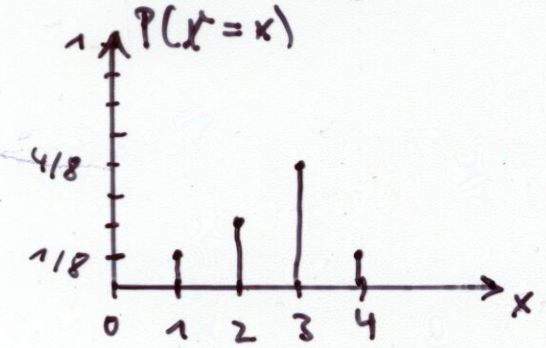
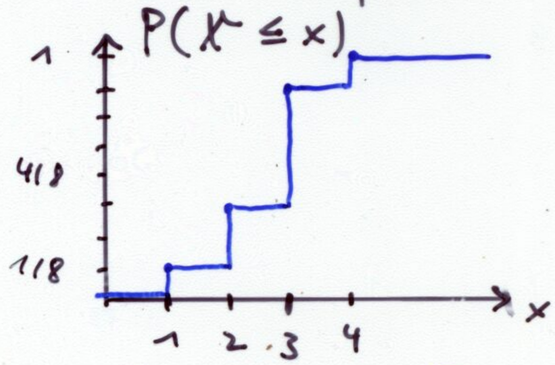
Bisher: Diskrete Werte

Wa.verteilung — Kum. Vert. fkt.





z.B. $P(X=3) = 4/8$ — z.B. $P(X \leq 3) = 7/8$

Neu: Kontinuierliche Werte

Problem:

ZV X_0 uniform auf Wertebereich $W_0 = \{0, 1, \ldots, 9\} \Rightarrow P(X_0 = x) = 1/10$

X_1 —"— $W_1 = \{0{,}0;\ 0{,}1;\ \ldots;\ 9{,}9\} \Rightarrow P(X_1 = x) = 1/100$

X_2 —"— $W_2 = \{0{,}00;\ 0{,}01;\ \ldots;\ 9{,}99\} \Rightarrow P(X_2 = x) = \frac{1}{1000}$

$\vdots$

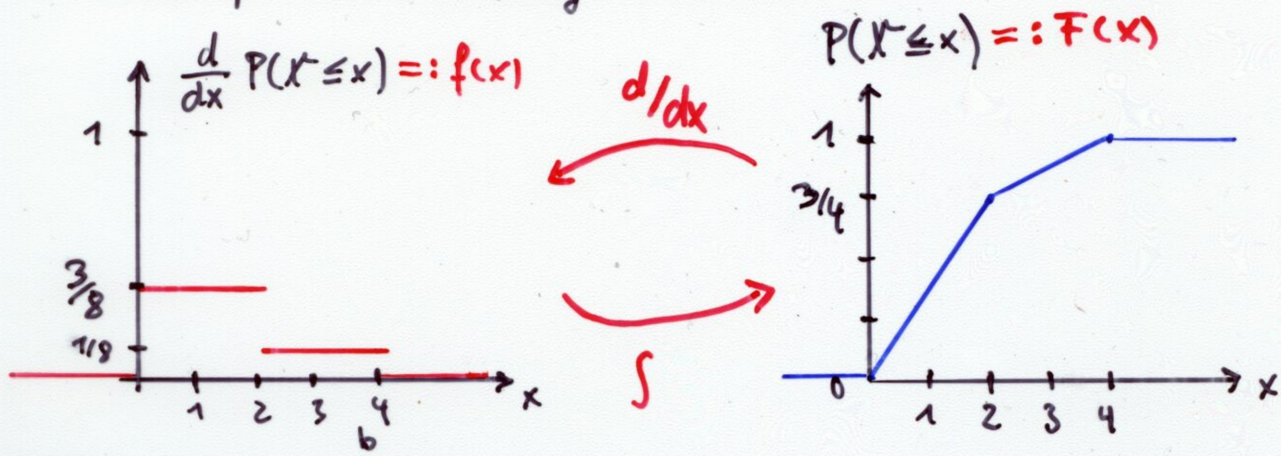
X_i —"— $P(X_i = x) = \frac{1}{10^{i+1}}$

X_∞ $W_\infty = [0, 10]$ $P(X_\infty = x) = 0$ ↯

⇒ Wa. verteilung ist nutzlos bei kontinuierlichen Werten

Aber: Kum. Vert. fkt. ist weiter OK

Ersatz für Wa.verteilung: Wa.dichte



$P(a \leq X \leq b) = \int_a^b f(x)\,dx \iff P(a \leq X \leq b) = P(X \leq b) - P(X \leq a)$

