



The Distribution of Wars in Time Author(s): Lewis F. Richardson Source: Journal of the Royal Statistical Society, Vol. 107, No. 3/4 (1944), pp. 242-250 Published by: Blackwell Publishing for the Royal Statistical Society Stable URL: <u>http://www.jstor.org/stable/2981216</u> Accessed: 12/10/2011 06:14

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THE DISTRIBUTION OF WARS IN TIME

By LEWIS F. RICHARDSON

A LIST of "Wars of modern civilization" from 1480 to 1940 A.D. has been published by Quincy Wright (1942, Appendix XX). He states that his list:

"is intended to include all hostilities involving members of the family of nations, whether international, civil, colonial, or imperial, which were recognized as states of war in the legal sense or which involved over 50,000 troops. Some other incidents are included in which hostilities of considerable but lesser magnitude, not recognized at the time as legal states of war, led to important legal results such as the creation or extinction of states, territorial transfers, or changes of government."

As a preliminary to any statistical treatment of Wright's list, a decision had to be made concerning four of the largest wars, namely the Thirty Years War, the French Revolution, the Napoleonic Wars, and the First World War, because Wright lists them both as wholes and as parts. In order to make the wars in the collection less unequal in size, I included all the parts and omitted the wholes, except two wholes which are shown as beginning before any of their named parts. The First World War is thus reckoned as five wars.

Each calendar year can be characterized by the number 0, 1, 2, 3, 4 of wars which began in it, as shown in Appendix I. The number of years of each character has been counted (by three observers, M. W., E. D. R. and L. F. R.) with the following result:

ΤA	BL	E	I

Years from 1500 to 1931 A.D.

Number of outbreaks in the year Number of such years 223 Poisson law 216.2	142 149·7	48 51·8	3 15 12·0	4 4 2·1	>4 0 0·3	432 432·1
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By the Poisson law is here meant the statement that there were

$$Ne^{-\lambda}\lambda^{x}/x!$$
 (1)

years each containing exactly x outbreaks of war. The law was fitted to the observations by first equating N to the observed total number 432 years; and then determining λ by R. A. Fisher's Principle of Maximum Likelihood, according to which λ equals the mean number of outbreaks per year; that is

$$\lambda = 299/432$$
 (2)

It is seen at a glance that there is a considerable resemblance between the historical facts and the Poisson law. This resemblance suggests that we may perhaps find out something about the causes of wars by thinking of their outbreaks in connection with other phenomena known to be described at least approximately by the Poisson law. Such include: factory accidents, deaths by kick from a horse, and the emission of alpha particles from radioactive substances. In the standard deduction of the Poisson law from the

sequence of terms of $(p + q)^k$ where p + q = 1, by making pk constant while $k \rightarrow \infty$, there is no mention of time. We are at liberty to suppose that the occasions on each of which the probability of the event is p, are distributed in time as may best suit the phenomena under consideration. For the alpha particles it is suitable to suppose that the occasions are uniformly spread in time t, so that the probability of a particle escaping during the differential dt is λdt . For the wars that is not quite suitable, because there is a seasonal effect: wars in the north temperate zone have ordinarily begun in spring or summer (Wright, 1943, p. 224). Again for the wars we need not insist on there being infinitely many occasions. The limit as the index $k \rightarrow \infty$ is preferred in theory because it leads to a simple formula, and not because of any better agreement with observation than, say, k = 1000. But maybe k = 20 or 100 might be large enough to explain the observations on wars. The hypothesis so far is that in any year there were the same large number k of occasions on which a war might have broken out, and the same small probability p of its doing so on each occasion, so that

$$pk = \lambda$$
 (3)

This statistical and impersonal view of the causation of wars is in marked contrast with the popular belief that a war can usually be blamed on one or two named persons. But there are similar contrasts in other social affairs; the statistics of marriage, for example, are in contrast with any biographer's account of the incidents that led two named persons to marry each other.

The main result has now been stated. It is evidently of considerable interest, and may well move us to make a critical examination of details. Let us turn to such inter-related questions as: whether the deviations from the Poisson law are significant; whether Quincy Wright's definition of a war is the most suitable for statistical purposes; and whether λ fluctuates over periods such as a generation.

