Solution to Series 6

1. **a)** An experimenter wishes to compare four treatments in blocks of two runs. Find a BIBD with six blocks. We have:

$$
\begin{aligned}
n &= 4 \\
b &= 6 \\
k &= 2 \\
r &= \frac{kb}{n} = \frac{12}{4} = 3. \\
\lambda &= \frac{r(k-1)}{n-1} = 1
\end{aligned}
$$

We find the BIBD: (Note that $\lambda = 1$ implies that any combination of 2 factors can appear just once).

	1	2	3	4
1	x	x		
2	x		x	
3	x			x
4		x	x	
5		x		x
6			x	x

b) An experimenter wishes to compare seven treatments in blocks of three runs. Find a BIBD with seven blocks. We have:

$$
\begin{aligned}
n &= 7 \\
b &= 7 \\
k &= 3 \\
r &= \frac{kb}{n} = \frac{21}{7} = 3. \\
\lambda &= \frac{r(k-1)}{n-1} = 1
\end{aligned}
$$

We find the BIBD. (Note that $\lambda = 1$ implies that any combination of 2 factors can appear just once).

	1	2	3	4	5	6	7
1	x	x	x				
2	x			x	x		
3	x					x	x
4		x		x		x	
5		x			x		x
6			x	x			x
7			x		x	x	

2. Analyze these data in a split plot anova. First, draw the corresponding ANOVA skeleton by hand. Then, fit the data using R and interpret your results. Finally, Plot the data and answer: Is the new treatment significantly worst or better than the old one?

We have the following model:

Stratum	Source	df		F
Main plots	Treatment	1		$MS_{TR}/MSres - main$
	Residual	19		
	Total		20	
Sub-plots	Time	1		$MS_{Time}/MSres - sub$
	TR:Time	1		$MS_{TR:Time}/MSres - sub$
	Residual	19		$MS_{TR:Time}/MSres - sub$
	Total		21	
	Total		41	

With the R -function we obtain:

```
> Sh.fit <- aov(Y~Time*Treatment+Error(Subject/Time),data=Sh)
> summary(Sh.fit)

Error: Subject
          Df Sum Sq Mean Sq F value Pr(>F)
Treatment  1    847   847.5   3.627 0.0721 .
Residuals 19   4440   233.7
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Error: Subject:Time
               Df Sum Sq Mean Sq F value   Pr(>F)
Time            1  542.9   542.9   15.14 0.000982 ***
Time:Treatment  1  407.4   407.4   11.36 0.003209 **
Residuals      19  681.2    35.9
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

`Time` and interaction `Time:Treatment` are significant. A plot also shows that the new treatment improves response values after surgery, whereas the rates are unchanged with a standard operation. The new operation is therefore superior to the standard treatment.

3. A market investigation explores the potential of three new types of pizzas in six different packings. 90 consumers assess the products on a 0–10 scale. What type of design is used and how does the skeleton anova look like if

Let

$$\begin{aligned} A &= \textit{packing} \\ B &= \textit{pizza} \end{aligned}$$

a) each person rates the six packings of just one type of pizza,
This is a split plot design with persons as main plots and the ratings of different packings as subplots.

Strata	Source	df	MS	F
Person	B	2	MS_B	$MS_B/MS_{res-main}$
	Residual	87	$MS_{res-main}$	
Subplots	A	5	MS_A	$MS_A/MS_{res-sub}$
	AB	10	MS_{AB}	$MS_{AB}/MS_{res-sub}$
	Residual	435	$MS_{res-sub}$	
	Total	539		

b) each person rates exactly one pizza in one packing, This is a factorial design.

Source	df	MS	F
A	5	MS_A	MS_A/MS_{res}
B	2	MS_B	MS_B/MS_{res}
AB	10	MS_{AB}	MS_{AB}/MS_{res}
Residual	72	MS_{res}	
Total	89		

c) each person rates every pizza in every packing? This is a complete block design with persons as blocks.

Source	df	MS	F
Blocks	89	MS$_{blocks}$	
A	5	MS$_A$	MS$_A$/MS$_{res}$
B	2	MS$_B$	MS$_B$/MS$_{res}$
AB	10	MS$_{AB}$	MS$_{AB}$/MS$_{res}$
Residual	1513	MS$_{res}$	
Total	1619		

4. Using R and the function `lm` we obtain:

```
> d.st <- lm(formula=Pu~T1+Pr1,data=d)
> d.st$coefficients

(Intercept)          T1          Pr1
      84.10       -0.85         0.25
```

This can be interpreted as follows:

$$\hat{y} \quad = \quad 84.10 - 0.85 \cdot T + 0.25 \cdot P \ ,$$

By letting $\hat{y}$ constant we obtain an equation for the contour lines, i.e. contour lines satisfy the equation

$$P \quad = \quad \frac{0.85}{0.25} \cdot T + constant = m_0 T + c \ .$$

The direction of steepest ascent is then:

$$-\frac{1}{m_0} = -\frac{5}{17} \ .$$