Biochemical Experiment

Serum levels after four medical treatments. Only four people can be treated per day, one for each medication.

	Day							
	1	2	3	4	5	6	7	8
Treat.								
I	4.4	5.3	5.3	1.8	3.7	6.5	5.4	5.2
II	2.8	3.3	7.0	2.6	5.9	5.4	6.9	6.8
III	4.8	1.9	4.3	3.1	6.2	5.7	6.2	7.9
IV	6.8	8.7	7.2	4.8	5.1	6.7	9.3	7.9

Block Design

Subjects

Randomisation

↙ ↓ ↘

	Block 1	Block 2	...	Block J
Group 1	×	×		×
Group 2	×	×		×
Group 3	×	×		×
⋮	⋮	⋮	⋮	⋮
Group I	×	×		×

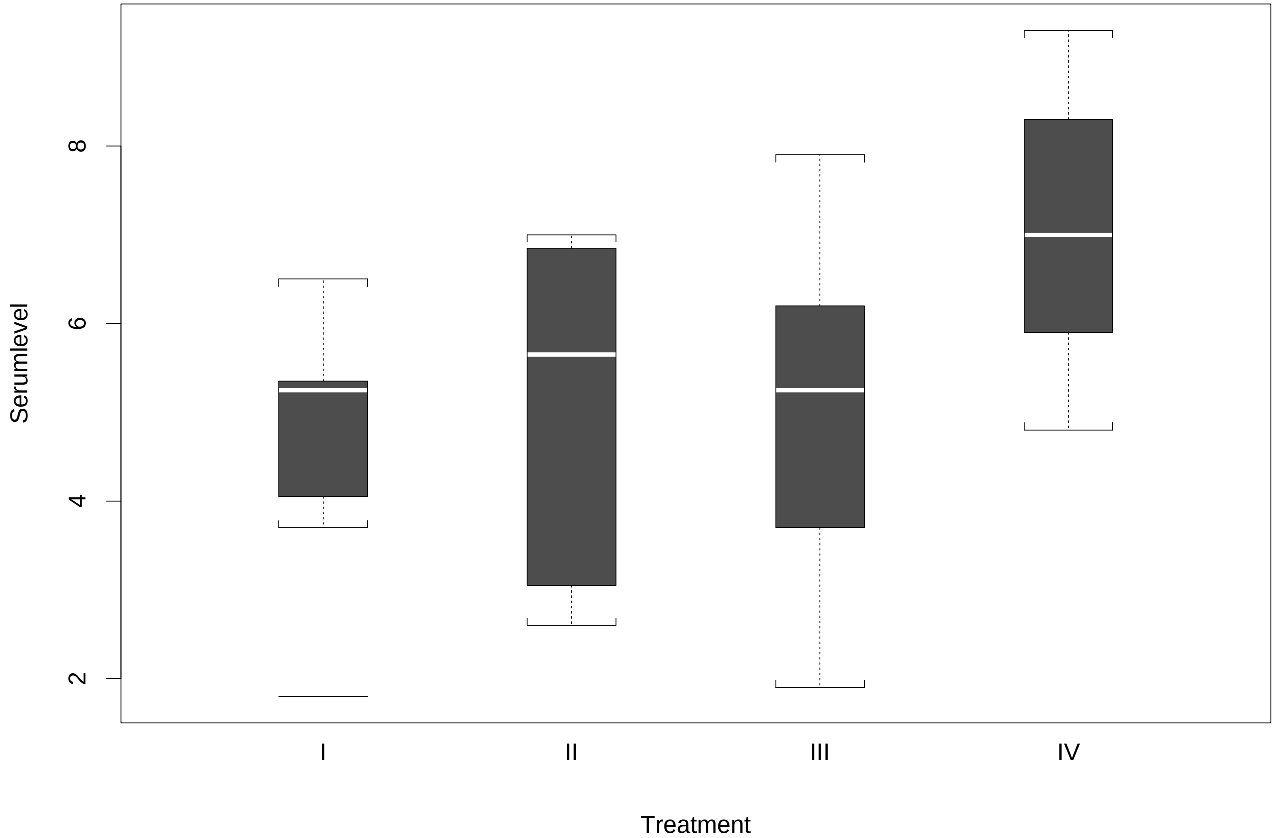
Block Randomisation

R: sample(rep(1:8,4)), sample(4) or sample(32)

