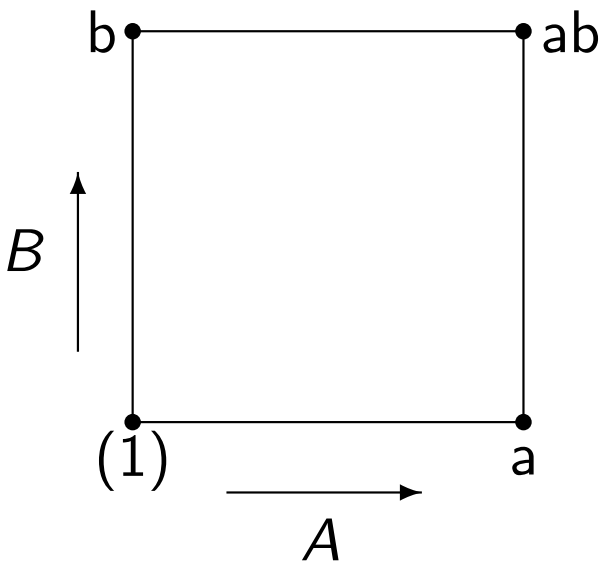
2^k Factorials

2^k Factorials

- Experiments with many factors
- Each factor has only two levels: high (+) and low(–)
- 2^k runs for a complete replicate with k factors

2^2- Design

run	A	B	Treatment
1	−	−	(1)
2	+	−	a
3	−	+	b
4	+	+	ab



Estimation of main effects and interaction

$$\begin{aligned}
\hat{A} &= \bar{y}_{A+} - \bar{y}_{A-} = \frac{1}{2n}(ab + a - b - (1)) \\
\hat{B} &= \bar{y}_{B+} - \bar{y}_{B-} = \frac{1}{2n}(ab + b - a - (1)) \\
\widehat{AB} &= \frac{1}{2n}((ab - b) - (a - (1))) = \frac{1}{2n}(ab + (1) - a - b)
\end{aligned}$$

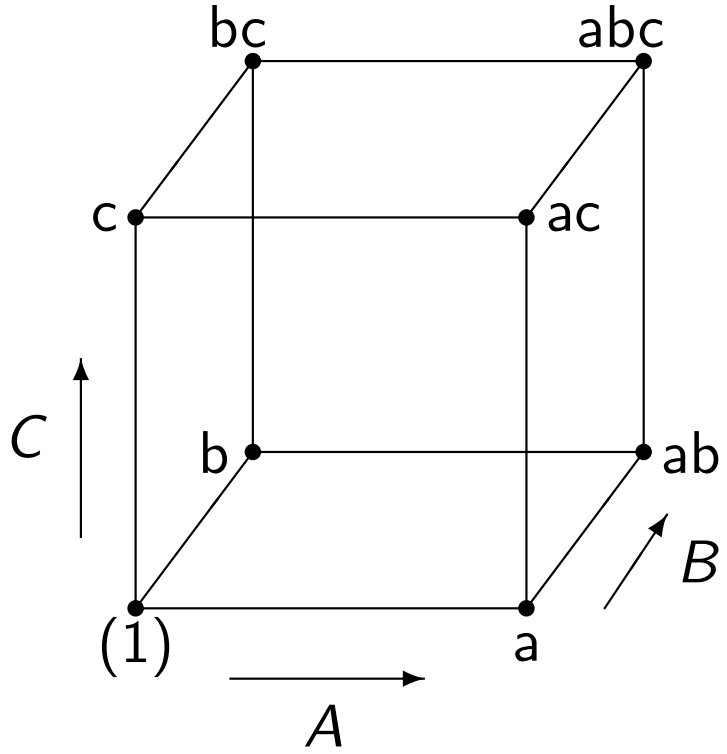
(n replicates, same notation for totals)

Algebraic signs for calculating effects

Treatment	I	A	B	AB
(1)	+	−	−	+
a	+	+	−	−
b	+	−	+	−
ab	+	+	+	+

2^3—Design

run	A	B	C	Treatment
1	−	−	−	(1)
2	−	−	+	c
3	−	+	−	b
4	−	+	+	bc
5	+	−	−	a
6	+	−	+	ac
7	+	+	−	ab
8	+	+	+	abc



Estimation of effects

Main effect A:

$$\hat{A} = \bar{y}_{A+} - \bar{y}_{A-} = \frac{1}{4n}(a - (1) + (ab - b) + (ac - c) + (abc - bc))$$

Interaction effect of AB: half the difference between the effect of A at the different levels of B.

$$\widehat{AB} = \frac{1}{4n}((ab - b) + (abc - bc) - ((a - (1)) + (ac - (c)))$$

Interaction ABC: half the difference between the interaction effect AB at the different levels of C.

$$\widehat{ABC} = \frac{1}{4n}((abc - bc) - (ac - (c)) - ((ab - b) - (a - (1)))$$

Algebraic signs for calculating effects

Treatment	I	A	B	AB	C	AC	BC	ABC
(1)	+	−	−	+	−	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	+	−	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	−	+	+	−	−
bc	+	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	+

Blocking in Factorials

run	A	B	C	D
1	−	−	−	1
2	−	−	+	1
3	−	+	−	1
4	−	+	+	1
5	+	−	−	2
6	+	−	+	2
7	+	+	−	2
8	+	+	+	2

What is wrong with this design?