For the purpose of applying Karl Pearson's χ^2 test to the deviations between theory and observation as shown in Table I, the years containing 3 or more outbreaks have been grouped together, so as to avoid small frequencies, thus making four groups altogether. A slight approximation was admitted by not redetermining λ . Then $\chi^2 = 2.4$. There are two constraints, one for N and one for λ . So Fisher's number of degrees of freedom is 4 - 2 = 2. Accordingly $P(\chi^2) = 0.3$; indicating that the deviations can reasonably be dismissed as chance.

Which of the alternative theoretical foundations of the χ^2 test are historians likely to accept? Will their detailed knowledge of fact permit them to imagine a hypothetical infinite collection of wars, from which the actual wars are a random sample? Or will they be able to experience degrees of belief that behave according to reasonable quantitative rules? Personally, I have difficulties both ways, and hope for some new foundation, not yet revealed. In the meantime methods like χ^2 may almost be regarded as justified, apart from any fundamental theory, by their credible application to a great variety of phenomena.

Founding on the doctrine of reasonable belief, Jeffreys (1939, pp. 256 to 260) has provided a method for testing whether a distribution deviates significantly from the Poisson law in the direction of the negative binomial. This test can deal with small numbers; an advantage over χ^2 . Jeffreys' formula (p. 258 (20)), when applied to Table I, yields k = 2.77; and this, according to Jeffreys

(p. 357), indicates that we need not further consider the negative binomial. It was otherwise with factory accidents. (Greenwood and Yule, 1920, quoted by Kendall, 1943, pp. 124–126.)

In order to search for other types of deviation from the Poisson law it is desirable to group whole numbers of years together so as to form successive "cells" of equal duration. Let the duration of a cell be τ years. Let there be *n* consecutive cells.

For example Wright's data can be arranged thus:

n = 144 cells of $\tau = 3$ years each. 0 1 2 3 5 Total x ... 4 6 ≱7 18 39 37 29 12 5 2 2 f(x)0 144 37.5 38.9 26.9 14.0 5.8 2.0 0.6 0.2144.0 Poisson ... 18.1

TABLE II.

The row headed "Poisson" contains the numbers

$$\frac{N}{\tau}e^{-\tau\lambda}(\tau\lambda)^{x}/x! \qquad (6)$$

a formula which can be deduced from (3).

The Method of Maximum Likelihood gives $\tau \lambda = 299/n = 299\tau/432$ for every τ , so that λ *automatically* comes to the same value 299/432 whatever cells we take. But the agreement of f(x) with the Poisson law is not automatic, and so is a genuine test. In Table II the agreement again appears good to inspection.

The run of 432 years was chosen because $432 = 3^3 \times 2^4$, and so can be divided into cells of a great variety of durations. For example:

TABLE III

n = 8 cells of $\tau = 54$ years each.

Here f(x) vanishes for all except the following x.

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccc} 31 & 32 \\ 1 & 1 \end{array}$	35 42 1 1	44 1	45 1	54 1	Total 8
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In Table III f(x) is no longer a useful conception. It would be more convenient to regard the sample as specified by the eight numbers in the row headed x.

This type of specification can be made general. For any τ let the sample consist of the *n* numbers

For $\tau = 1$ many of the 432 numbers coincide; for example, 223 of them are zero; but that causes no unclarity.

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A concise test for rejecting a Poisson law.

The χ^2 test cannot be applied to compare the observations in Table III with their Poisson expectations, because all the numbers are small.

In a Poisson population the variance is equal to the mean. This equality is intimately related to rarity or improbability. For the Poisson law is the limit of the term-sequence of the expansion of the binomial $N(p + \chi)^k$ where p + q = 1 as $p \rightarrow 0$ and $pk \rightarrow \lambda$. But for the binomial the mean is kp and the variance kpq. So the equality of the mean and variance of a binomial law, for any k, entails that q = 1, p = 0; that is zero probability.

