{\Lambda}_\theta & \mathbf{X}'\mathbf{X} \end{bmatrix}}_{\begin{bmatrix} \mathbf{L}_\theta & 0 \\ \mathbf{R}'_{\mathbf{ZX}} & \mathbf{R}'_{\mathbf{X}} \end{bmatrix} \begin{bmatrix} \mathbf{L}'_\theta & \mathbf{R}_{\mathbf{ZX}} \\ 0 & \mathbf{R}_{\mathbf{X}} \end{bmatrix}} \begin{bmatrix} \hat{u} \\ \hat{\beta}_\theta \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda}'_\theta\mathbf{Z}'y \\ \mathbf{X}'y \end{bmatrix}$$

$$\mathbf{L}_\theta\mathbf{R}_{\mathbf{ZX}} = \boldsymbol{\Lambda}'_\theta\mathbf{Z}'\mathbf{X}$$
$$\mathbf{R}'_{\mathbf{X}}\mathbf{R}_{\mathbf{X}} = \mathbf{X}'\mathbf{X} - \mathbf{R}'_{\mathbf{ZX}}\mathbf{R}_{\mathbf{ZX}}$$

What do we mean by "prediction"?

- Estimation of effects in the model
  → prediction = linear combination of estimated effects
- Marginal vs. Conditional predictions
- What is needed?
  - Marginal vs. Conditional predictions
  - In Example: variety or nitrogen prediction?

Is there a general strategy?

Problem: Each situation needs to be analyzed “by hand”!

Questions that might arise:

- In- or exclusion of random model terms from the prediction?
- Different weighting schemes?
- etc.

Predictions

2 types of prediction:

- → Prediction of *mixed effects*
- → Prediction of a *future observation*

Linear mixed model

$$\boxed{y = X\beta + Zb + \varepsilon = W\gamma + \varepsilon}$$

$y \cong$ $n \times 1$ vector of observations

$X \cong$ $n \times p$ matrix associating observations with the appropriate combination of fixed effects

$\beta \cong$ $p \times 1$ vector of fixed effects

$Z \cong$ $n \times q$ matrix associating observations with the app. comb. of random effects

$b \cong$ $q \times 1$ vector of random effects

$\varepsilon \cong$ $n \times 1$ vector of residual errors

$W, \gamma \cong$ combined design matrix resp. vector of effects

Prediction of Mixed Effects

"all parameters are known", i.e. fixed effects + variance components are known

Consider : $\gamma = \mathbf{x}'\beta + \mathbf{z}'b = \mathbf{x}'\beta + \xi$

$\mathbf{x}, \mathbf{z}$: known vectors

β, b : vector of fixed/random effects

Best predictor for ξ (as MSE): $\hat{\xi} = \mathrm{E}\left[\xi \mid y\right] = \mathbf{z}'\mathrm{E}\left[b \mid y\right]$

Assumption

$$\begin{bmatrix} b \\ y \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ X\beta \end{bmatrix}, \begin{bmatrix} \operatorname{cov}(b) & \operatorname{cov}(b)\mathbf{Z}' \\ \mathbf{Z}\operatorname{cov}(b) & \mathbf{V} \end{bmatrix} \right)$$

$$\mathbf{R} = \operatorname{cov}(\epsilon),\ \mathbf{V} = \operatorname{cov}(y) = \mathbf{Z}\operatorname{cov}(b)\mathbf{Z}' + \mathbf{R}$$

$$\Rightarrow E\left[b \middle| y \right] = \operatorname{var}(b)\mathbf{Z}'\mathbf{V}^{-1}\left(y - \mathbf{X}\beta \right)$$

$$\Rightarrow \hat{\xi} = \mathbf{x}'\operatorname{var}(b)\mathbf{Z}'\mathbf{V}^{-1}\left(y - \mathbf{X}\beta \right)$$

Best linear predictor of γ is then $\boxed{\hat{\gamma} = \mathbf{x}'\beta + \mathbf{z}'\operatorname{cov}(b)\mathbf{Z}'\mathbf{V}^{-1}\left(y - \mathbf{X}\beta \right)}$

Example: IQ-Test

- $\text{IQ} \sim N(100, 15^2)$
- Estimate the true IQ of a student scoring 130 in a test
- $\text{score} | \text{IQ} \sim N(\text{IQ}, 5^2)$
- model: $y = \mu + b + \varepsilon, \quad y \cong$ student test score,

  $b \cong$ realization of a random effect

- Predict: $\mu + b \cong$ student's true IQ (unobservable)

Result: $\boxed{\widehat{\text{IQ}} = 127}$

Prediction of Mixed Effects

- Fixed effects + variance components unknown
- 20 Students
- each student: 5 tests
- Computations as described before
- See R-File: rf_prediction3.R

Prediction of Future Observations

AIM: construct **prediction intervals**, i.e. an interval in which future observation will fall with a certain probability given what has been observed.

