# Solution to Series 6

**1. a)** An experimenter wishes to compare four treatments in blocks of two runs. Find a BIBD with six blocks. We have:

$$
\begin{aligned}
n &= 4 \\
b &= 6 \\
k &= 2 \\
r &= \frac{kb}{n} = \frac{12}{4} = 3. \\
\lambda &= \frac{r(k-1)}{n-1} = 1
\end{aligned}
$$

We find the BIBD: (Note that $\lambda = 1$ implies that any combination of 2 factors can appear just once).

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | x | x | | |
| 2 | x | | x | |
| 3 | x | | | x |
| 4 | | x | x | |
| 5 | | x | | x |
| 6 | | | x | x |

**b)** An experimenter wishes to compare seven treatments in blocks of three runs. Find a BIBD with seven blocks. We have:

$$
\begin{aligned}
n &= 7 \\
b &= 7 \\
k &= 3 \\
r &= \frac{kb}{n} = \frac{21}{7} = 3. \\
\lambda &= \frac{r(k-1)}{n-1} = 1
\end{aligned}
$$

We find the BIBD. (Note that $\lambda = 1$ implies that any combination of 2 factors can appear just once).

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | x | x | x | | | | |
| 2 | x | | | x | x | | |
| 3 | x | | | | | x | x |
| 4 | | x | | x | | x | |
| 5 | | x | | | x | | x |
| 6 | | | x | x | | | x |
| 7 | | | x | | x | x | |

**2.** Analyze these data in a split plot anova. Are you comfortable with the assumptions?

We have the following model:

| Stratum | Source | df | F |
|---|---|---|---|
| Main plots | Treatment | 1 | $MS_{TR}/MSres - main$ |
| | Residual | 19 | |
| | Total | 20 | |
| Sub-plots | Time | 1 | $MS_{Time}/MSres - sub$ |
| | TR:Time | 1 | $MS_{TR:Time}/MSres - sub$ |
| | Residual | 19 | $MS_{TR:Time}/MSres - sub$ |
| | Total | 21 | |
| | Total | 41 | |

With the R -function we obtain:

```
> Sh.fit <- aov(Y~Time*Treatment+Error(Subject/Time),data=Sh)
> summary(Sh.fit)

Error: Subject
          Df Sum Sq Mean Sq F value Pr(>F)
Treatment  1    847   847.5   3.627 0.0721 .
Residuals 19   4440   233.7
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Error: Subject:Time
               Df Sum Sq Mean Sq F value   Pr(>F)
Time            1  542.9   542.9   15.14 0.000982 ***
Time:Treatment  1  407.4   407.4   11.36 0.003209 **
Residuals      19  681.2    35.9
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

`Time` and interaction `Time:Treatment` are significant. A plot also shows that the new treatment improves response values after surgery, whereas the rates are unchanged with a standard operation. The new operation is therefore superior to the standard treatment.

**3.** A market investigation explores the potential of three new types of pizzas in six different packings. 90 consumers assess the products on a 0–10 scale. What type of design is used and how does the skeleton anova look like if

Let

$$\begin{aligned} A &= packing \\ B &= pizza \end{aligned}$$

**a)** each person rates the six packings of just one type of pizza,
This is a split plot design with persons as main plots and the ratings of different packings as subplots.

| Strata | Source | df | MS | F |
|---|---|---|---|---|
| Person | B | 2 | $MS_B$ | $MS_B/MS_{res-main}$ |
| | Residual | 87 | $MS_{res-main}$ | |
| Subplots | A | 5 | $MS_A$ | $MS_A/MS_{res-sub}$ |
| | AB | 10 | $MS_{AB}$ | $MS_{AB}/MS_{res-sub}$ |
| | Residual | 435 | $MS_{res-sub}$ | |
| | Total | 539 | | |

**b)** each person rates exactly one pizza in one packing, This is a factorial design.

| Source | df | MS | F |
|---|---|---|---|
| A | 5 | $MS_A$ | $MS_A/MS_{res}$ |
| B | 2 | $MS_B$ | $MS_B/MS_{res}$ |
| AB | 10 | $MS_{AB}$ | $MS_{AB}/MS_{res}$ |
| Residual | 72 | $MS_{res}$ | |
| Total | 89 | | |

**c)** each person rates every pizza in every packing? This is a complete block design with persons as blocks.

| Source | df | MS | F |
|---|---|---|---|
| Blocks | 89 | $\text{MS}_{blocks}$ | |
| A | 5 | $\text{MS}_A$ | $\text{MS}_A/\text{MS}_{res}$ |
| B | 2 | $\text{MS}_B$ | $\text{MS}_B/\text{MS}_{res}$ |
| AB | 10 | $\text{MS}_{AB}$ | $\text{MS}_{AB}/\text{MS}_{res}$ |
| Residual | 1513 | $\text{MS}_{res}$ | |
| Total | 1619 | | |

**4.** Using R and the function `lm` we obtain:

```
> d.st <- lm(formula=Pu~T1+Pr1,data=d)
> d.st$coefficients

(Intercept)          T1          Pr1
      84.10       -0.85         0.25
```

This can be interpreted as follows:

$$\hat{y} \quad = \quad 84.10 - 0.85 \cdot T + 0.25 \cdot P \ ,$$

By letting $\hat{y}$ constant we obtain an equation for the contour lines, i.e. contour lines satisfy the equation

$$P \quad = \quad \frac{0.85}{0.25} \cdot T + constant = m_0 T + c \ .$$

The direction of steepest ascent is then:

$$-\frac{1}{m_0} = -\frac{5}{17} \ .$$