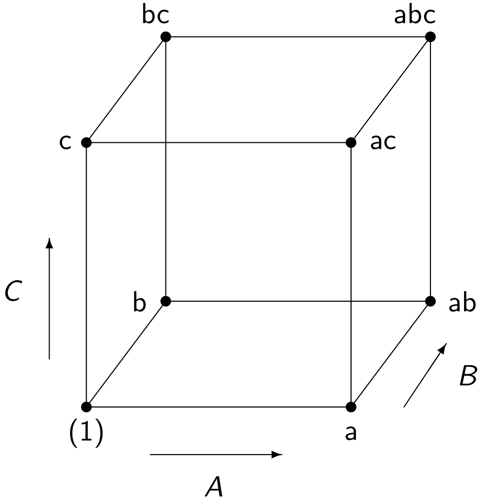
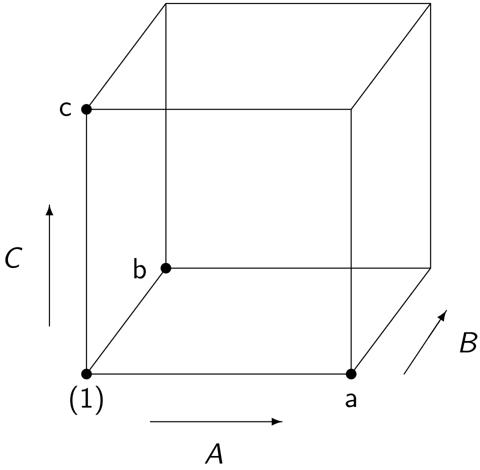
Fractional Factorials

- Too many runs for many factors
- Ignore some high-order interactions and run only a fraction of all possible runs
- How to choose the runs?

Full 2^3 factorial

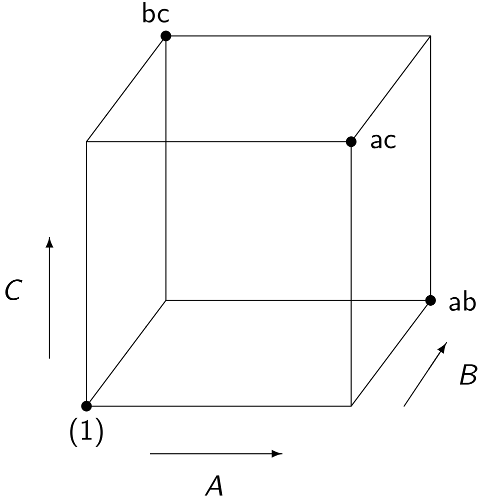


Half-replicate



(1)
a
b
c

Optimal coverage



(1)
ab
ac
bc

2^{3-1} design

run	A	B	C	AB	AC	BC	ABC
(1)	−	−	−	+	+	+	−
ab	+	+	−	+	−	−	−
ac	+	−	+	−	+	−	−
bc	−	+	+	−	−	+	−

$$\hat{C} = -\hat{AB}, \hat{B} = -\hat{AC}, \hat{A} = -\hat{BC}, \hat{I} = -\hat{ABC}$$

Leaf spring experiment

- An experiment to improve a heat treatment process on truck leaf springs.
- The heat treatment consists of heating in a high temperature oven, processing by a forming machine, and cooling in an oil bath.
- The response, the height of an unloaded spring, should be 8.0.
- half fraction of a 2^5 design is used to study 5 factors.

Factors and levels

Factor		Level −	Level +
A	heat temperature (°F)	1840	1880
B	heating time (seconds)	23	25
C	transfer time (seconds)	10	12
D	hold down time (seconds)	2	3
E	oil temperature (°F)	130-150	150-170

Why using fractional factorials?

- 2^5 design has 32 runs to estimate the overall mean and

Main	Interactions			
Effects	2-Factor	3-Factor	4-Factor	5-Factor
5	10	10	5	1

- 4-factor, 5-factor and even 3-factor interactions are not likely to be important. There are $10+5+1 = 16$ such effects, half of the total runs!
- use a half-replicate. What price is to pay?

Design matrix

Treatment	A	B	C	D	E
(1)	−	−	−	−	−
ab	+	+	−	−	−
ac	+	−	+	−	−
bc	−	+	+	−	−
ad	+	−	−	+	−
bd	−	+	−	+	−
cd	−	−	+	+	−
abcd	+	+	+	+	−
e	−	−	−	−	+
abe	+	+	−	−	+
ace	+	−	+	−	+
bce	−	+	+	−	+
ade	+	−	−	+	+
bde	−	+	−	+	+
cde	−	−	+	+	+
abcde	+	+	+	+	+

$I = ABCD$ is the defining relation
$D = ABC$: D is aliased with the interaction ABC.

Aliasing structure

The complete aliasing structure is:

$$
\begin{aligned}
I &= ABCD & AD &= BC \\
A &= BCD & AE &= BCDE \\
B &= ACD & BE &= ACDE \\
C &= ABD & CE &= ABDE \\
D &= ABC & DE &= ABCE \\
E &= ABCDE & ABE &= CDE \\
AB &= CD & ACE &= BDE \\
AC &= BD & ADE &= BCE
\end{aligned}
$$

Construction method I

To construct a 2^{k-1} design choose one block of a 2^k design divided into two blocks.
Ex: k=4, confound the ABCD interaction with blocks and take the principal block as half replicate.

(1)
ab
ac
bc
ad
bd
cd
abcd

2^{4-2} Design

Choose two confounding interactions: AB und CD.
ABCD is also confounded with blocks.

(1)
... ab
... cd
... abcd

Aliasing structure:

$I = AB, CD, ABCD$

$A = B, ACD, BCD$

$C = ABC, D, ABD$

$AC = BC, AD, BD$

Construction method II

To construct a 2^{4-1} design start with a 2^3 design and identify the fourth factor with the ABC interaction.

Treatment	I	A	B	AB	C	AC	BC	ABC=D	
(1)	+	−	−	+	−	+	+	−	(1)
a	+	+	−	−	−	−	+	+	ad
b	+	−	+	−	−	+	−	+	bd
ab	+	+	+	+	−	−	−	−	ab
c	+	−	−	+	+	−	−	+	cd
ac	+	+	−	−	+	+	−	−	ac
bc	+	−	+	−	+	−	+	−	bc
abc	+	+	+	+	+	+	+	+	abcd

Resolution of a design

- Resolution = length of shortest word among the $2^l - 1$ words used in the defining relations.
- In any resolution III design, main effects are not confounded with other main effects.
- In any resolution IV design, main effects are not aliased with any other main effect or 2-factor interactions.
- In any resolution V design, the main effects are not aliased with any other main effect, 2-factor or 3-factor interactions. The two-factor interactions are not aliased with any other 2-factor interaction.