Examples:

1. Longitudinal studies
   - Prediction of a fut. obs. from an individual not previously observed
   - Less interest to predict another observation from an observed individual as the studies often aim at applications to a larger population
   - E.g.: drugs going to the market after clinical trials
2. Surveys
   - 2-step survey:
   - A) number of families randomly selected
   - B) some family members of each family are interviewed
   - Prediction for a non-selected family

Prediction Intervals

a. Assumption: fut. obs. has certain distribution
   - Distribution defined up to a finite number of unknown parameters
   - Estimate parameter → obtain prediction interval
   - BUT: if distribution assumption fails, the interval might be wrong

b. Distribution-free
   - Normality is not assumed
   - Distribution-free approach
   - Assumption: future observation is independent of current ones

c. Markov-Chains, Montecarlo

d. et cetera…

Confidence vs. Prediction Intervals

Confidence Interval (CI)

- Interval estimate of population parameter
- ↔ unobservable population parameter
- predict distribution of estimate of unobservable quantity of interest (e.g. true pop. mean)

Prediction Interval (PI)

- Interval estimate of future observation
- ↔ future observation
- Predict distribution of individual future points

Confidence vs. Prediction Intervals (math.)

Fixed effect model $y = X\beta + \varepsilon, \quad \varepsilon \sim N\left(0, \sigma^2\right)$

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$\operatorname{var}\left(\hat{\beta}\right) = \sigma^2 (X'X)^{-1}$$

$$\operatorname{var}\left(\hat{\beta} - \beta\right) = \sigma^2 (X'X)^{-1}$$

$$\left.\begin{array}{ll} \text{true} & \breve{y}_i = x_i'\beta \\ \text{observed} & y_i = x_i'\beta + \varepsilon \\ \text{modelled} & \hat{y}_i = x_i'\hat{\beta} \\ \text{predicted} & \hat{y}_{n+1} = x_{n+1}'\hat{\beta} \end{array}\right\} \text{Interval: } \hat{y} \pm 1.96 \cdot \operatorname{var}\begin{cases} \text{CI} \\ \text{PI} \end{cases}$$

Normal Approximation

Confidence vs. Prediction Intervals (math.)

CI: $\operatorname{var}(\hat{y}_i) = \sigma^2 x_i' (X'X)^{-1} x_i$

PI: $\operatorname{var}(\hat{y}_{n+1} - y_{n+1}) = \sigma^2 \left(x_{n+1}' (X'X)^{-1} x_{n+1} + 1 \right)$

Confidence Interval: $\hat{y} \pm 1.96 \cdot \sigma^2 x_i' (X'X)^{-1} x_i$

Prediction Interval: $\hat{y} \pm 1.96 \cdot \sigma^2 \left(x_{n+1}' (X'X)^{-1} x_{n+1} + 1 \right)$

Confidence vs. Prediction Intervals

How do we construct Prediction Intervals for a more general model?

Mixed effects model: $y = X\beta + Zb + \varepsilon$

Prediction Intervals

Model: $y = X\beta + Zb + \varepsilon, \quad b \perp \varepsilon$

n Observations

$$b \sim N\left(0, \boldsymbol{\Sigma}_\theta\right)$$

$$\varepsilon \sim N\left(0, \sigma^2 \mathbf{I}_n\right)$$

$$\mathbf{V} = \mathbf{Z}\boldsymbol{\Sigma}_\theta \mathbf{Z'} + \operatorname{cov}(\varepsilon)$$

$$\operatorname{cov}\left(b\right) = \boldsymbol{\Sigma}_\theta = \sigma_1^2 \mathbf{I}_q \quad \operatorname{cov}\left(\varepsilon\right) = \sigma^2 \mathbf{I}_n$$

