## Series 7

## 1. Poisson Regression

In an experiment one is interested in assessing the concentration of nematodes (a species of worms) in a certain liquid. Three samples of $20 \mu \mathrm{l}$ each were generated and thinned with an equal amount of water. From each of the thinned probes the researchers generated 15 subsamples with a concentration of $40 \mu \mathrm{l}, 20 \mu \mathrm{l}$ and $20 \mu \mathrm{l}$, respectively. The table shows the counted number of nematodes for each of the 45 subsamples.

| Sample | volume | Number of nematodes in each subsample |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $40 \mu \mathrm{l}$ | 31 | 28 | 33 | 38 | 28 | 32 | 39 | 27 | 28 | 39 | 21 | 39 | 45 | 37 | 41 |
| 2 | $20 \mu \mathrm{l}$ | 14 | 16 | 18 | 9 | 21 | 21 | 14 | 12 | 13 | 13 | 14 | 20 | 24 | 15 | 24 |
| 3 | $20 \mu \mathrm{l}$ | 18 | 13 | 19 | 14 | 15 | 16 | 14 | 19 | 25 | 16 | 16 | 18 | 9 | 10 | 9 |

a) Propose a regression model for the dependent variable "number of nematodes" and the independent variable sample.
b) Is there any difference among the three samples?
c) Could one also use the log-volume instead of the three groups as explanatory variable? How would the corresponding model look like?
d) Would $\lambda=c \cdot v o l$ also be an appropriate model? Why? Hint: Is the coefficient of $\log ($ vol $)$ significantly different from 1 ?
e) Calculate a model in which you fix $\log (\mathrm{vol})$ at 1 .

Hint: With the offset comand you can constrain the coefficient of a covariate to 1. E.g., when writing offset $\left(x_{1}\right)$ instead of $x_{1}$ in the model formula in R the coefficient of $x_{1}$ is fixed at 1 and not estimated.
Compare the different models.

## 2. Multinomial Logit Model

Do people with the freedom to choose a different investment strategy for their pension make that choice when they have a saying in that investment strategy? The data set pension.dta can be read into $R$ with the read.dta command from the library foreign. The link for the data set is

```
read.dta("http://fmwww.bc.edu/ec-p/data/wooldridge2k/PENSION.DTA")
```

The data set contains observations of 226 subjects on the following variables.

```
            id Identification number of the person
    pyears Number of years in pension plan
            age Age in years
    choice Freedom to choose investment strategy 1=yes, 0=no
    prftshr Profit sharing plan 1=yes, 0=no
    female Sex 1=female 0=male
married married 1=yes, 0=no
    black Ethnic background 1=African-American 0= not African-American
    educ Years of education in school
    finc25 Income }\leq25,000
            ... ...
wealth89 Net assets 1989 in 1000 $
    pctstk Investment strategy 0=mainly obligations, 50= mixed, 100=mainly stocks
```

a) What is the relationship between freedom to choose the investment strategy and the de-facto chosen investment strategy whilst neglecting all other variables?
b) Is there a relationship between income and investment strategy?
c) Construct a new income variable with three levels: Levels: up to $25,000,25,001$ up to 50,0000 , over 50,000.
Hint: Partitioning a continuous variable into several categories is convenient for modelling a non-linear effects.
d) Fit a nominal logit model with pctstk $=50$ as reference category.
e) Is the variable choice significant? Interpret the coefficients of choice using odds.
f) How large is the probability for each of the three investment strategies to be chosen for a 60 year old white male, single, with 13.5 years of school education, an income of over $50,000 \$$, net assets of $200,000 \$$ and a profit-sharing plan in the two cases that he either does or does not have the freedom to choose his investment strategy.

## 3. Logistic Regression for Binary Data

A car manufacturer instructed a market research company to analyze which families are going to buy a new car next year using a logistic regession model. Data stems from a random sample of 33 families from an agglomeration area. Assessed variables cover the yearly household income (in 1000 US \$) and the age of the oldest car in the family (in years). 12 months later, interviewers assessed which families had bought a new car in the meantime. The data is available in the file car.dat and can be read in with following command.

```
read.table("http://stat.ethz.ch/Teaching/Datasets/car.dat",header=T)
```

a) Perform a logistic regression. Report the regression equation.
b) Estimate $\exp \hat{\beta}_{\text {income }}$ and $\exp \hat{\beta}_{\text {age }}$ and interpret the values.
c) How large is the estimated probability that a family with a yearly household income of 50000 US $\$$ and whose oldest car is 3 years old will buy a new car?
d) Do the residual plots show any abnormalities?
e) Is the variable age required in the model?
f) Is there a non-negligible interaction between income and age?

## 4. Logistic Regression for Binomial Data

In this task we analyze the example concerning hypertension from Altman (1991). First, we need to enter the data. This is done as follows:

```
> no.yes <- c("No", "Yes")
> smoking <- gl(2,1,8, no.yes)
> obesity <- gl(2,2,8, no.yes)
> snoring <- gl(2,4,8, no.yes)
> n.total <- c(60, 17, 8, 2, 187, 85, 51, 23)
> n.hyper <- c(5, 2, 1, 0, 35, 13, 15, 8)
```

Here, the function gl creates a factor with given levels. The factors smoking, obesity and snoring have an obvious meaning. n.total is the number of observations and n.hyper is the number of people with hypertension in each group.
a) In order to fit a binomial logistic regression model construct a response matrix with two columns containing the number of people with and without hypertension, respectively.
b) Fit a binomial logistic regression model to the data.
c) Does this model fit well? Assess the goodness-of-fit via the residual deviance.
d) Which variables significantly influence the occurence of hypertension?
e) Try to find a suitable model. Perform likelihood-ratio tests to achieve this goal.
f) Compare the observed and fitted proportions for hypertension under model e). What is striking here? Additionally, calculate the expected and observed counts.

Preliminary discussion: Monday, December 12.
Deadline: Monday, December 19.