These considerations suggest that we should test our observed sample (x_1, x_2, \ldots, x_n) by forming the *statistic g defined by*

$$m = \frac{1}{n} \sum_{i=1}^{i=n} x_i, \qquad g = \frac{1}{m(n-1)} \sum_{i=1}^{i=n} (x_i - m)^2. \quad . \quad . \quad (8) \quad (9)$$

Then g has the following pleasant properties. All or most of them are known, but a collection may be helpful.

(ii) Provided that the mean *m* is large (say m > 5) the distribution of (n-1)g in samples from a Poisson population is the same as that of χ^2 for n-1 degrees of freedom. More explicitly let $L_{>g}$, in which the inequality sign appears in the suffix, denote the probability that in samples of *n* from a Poisson population, *g* would be greater than its observed value . . (11) Then

$$L_{>q} = P(\chi^2, n-1)$$
 (12)

where $\chi^2 = (n-1)g$ and P is the probability for n-1 degrees of freedom; given, say, in Fisher's table.

It is thus quite distinct from the χ^2 applied above to Table I. Both Fisher (1936, p. 60) and Jeffreys (1939, p. 259) mention this special form, namely, $\chi^2 = \frac{1}{m} \sum_i (x_i - m)^2$ in connection with the Poisson law. The simple hypothesis (13) is certainly consistent with the Poisson law, but can hardly be said to be equivalent to it; for the type of chance deviations is not specified in (13). Yet there is another hidden connection. For the Poisson law, or something approximately equivalent to it, is used in some proofs of $P(\chi^2)$ (e.g., Kendall, 1943, pp. 290–291). The forms and usages of χ^2 are so multifarious that in order to avoid confusion it is desirable to have here a distinctive symbol such as g defined in (9).

(iv) Just where the property (ii) is lacking because the mean m is not large, the gap is made good because in the present data n is then large, and we can prove that for samples from a Poisson population

var
$$g = \frac{2}{n}$$
, for large n (14)

lall's (1943) "Advanced Theory of Statistics," Ch. 9.

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(v) Jeffreys' test for a deviation from the Poisson law towards the negative binomial is a function of n and g, as he indicates (1939, pp. 258–9).

(vi) There are various simple relations between g and a coefficient Q^2 introduced by Lexis, but variously interpreted by his successors (Keynes, 1921, Ch. XXXII). Lexis included a factor to represent (1 - p) in the binomial variance. This factor is purposely omitted from g. Irwin (1932, p. 506-8) explains the relation of Q^2 to one of the χ^2 . Fisher (1936, p. 83) puts Q where the others put Q^2 .

(vii) If the observed g is incredible for a sample of n from a Poisson population then the Poisson law must be rejected. But conversely, if g is credible, all that we can say is that so far there is no objection to a Poisson law; a more searching test might still reject it.

Resumed discussion of the distribution of wars among cells of various durations.

In Table III the distribution among cells of 54 years gives g = 3.56, $L_{>g} < 0.001$. We must therefore reject both the Poisson law, and the simple law (13) of uniformity apart from chance. Some historical explanation of the departures from uniformity, or at least of their extremes, is therefore required. The 54 year cells that contained most and least outbreaks, extended from 1824 to 1877 A.D. for most, and from 1716 to 1769 A.D. for least.

For the same data arranged as in Table I we have g = 1.093 for n = 432. But by (10) and (14) we expect, from a Poisson population, $g = 1 \pm \sqrt{\frac{2}{432}} = 1 \pm 0.068$, the number after the ambiguous sign being the standard deviation. The hypothesis (13) of uniformity-apart-from-chance, and the Poisson law are so far both quite credible for cells of $\tau = 1$ year. We have indeed already verified the Poisson law more thoroughly by a different χ^2 applied to Table I.

How is it that deviations from uniformity-apart-from-chance show so clearly in 54 year cells, but hardly at all in 1 year cells? For much the same reason that the fat and the lean become inconspicuous when the meat is minced.

To push this type of analysis to its extreme in the other direction let us enquire whether wars on the average became more frequent.

TABLE IV

n = 2 cells of $\tau = 216$ years each.