Estimate β and b : $\hat{\beta} = \left(\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{X}\right)^{-1} \mathbf{X}'\hat{\mathbf{V}}^{-1} y$

$$\tilde{b} = \hat{\sigma}_1^2 \mathbf{Z}'\hat{\mathbf{V}}^{-1}\left(y - \mathbf{X}\hat{\beta}\right)$$

$$\text{where } \hat{\mathbf{V}} = \hat{\sigma}_1^2 \mathbf{Z}\mathbf{Z}' + \hat{\sigma}^2 \mathbf{I}_n$$

Prediction Intervals

Marginal:

$$\operatorname{cov}\left(\hat{y}_i\right)=\operatorname{cov}\left(x_i'\hat{\beta}\right)=x_i'\operatorname{cov}\left(\hat{\beta}\right)x_i$$

$$\operatorname{cov}\left(\hat{y}_{n+1}-y_{n+1}\right)=\operatorname{cov}\left(x_{n+1}'\hat{\beta}-\left(x_{n+1}'\beta+z_{n+1}'b+\varepsilon\right)\right)=$$

$$=x_{n+1}'\operatorname{cov}\left(\hat{\beta}-\beta\right)x_{n+1}+z_{n+1}'\mathbf{\Sigma}_\theta z_{n+1}+\boldsymbol{\sigma}^2\mathbf{I}_n$$

Conditional (on all random effects):

$$\operatorname{cov}\left(\hat{y}_{n+1}-y_{n+1}\middle|b=\tilde{b}\right)=\sigma^2\left(x_{n+1}\left(\mathbf{X}'\mathbf{X}\right)^{-1}x_{n+1}'+1\right)$$

Prediction Intervals

Marginal:

Confidence Interval: $\hat{y} \pm 1.96 \cdot x_i' \operatorname{cov}\left(\hat{\beta}\right) x_i$

Prediction Interval: $\hat{y} \pm 1.96 \cdot \left(x_{n+1}' \operatorname{cov}\left(\hat{\beta} - \beta\right) x_{n+1} + z_{n+1}' \boldsymbol{\Sigma}_\theta z_{n+1} + \sigma^2 \mathbf{I}_n \right)$

Conditional on all random effects:

Prediction Interval: $\hat{y} \pm 1.96 \cdot \sigma^2 \left(x_{n+1} \left(X'X \right)^{-1} x_{n+1}' + 1 \right)$

Prediction Intervals

There is a difference between marginal and conditional predictions!

→Which one is of interest?

The prediction process

Prediction:

• is a linear function of the best linear (unbiased) predictor of random effects with the best linear (unbiased) estimator of fixed effects in the model

• is typically associated with a combination of explanatory variables

• either averaged over, ignoring, or at a specific value of other explanatory variables in the model

The prediction process

Partition of the explanatory variables (e.v.) into 3 sets:

1. Classifying set
   - e.v. for which predicted values are required
2. Averaging set
   - e.v. which have to have averaged over
3. Rest
   - e.v. which will be ignored

The role of fixed and random effects with respect to prediction

Fixed Terms

• have associated set of effects (parameters) which have to be estimated

Random Terms

• associated effects are normally distributed with 0 mean and co-variance matrix

• co-variance matrix is function of (usually) unknown parameters

• error terms due to randomization or other structure of the data

How to deal with Random Factor Terms

1. Evaluate at a given value(s) specified by user
2. Average over the set of random effects
   - Prediction specific to / conditional on the random effects observed
   - → „Conditional prediction” w.r.t. the term
3. Omit the random term from the model
   - Prediction at the population average (zero)
   - substitutes the assumed pop. mean for an unknown random effect
   - → „Marginal prediction” w.r.t. the term

How to deal with Fixed Factors

- no pre-defined population average
- no natural interpretation for a prediction derived by omitting a fixed term from the fitted values
- average over all the present levels to give a conditional average
- or: user should specify the value(s)