Kennzahlen

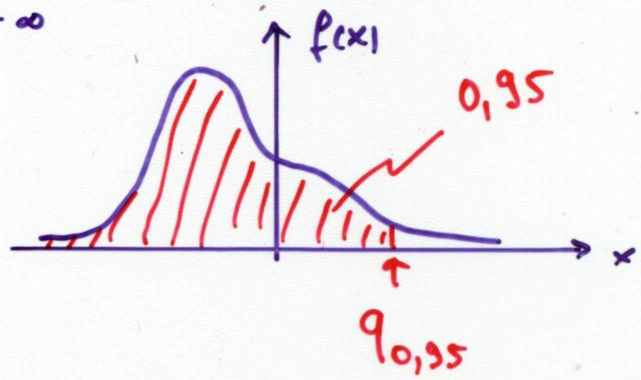
(wie im diskreten Fall; $\Sigma \to \int$)

$$E[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$Var(X) = E\left[(X - E(X))^2\right] = \int_{-\infty}^{\infty} (x - E[X])^2 f(x)\, dx$$

$$\sigma_X = \sqrt{Var(X)}$$

α-Quantil: $q_\alpha = F^{-1}(\alpha)$



Spezialfälle:

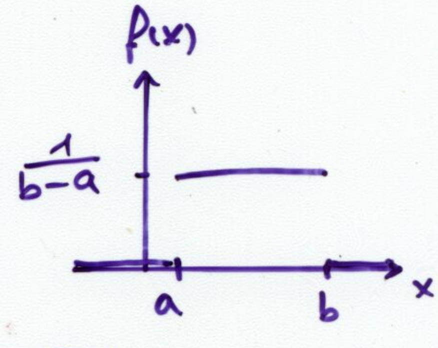
z_α: α-Quantil der Normalverteilung

$t_{\alpha;\, df}$: α-Quantil der t-Verteilung mit df Freiheitsgraden

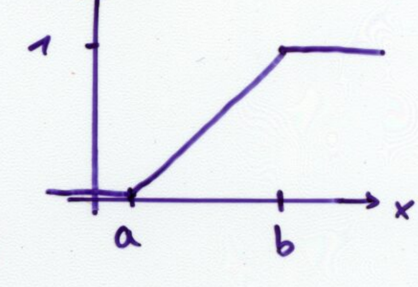
Beispiele

1) Uniforme Verteilung $X \sim Unif(a, b)$

$$f(x) = \begin{cases} 1/(b-a) & \text{falls } a \le x \le b \\ 0 & \text{sonst} \end{cases}$$



$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$

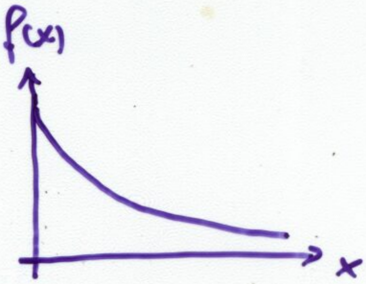


Bsp: Wartezeit auf Zürich-Tram: Unif(0, 7)

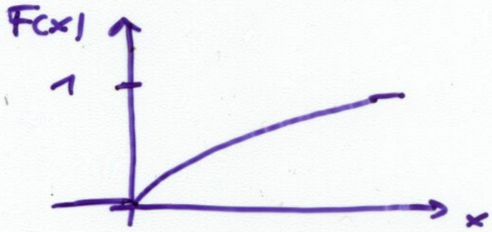
2) Exponentialverteilung $X \sim Exp(\lambda)$

„Wartezeit in Schlange"

$$f(x) = \begin{cases} \lambda \exp(-\lambda x) & x \geq 0 \\ 0 & x < 0 \end{cases}$$



$$F(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

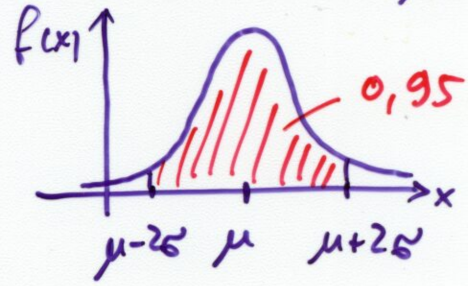


Bsp: Wartezeit bis radioaktiver Atomkern zerfällt

3) Normalverteilung (Gauss'sche Glockenkurve) $X \sim \mathcal{N}(\mu, \sigma^2)$

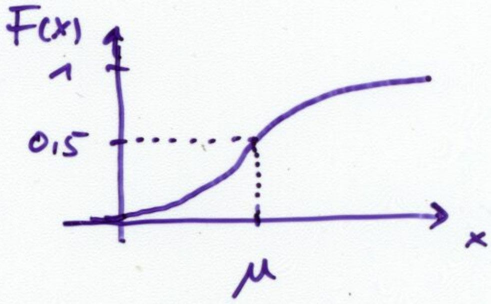
„Messfehler"

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



$F(x) =$? => Tabelle für $\mathcal{N}(0,1)$

od. Computer



Bsp: Messfehler

Standard-Normalverteilung: $\mathcal{N}(0,1)$ $(\mu = 0, \sigma = 1)$

Dichte: $\varphi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$

Kum. Vertflkt: $\Phi(x) = \int_{-\infty}^{x} \varphi(y)\, dy$ → tabelliert

Konvention: Std. normalverteilte Zufallsvariablen werden mit Z bezeichnet