	Dates	1500 to 1715 a.d.	1716 to 1931 a.d.
x		$x_1 = 143$	$x_2 = 156$

Here g = 0.565 for one degree of freedom, and so by (12) and Yule's table (Kendall, 1943, p. 444), $L_{>g} = 0.45$.

That is to say Wright's list indicates an absence of any steady drift towards more or fewer wars

Cells of some other durations have also been examined by the same tests. The results are collected in the following *summary*.

TABLE V

Number of cells, n	Duration of each cell, $ au$ years	g	$L_{>g}$	Was distribution uniform apart from chance ?	Did Poisson law hold ?
2 4 8 16 48 144 432	216 108 54 27 9 3 1	0.565 3.29 3.56 2.249 1.538 1.058 1.093	$\begin{array}{c} 0.45 \\ 0.021 \\ < 0.001 \\ 0.00_3 \\ 0.010 \\ * \\ 0.28 \\ 0.09 \end{array}$	yes unlikely no no yes credible	evidence lacking unlikely no no yes yes

Distribution of 299 outbreaks of war among the 432 years extending from 1500 A.D. to 1931 A.D. according to Ouincy Wright's list.

* By Wilson and Hilferty's extension of $P(\chi^2)$ tables.

The search for periods differs from the present type of analysis in various To begin with, the former is applied to the sums of the columns, respects. but the latter to the sums of the rows, of a Buys-Ballot table (Kendall, 1945, Art. 3). Nevertheless, a brief allusion to periods may be in place here.

Quincy Wright (1942, Ch. IX) has discussed fluctuations in the intensity of war. He remarks (p. 230) that: "A fifty-year war period has often been noticed." But he does not mention any test of its statistical significance. The need for caution concerning this alleged period has been emphasized by Kendall (1945, Art. 57).

Another collection of wars, defined more objectively

What one person might call a war, another might call a mutiny, or even an incident. The great obstacle to any scientific study of quarrels is contradictory evidence from the opposing sides. Wright's definition of a war involves the notions of its "legality" and "political importance" which are always matters of opinion, and can be permanently controversial. By contrast, the number of persons who died because of the quarrel, although often deliberately mis-stated, is in almost all cases ascertainable by ten years after the end of the fighting within a factor of three times more or less, which is small in comparison with the whole range. The war dead were taken to include all those, on both sides, whether armed personnel or civilians, who were killed fighting, or drowned by enemy action, or who died of wounds or from poison gas, or from starvation in a siege, or from other malicious acts of their enemies. Moreover, deaths from disease or exposure of armed personnel during a campaign were included: but not civilian deaths by epidemic disease in places far distant from the geographical location of the fighting. Before I saw Wright's list I had almost completed a card-index of fatal quarrels classified according as the number of war-dead were of the order of 10⁷ or 10⁶ or 10⁵ or 10⁴. The last group, which is the most numerous, concerns us now. Its precise range is intended to be from 10^{3·5} to 10^{4·5} in war-dead. In popular language it consists of small wars, revolts, insurrections and incidents, in any part of the world, regardless of their legality or their political importance. They were collected from sources too numerous to be specified here, but a list of the dates is set out in Appendix II. Wright's list, which begins in 1480 A.D., has the great merit of being longer than mine, which begins only with 1820 A.D. Where к

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the two lists overlap, there are many divergencies between them, both of inclusion and exclusion. Yet Poisson's law shows in both. The 110 calendar years from 1820 to 1929 contained on my list 63 beginnings and, as it chanced, also 63 endings of fatal quarrels for which the war-dead were more than $10^{3\cdot 5}$ and fewer than $10^{4\cdot 5}$. When these are sorted, as in Table I, into cells of one year the result is:

TABLE	٧I
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$x \dots \dots \\ f(x) \begin{cases} \text{beginnings} \\ \text{ends} \dots \end{cases}$ Poisson	···· ·	$ \begin{array}{c cccc} & 0 \\ & 63 \\ & 62 \\ & 62 \cdot 0 \end{array} $	$ \begin{array}{c} 1 \\ 35 \\ 34 \\ 35 \cdot 5 \end{array} $	2 9 13 10·2	3 2 1 1·9	4 1 0 0·3	>4 0 0 0·0	sum 110 110 109· 9
-----------------------------------------------------------------------------------------------------	--------	---------------------------------------------------------------------------	----------------------------------------------------------------------------------------------	----------------------	--------------------	--------------------	---------------------	------------------------------------

The agreement with Poisson's law of improbable events draws our attention to the existence of a persistent background of probability. If the beginnings of wars had been the only facts involved, we might have called it a background of pugnacity. But, as the ends of wars have the same distribution, the background appears to be composed of a restless desire for change.