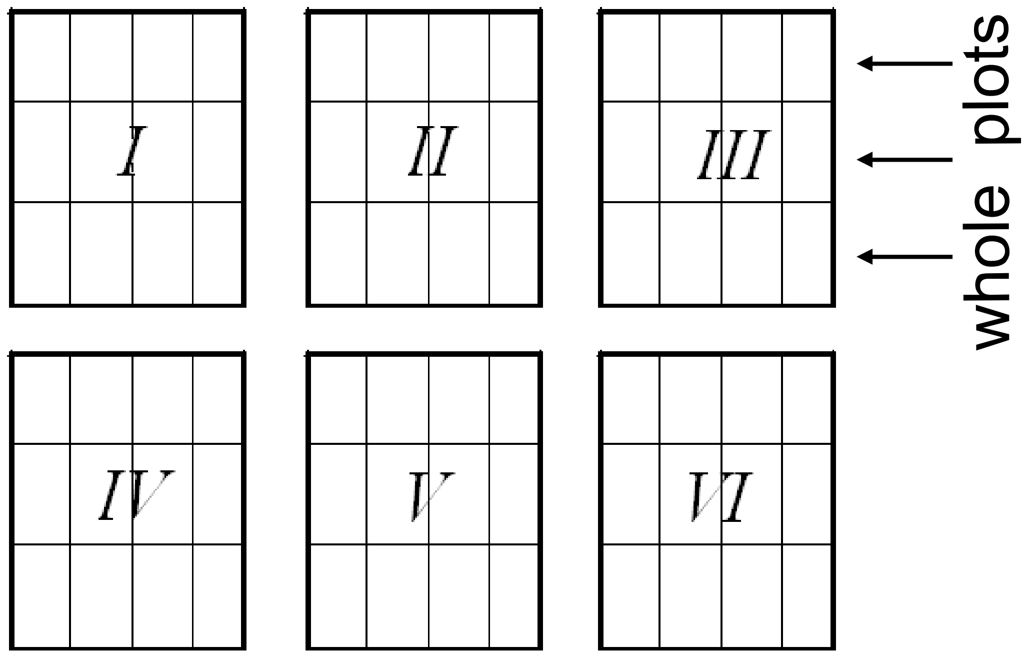
4 conceptual steps for the prediction process

1. Choose e.v. and their respective values for which predictive margins are required, i.e. determine the classifying set
2. Determine which variables should be averaged over, i.e. determine the averaging set
3. Determine terms that are needed to compute parameters and estimations
4. Choose the weighting for taking means over margins (for the averaging set)

Split-Plot Design

Experiment:

- 4 levels of nitrogen
- 3 oat varieties
- 6 "tries", i.e. 6 blocks
- 4 subplots
- 3 whole-plots
- random allocation of nitrogen within a block



Fixed effects:

- treatment combination

Random effects:

- blocking factors (source of error variation)

AIM: estimate the performance of each treatment combination within the experiment

The Data-Set

Yields from a split-plot field trial of oat varieties and nitrogen application rates

		Nitrogen						Nitrogen			
Block	Variety	0.0	0.2	0.4	0.6	Block	Variety	0.0	0.2	0.4	0.6
	GR	111	130	157	174		GR	74	89	81	122
I	M	117	114	161	141	IV	M	64	103	132	133
	V	105	140	118	156		V	70	89	104	117
	GR	61	91	97	100		GR	62	90	100	116
II	M	70	108	126	149	V	M	80	82	94	126
	V	96	124	121	144		V	63	70	109	99
	GR	68	64	112	86		GR	53	74	118	113
III	M	60	102	89	96	VI	M	89	82	86	104
	V	89	129	132	124		V	97	99	119	121

The model: components

$$\textit{fixed} \sim constant + variety + nitrogen + variety : nitrogen$$
$$\textit{random} \sim blocks + blocks : wplots$$
$$\textit{residual} \sim blocks : wplots : splots$$

Random terms:

- error terms used in estimation of treatment effects
- Not otherwise relevant to the prediction of treatment effects

Conditional vs. Marginal prediction for the random effects

Conditional prediction:

- gives a prediction specific to the blocks and plots used in the experiment
- appropriate to inference for the specific instance that occurred in the dataset

Marginal prediction:

- the prediction corresponds to the yields expected from a similar experiment laid out using different blocks and plots
- appropriate when inference is required for members of the wider population

The model

$$y_{ijk} = \mu + b_i + v_{r(ij)} + w_{ij} + n_{s(ijk)} + vn_{rs} + e_{ijk}$$

$y_{ijk} \cong$ yield on block $i = 1,\ldots,6$, whole-plot $j = 1,..3$, sub-plot $k = 1,\ldots,4$