Example

```
> data
    y  A  B  C
1  13 -1 -1 -1
2  63 -1 -1  1
3  91 -1  1 -1
4 113 -1  1  1
5 119  1 -1 -1
6 125  1 -1  1
7 137  1  1 -1
8 139  1  1  1
```

Example continued

```
> mod1=aov(y~A*B*C)
> summary(mod1)
      Df Sum of Sq Mean Sq
A      1      7200    7200
B      1      3200    3200
C      1       800     800
A:B    1      1152    1152
A:C    1       512     512
B:C    1       128     128
A:B:C  1        72      72

> mod1$coef
 (Intercept)   A   B   C A:B A:C B:C A:B:C
         100  30  20  10 -12  -8  -4     3
```

with blocking

```
> mod2=aov(y~D+A*B*C)
> summary(mod2)
            Df Sum of Sq Mean Sq
D            1      7200    7200
B            1      3200    3200
C            1       800     800
A:B          1      1152    1152
A:C          1       512     512
B:C          1       128     128
A:B:C        1        72      72
> mod2$coef
 (Intercept)  D  A  B  C A:B A:C B:C A:B:C
         100 30 NA 20 10 -12  -8  -4     3
```

A little bit better:

run	A	B	C	D
1	−	−	−	2
2	−	−	+	1
3	−	+	−	1
4	−	+	+	2
5	+	−	−	2
6	+	−	+	1
7	+	+	−	1
8	+	+	+	2

Blocks confounded with ABC

run	A	B	C	D
1	−	−	−	1
2	−	−	+	2
3	−	+	−	2
4	−	+	+	1
5	+	−	−	2
6	+	−	+	1
7	+	+	−	1
8	+	+	+	2

Construction method

- Choose an interaction to be confounded with blocks
- The principal block consists of (1) and all treatments which have an even number of letters in common with the chosen interaction.
- $\mathbf{2^k}$ design in $\mathbf{2^l}$ blocks: choose l confounded interactions. The principal block consists of (1) and all treatments which have an even number of letters in common with the chosen interactions. For the other blocks multiply the principal block with a letter not included yet.

Partial confounding

2^3 design in 2 blocks: [(1),ab,ac,bc] and [a,b,c,abc] Take four replicates to get sufficient precision, confound a different interaction in each replicate.

I:	[(1),ab,ac,bc]	and	II:	[a,b,c,abc]	ABC confounded
III:	[(1),a,bc,abc]	and	IV:	[b,c,ab,ac]	BC confounded
V:	[(1),b,ac,abc]	and	VI:	[a,c,ab,bc]	AC confounded
VII:	[(1),c,ab,abc]	and	VIII:	[a,b,ac,bc]	AB confounded

Main effects are estimated from 8 blocks, interactions from 6 blocks.

Sterilisation procedures

Surgical equipment is compared at 4 different sites. Sterilisation procedures are combinations of 4 factors: oxidants A and B (present or not), time in autoclave C (5 min, 10 min) and heat level D. Response is average bug counts per mm^2. Data are:

Site 1		Site 2		Site 3		Site 4	
(1)	52.5	a	52.1	c	56.0	abc	42.1
b	49.5	ab	44.7	bc	49.8	ac	51.1
acd	50.3	cd	57.2	ad	52.1	bd	49.6
abcd	36.6	bcd	51.1	abd	42.9	d	55.3

Anova table I

	Df	Sum Sq	Mean Sq
blocks	3	35.217	11.739
A	1	150.676	150.676
B	1	227.256	227.256
C	1	1.266	1.266
D	1	0.456	0.456
A:B	1	20.931	20.931
B:C	1	5.881	5.881
B:D	1	5.176	5.176
A:B:C	1	0.391	0.391
A:B:D	1	0.951	0.951
A:C:D	1	1.051	1.051
B:C:D	1	0.001	0.001
A:B:C:D	1	2.031	2.031

Anova table II

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
blocks	3	35.217	11.739	13.2700	0.008127	**
A	1	150.676	150.676	170.3271	4.711e-05	***
B	1	227.256	227.256	256.8949	1.722e-05	***
C	1	1.266	1.266	1.4307	0.285274	
D	1	0.456	0.456	0.5150	0.505084	
A:B	1	20.931	20.931	23.6604	0.004616	**
B:C	1	5.881	5.881	6.6476	0.049535	*
B:D	1	5.176	5.176	5.8506	0.060206	.
Residuals	5	4.423	0.885			