Subjects

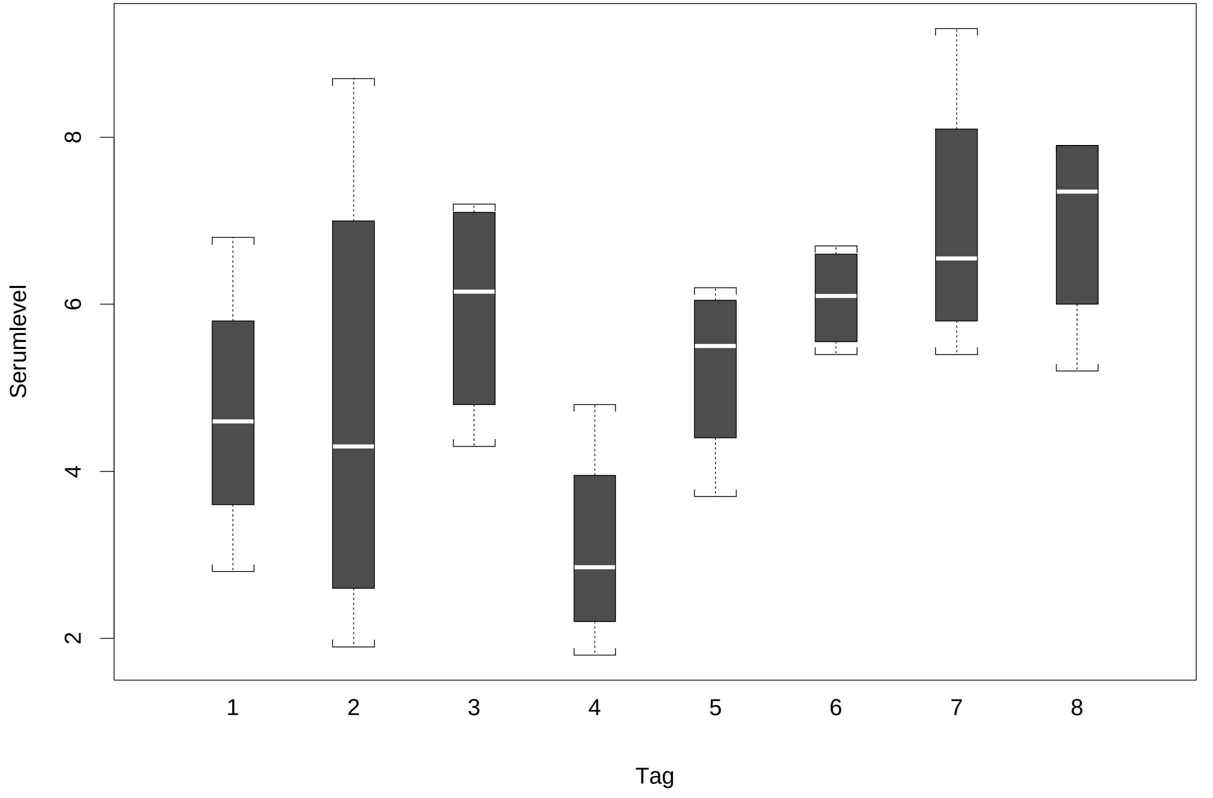
	Day							
Treatment	1	2	3	4	5	6	7	8
I	13	3	26	23	4	28	20	21
II	24	18	6	10	9	25	32	1
III	19	7	8	22	27	30	16	14
IV	2	11	15	12	31	17	29	5

Serum levels by Treatment



Mean: 4.7 5.09 5.01 7.06

Serum levels by Day



Mean: 4.7 4.8 5.95 3.08 5.23 6.07 6.95 6.95

Randomized Complete Block Design

- Each treatment in each block equally often.
- Model:

$$Y_{ij} = \mu + A_i + b_j + \epsilon_{ij} \tag{2}$$

  b_j: Effect of block j
- Fixed-Effects Model:
  $\sum A_i = 0,\ \sum b_j = 0,\ \epsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$

  Mixed Model:
  $\sum A_i = 0,\ b_j \sim \mathcal{N}(0, \sigma_b^2),\ \epsilon_{ij} \sim \mathcal{N}(0, \sigma_e^2)$
  all b_j and ϵ_{ij} independent.

  Random-Effects Model: all factors are random

Block effects: fixed or random?

fixed: a few levels, interest in levels themselves

random: levels are chosen from a population, interest in variability, blocks for reduction of variability

$$SS_{tot} = SS_{treat} + SS_{blocks} + SS_{res}$$

Source	SS	df	MS	F
Blocks	47.3	$J-1=7$	6.75	
Treatments	27.9	$I-1=3$	9.29	...
Residual	35.3	$(I-1)(J-1)=21$	1.68	
Total	110.4	$N-1=31$		

Expected mean squares

Fixed-effects model

$$
\begin{aligned}
E(MS_{res}) &= \sigma^2 \\
E(MS_{treat}) &= \sigma^2 + J\sum A_i^2/(I-1) \\
E(MS_{block}) &= \sigma^2 + I\sum b_j^2/(J-1)
\end{aligned}
$$

Mixed-effects model

$$
\begin{aligned}
E(MS_{res}) &= \sigma_e^2 \\
E(MS_{treat}) &= \sigma_e^2 + J\sum A_i^2/(I-1) \\
E(MS_{block}) &= \sigma_e^2 + I\sigma_b^2
\end{aligned}
$$

F Tests

Fixed-effects Model:

$H_0 : A_i = 0 \quad \forall i, \qquad F = \frac{MS_{treat}}{MS_{res}} \sim F_{I-1,(I-1)(J-1)}$

($H_0 : b_j = 0 \quad \forall j, \qquad F = \frac{MS_{blocks}}{MS_{res}} \sim F_{J-1,(I-1)(J-1)}$)

Mixed Model:

$H_0 : A_i = 0 \quad \forall i, \qquad F = \frac{MS_{treat}}{MS_{res}}$ as above

$H_0 : \sigma_b^2 = 0 \qquad F = \frac{MS_{blocks}}{MS_{res}}$ usually not tested

$MS_{blocks} >> MS_{res}$: Blocking good

$MS_{blocks} \leq MS_{res}$: Blocking not necessary

Example:

$F_A = 5.53$ Medication significant

$MS_{blocks} = 6.75 > MS_{res} = 1.68$ Blocking good