$\mu \cong$ overall constant

$b_i \cong$ effect of block

$v_{r(ij)} \cong$ effect of variety $r(ij)$

$r(ij) \cong$ randomization of varieties to whole-plots

$w_{ij} \cong$ effect of whole-plot j in block i

$n_{s(ijk)} \cong$ effect of nitrogen level s

$s(ijk) \cong$ randomization of nitrogen levels to sub-plots

$vn_{rs} \cong$ interaction of variety level r with nitrogen level s

$e_{ijk} \cong$ residual error for block i, whole-plot j, sub-plot k

Assumption

$$y_{ijk} = \mu + b_i + v_{r(ij)} + w_{ij} + n_{s(ijk)} + vn_{rs} + e_{ijk}$$

For the following terms we assume a normal distribution

$$b = (b_1, b_2, \ldots, b_6),\ w = (w_{11}, w_{12}, \ldots, w_{63}) \text{ and } e = (e_{111}, e_{112}, \ldots, e_{634})$$

$$\begin{bmatrix} b \\ w \\ e \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_b^2 I_6 & 0 & 0 \\ 0 & \sigma_w^2 I_{18} & 0 \\ 0 & 0 & \sigma^2 I_{72} \end{bmatrix} \right)$$

$$y_{ijk} = \mu + b_i + v_{r(ij)} + w_{ij} + n_{s(ijk)} + vn_{rs} + e_{ijk}$$

ANOVA

- no $\text{variety} \times \text{nitrogen}$ interaction
- usually: drop non-significant terms

Prediction Process

$$y_{ijk} = \mu + b_i + v_{r(ij)} + w_{ij} + n_{s(ijk)} + vn_{rs} + e_{ijk}$$

Prediction of yield for each nitrogen level

- = general effect of different nitrogen applications
- → unweighted average across all varieties
- = prediction of yield for nitrogen level l for "average" block+whole-plot
- → marginal prediction wrt block+whole-plot

Calculation: ignore random terms:

$$\hat{\mu} + \hat{n}_l + \frac{1}{3}\sum_{j=1}^{3}\left(\hat{v}_j + \widehat{vn}_{jl}\right)$$

Prediction Process

$$y_{ijk} = \mu + b_i + v_{r(ij)} + w_{ij} + n_{s(ijk)} + vn_{rs} + e_{ijk}$$

Prediction specific to blocks+whole-plots in experiment

- → conditional prediction

Calculation: include random terms:

$$\hat{\mu} + \frac{1}{6}\sum_{i=1}^{6}\tilde{b}_i + \hat{n}_l + \frac{1}{18}\sum_{i=1}^{6}\sum_{j=1}^{3}\tilde{w}_{ij} + \frac{1}{3}\sum_{j=1}^{3}\left(\hat{v}_j + \widehat{vn}_{jl}\right)$$

Prediction Process

Explanatory variable	Set		Levels		Averaging	
	M	C	M	C	M	C
Variety	a	a	all	all	e	e
Nitrogen	c	c	all	all	n	n
Blocks	x	a	-	all	-	e
Wplots	x	a	-	all	-	e
splots	x	x	-	-	-	-

M : marginal pred.
C : conditional pred.

a : averaging set
c : classifying set
x : excluded

e : equal weights
n : none

Prediction Process

Model term	In prediction?	
	M	C
Constant	+	+
Variety	+	+
Nitrogen	+	+
Variety:nitrogen	+	+
Blocks	x	+
Blocks:wplots	x	+
Blocks:wplots:splots	x	x

+ : used
x : ignored

The resulting predictions

Predictions for nitrogen application levels with SE and SED, using marginal (M) or conditional (C) values of blocks+whole-plots

Nitrogen application	Prediction	SE	
		M	C
0.0 cwt/acre	**79.4**	7.18	3.14
0.2 cwt/acre	**98.9**	7.18	3.14
0.4 cwt/acre	**114.2**	7.18	3.14
0.6 cwt/acre	**123.4**	7.18	3.14
SED		4.44	4.44

The resulting predictions

$\text{variety} \times \text{nitrogen}$ predictions

Variety	Nitrogen application (cwt/acre)				Margin
	0.0	0.2	0.4	0.6	
Golden Rain	**80.00**	**98.50**	**124.83**	**124.83**	104.50
Marvellous	**86.67**	**108.50**	**117.17**	**126.83**	109.79
Victory	**71.50**	**89.67**	**110.83**	**118.50**	97.63
Margin	79.39	98.89	114.22	123.39	103.97

The resulting predictions

- SE smaller for conditional predictions
  - → because predictions are calculated conditional on the blocks+whole-plots observed!