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APPENDICES

The following raw material has been provided at the request of Dr. L. Isserlis so as to enable any reader to make his own analyses.

APPENDIX I. Extract from Prof. Quincy Wright's list (1942) of Wars of Modern Civilization.

The columns are in pairs. The first column of each pair contains the date A.D.; the second column contains the number of wars which began in that calendar year. Much further information such as names of wars, names of belligerents, names of treaties of peace, dates of ending, number of battles, and type of war, can be seen in Wright's book.

A.D.		A.D.		A.D.		A.D.		A.D.		A.D.		A.D.	
1482 83 84 85 86 87 88 89	1 0 0 0 0 0 0 0	1490 91 92 93 94 95 96 97	0 0 1 0 0 1 0 0	98 99 1500 01 02 03 04 05	0 0 0 0 0 0 0 0	06 07 08 09 1510 11 12 13	0. 0 1 2 2 1 0	14 15 16 17 18 19 1520 21	0 1 1 0 0 0 1 2	22 23 24 25 26 27 28 29	1 0 1 0 1 0 0 0	1530 31 32 33 34 35 36 37	0 2 2 1 1 0 1 1

APPENDIX I.—Continued.

A.D.	A.D.		A.D.		A.D.		A. D.		A.D.		A.D.	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 96\\ 97\\ 98\\ 99\\ 1600\\ 01\\ 02\\ 03\\ 04\\ 05\\ 06\\ 07\\ 08\\ 09\\ 1610\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 16\\ 21\\ 223\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ 1630\\ 31\\ 32\\ 33\\ 34\\ 35\\ 36\\ 37\\ 38\\ 39\\ 1640\\ 41\\ 42\\ 44\\ 45\\ 46\\ 447\\ 48\\ 49\\ 1650\\ 51\\ 22\\ 53\\ 36\\ 37\\ 38\\ 39\\ 1640\\ 41\\ 42\\ 45\\ 51\\ 52\\ 53\\ 36\\ 36\\ 37\\ 38\\ 39\\ 1640\\ 41\\ 42\\ 44\\ 45\\ 51\\ 52\\ 53\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 38\\ 36\\ 36\\ 37\\ 38\\ 38\\ 38\\ 36\\ 36\\ 36\\ 37\\ 38\\ 38\\ 38\\ 36\\ 36\\ 36\\ 36\\ 36\\ 36\\ 36\\ 36\\ 36\\ 36$	$\begin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ 4 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1$	$\begin{array}{c} 54\\ 55\\ 56\\ 57\\ 58\\ 59\\ 1660\\ 61\\ 62\\ 63\\ 64\\ 65\\ 66\\ 67\\ 68\\ 69\\ 1670\\ 71\\ 72\\ 73\\ 74\\ 75\\ 76\\ 77\\ 78\\ 79\\ 1680\\ 81\\ 82\\ 83\\ 84\\ 85\\ 86\\ 87\\ 88\\ 89\\ 91690\\ 91\\ 92\\ 93\\ 94\\ 95\\ 96\\ 97\\ 78\\ 88\\ 89\\ 99\\ 1700\\ 01\\ 10\\ 02\\ 03\\ 04\\ 05\\ 06\\ 07\\ 08\\ 99\\ 1710\\ 01\\ 11\\ 11\\ 11\\ 11\\ 11\\ 11\\ 11\\ 11\\ $	$\begin{array}{c} 2 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	$\begin{array}{c} 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 1720\\ 21\\ 222\\ 23\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ 1730\\ 31\\ 32\\ 33\\ 34\\ 35\\ 36\\ 37\\ 38\\ 9\\ 1740\\ 41\\ 422\\ 43\\ 36\\ 57\\ 58\\ 57\\ 58\\ 57\\ 58\\ 59\\ 1760\\ 61\\ 62\\ 66\\ 67\\ 68\\ 69\\ \end{array}$	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	$\begin{array}{c} 1770\\ 71\\ 72\\ 73\\ 74\\ 75\\ 76\\ 77\\ 78\\ 79\\ 1780\\ 81\\ 82\\ 83\\ 845\\ 86\\ 87\\ 88\\ 89\\ 1790\\ 91\\ 923\\ 94\\ 95\\ 96\\ 97\\ 98\\ 999\\ 1800\\ 01\\ 10\\ 20\\ 3\\ 04\\ 05\\ 06\\ 07\\ 98\\ 999\\ 1800\\ 01\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 1820\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ \end{array}$	$\begin{array}{c} 2 \\ 0 \\ 0 \\ 1 \\ 0 \\ 2 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0$	$\begin{array}{c} 28\\ 29\\ 1830\\ 31\\ 32\\ 33\\ 34\\ 435\\ 36\\ 37\\ 38\\ 39\\ 1840\\ 41\\ 423\\ 444\\ 455\\ 466\\ 47\\ 48\\ 49\\ 1850\\ 51\\ 52\\ 53\\ 545\\ 556\\ 57\\ 58\\ 59\\ 1860\\ 61\\ 623\\ 64\\ 65\\ 667\\ 68\\ 69\\ 1870\\ 711\\ 72\\ 73\\ 74\\ 75\\ 76\\ 77\\ 78\\ 9\\ 1880\\ 81\\ 82\\ 83\\ 84\\ 85\\ \end{array}$	$\begin{array}{c}1\\0\\3\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\$	86 87 88 99 92 93 94 95 96 97 98 99 99 1900 01 02 03 04 05 06 07 08 09 1910 11 12 13 14 15 167 17 22 23 24 5 26 27 28 91 930 31 32 334 35 36 37 38 39	$\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 &$

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APPENDIX II. Dates of beginning and ending of fatal quarrels which caused more than $10^{3\cdot 5}$ but fewer than $10^{4\cdot 5}$ deaths.

The columns are arranged in triplets, in each of which the first column contains the date A.D., the second column shows the number of fatal quarrels of the specified magnitude which began, and the third column the number which ended, in that calendar year.

The following list does not, and ought not to, agree with Wright's list; for its definition is different. Many further particulars await publication.

A.D.			A.D.			A.D.			A.D.		
$\begin{array}{c} 1820\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ 1830\\ 31\\ 32\\ 33\\ 34\\ 35\\ 36\\ 37\\ 38\\ 39\\ 1840\\ 41\\ 42\\ 43\\ 44\\ 45\\ 46\\ 47\\ 48\\ 49\\ \end{array}$	0 1 1 1 0 0 1 1 0 1 0 1 0 1 0 1 0 1 0 1	0 0 0 1 0 3 2 0 0 0 1 0 1 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\begin{array}{c} 1850\\ 51\\ 52\\ 53\\ 54\\ 55\\ 56\\ 67\\ 58\\ 59\\ 1860\\ 61\\ 62\\ 63\\ 64\\ 65\\ 66\\ 67\\ 78\\ 79\\ 73\\ 74\\ 75\\ 76\\ 77\\ 78\\ 79\end{array}$	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 2 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1$	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\$	1880 81 82 83 84 85 86 87 88 89 1890 91 92 93 94 95 96 97 98 99 1900 01 02 03 04 05 06 07 08 09	$\begin{array}{c} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\$	$ \begin{array}{c} 1\\ 1\\ 0\\ 2\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 1\\ 0\\ 0\\ 0\\ 0\\ 1\\ 0\\ 0\\ 0\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\$	1910 11 12 13 14 15 16 17 18 190 1920 21 22 23 24 25 26 27 28 29 1930 31 32 33 34 35 36 37	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	1 0 0 1 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0