- Using marginal values = „no information on block+whole-plot effect“

Special case: data missing

Data for all replicates of Golden Rain with 0 cwt nitrogen are missing

Yields from a split-plot field trial of oat varieties and nitrogen application rates

		Nitrogen						Nitrogen			
Block	Variety	0.0	0.2	0.4	0.6	Block	Variety	0.0	0.2	0.4	0.6
	GR	/	130	157	174		GR	/	89	81	122
I	M	117	114	161	141	IV	M	64	103	132	133
	V	105	140	118	156		V	70	89	104	117
	GR	/	91	97	100		GR	/	90	100	116
II	M	70	108	126	149	V	M	80	82	94	126
	V	96	124	121	144		V	63	70	109	99
	GR	/	64	112	86		GR	/	74	118	113
III	M	60	102	89	96	VI	M	89	82	86	104
	V	89	129	132	124		V	97	99	119	121

Special case: data missing

Variety	Nitrogen application (cwt/acre)				Margin
	0.0	0.2	0.4	0.6	
Golden Rain	???	**98.50**	**124.83**	**124.83**	
Marvellous	**86.67**	**108.50**	**117.17**	**126.83**	109.79
Victory	**71.50**	**89.67**	**110.83**	**118.50**	97.63
Margin		98.89	114.22	123.39	

Corresponding cell cannot be estimated without additional assumptions!

Special case: data missing

No significant variety main effect/ interactions present in the model

→ Approach chosen has no great influence on nitrogen prediction

→ consider variety predictions

Possible approaches

- set inestimable parameters to 0, average over all cells
- average over cells with data present
- average over levels of nitrogen for which all varieties are present

The resulting predictions

Variety predictions with ‘Golden Rain + 0 cwt nitrogen‘ plots set to missing value

Margin	Variety	Inestimable parameters zero	For data present	On nitrogen levels 0.2-0.6
104.50	Golden Rain	**105.67**	**112.67**	**112.67**
109.79	Marvellous	**109.97**	**109.79**	**117.50**
97.63	Victory	**97.63**	**97.63**	**106.33**

↑ *Complete data*

The resulting predictions

Variety	Inestimable parameters zero	For data present	On nitrogen levels 0.2-0.6
Golden Rain	**105.67**	**112.67**	**112.67**
Marvellous	**109.97**	**109.79**	**117.50**
Victory	**97.63**	**97.63**	**106.33**

In 2nd case: variety ordering has changed
→ prediction not comparable because of large nitrogen effect

Comparison

Data complete

Variety	Margin
Golden Rain	104.50
Marvellous	109.79
Victory	97.63
Margin	103.97

Data missing

Variety	Inest. param. zero	For data present	On nitrogen levels 0.2-0.6
Golden Rain	105.67	112.67	112.67
Marvellous	109.79	109.79	117.50
Victory	97.63	97.63	106.33

Other special cases: data missing

First entry in the data is missing (i.e. Victory, 0.0 cwt/acre, Block I)

Yields from a split-plot field trial of oat varieties and nitrogen application rates

		Nitrogen						Nitrogen			
Block	Variety	0.0	0.2	0.4	0.6	Block	Variety	0.0	0.2	0.4	0.6
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Data in Block I for all replicates with 0.0 cwt nitrogen are missing

Yields from a split-plot field trial of oat varieties and nitrogen application rates

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All Data for replicates with 0.0 cwt nitrogen are missing

Yields from a split-plot field trial of oat varieties and nitrogen application rates

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	GR	/	91	97	100		GR	/	90	100	116
II	M	/	108	126	149	V	M	/	82	94	126
	V	/	124	121	144		V	/	70	109	99
	GR	/	64	112	86		GR	/	74	118	113
III	M	/	102	89	96	VI	M	/	82	86	104
	V	/	129	132	124		V	/	99	119	121

Making the computations…

R-FILE!