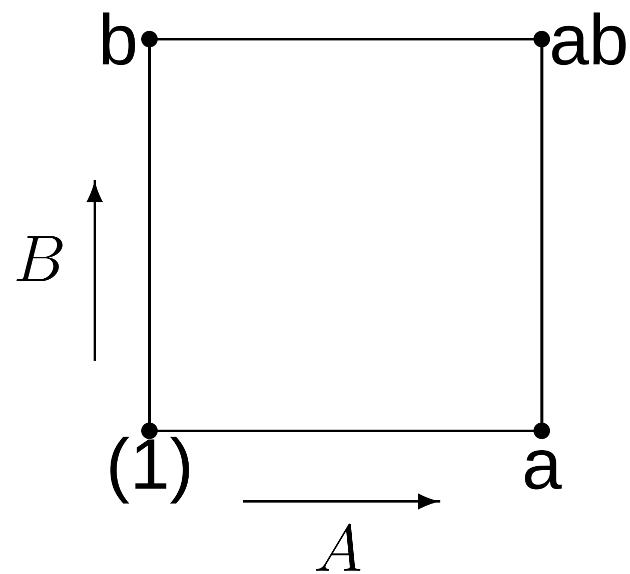
2^k *Factorials*

- Experiments with many factors
- Each factor has only two levels: high (+) and low(–)
- 2^k runs for a complete replicate with k factors
- Blocking in factorials

2^2- *Design*

run	A	B	Treatment
1	–	–	(1)
2	+	–	a
3	–	+	b
4	+	+	ab



Estimation of main effects and interaction

$$\begin{aligned}
\hat{A} &= \bar{y}_{A+} - \bar{y}_{A-} = \frac{1}{2n}(ab + a - b - (1)) \\
\hat{B} &= \bar{y}_{B+} - \bar{y}_{B-} = \frac{1}{2n}(ab + b - a - (1)) \\
\widehat{AB} &= \frac{1}{2n}((ab - b) - (a - (1))) = \frac{1}{2n}(ab + (1) - a - b)
\end{aligned}$$

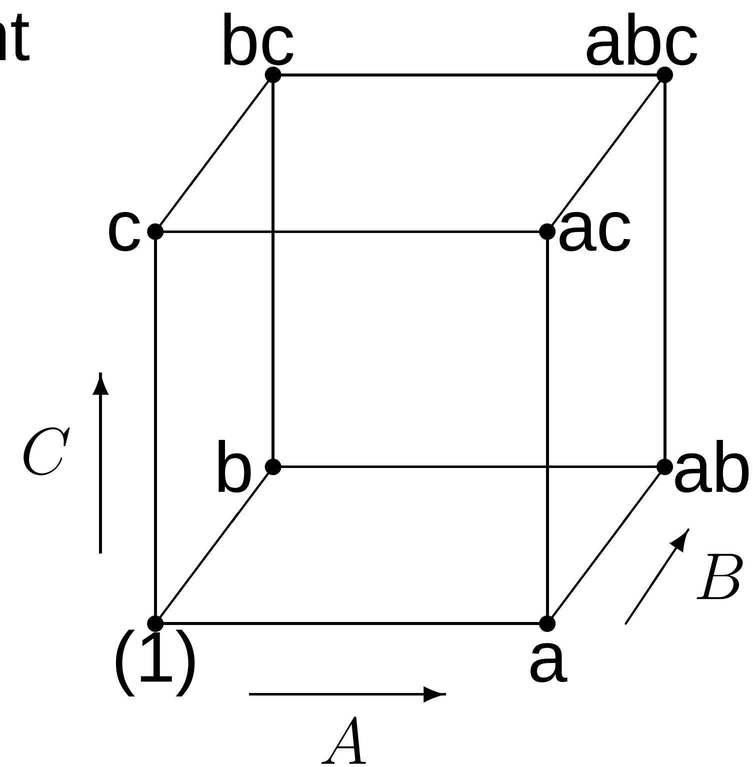
(n replicates, same notation for totals)

Algebraic signs for calculating effects

Treatment	I	A	B	AB
(1)	+	–	–	+
a	+	+	–	–
b	+	–	+	–
ab	+	+	+	+

2^3-*Design*

run	A	B	C	Treatment
1	–	–	–	(1)
2	–	–	+	c
3	–	+	–	b
4	–	+	+	bc
5	+	–	–	a
6	+	–	+	ac
7	+	+	–	ab
8	+	+	+	abc



Estimation of effects

Main effect A:

$$\hat{A} = \bar{y}_{A+} - \bar{y}_{A-} = \frac{1}{4n}(a - (1) + (ab - b) + (ac - c) + (abc - bc))$$

Interaction effect of AB: mean difference between the effect of A at the different levels of B.

$$\widehat{AB} = \frac{1}{4n}((ab - b) - (a - (1)) + (abc - bc) - (ac - (c))$$

Interaction ABC: mean difference between the interaction effect AB at the different levels of C.

$$\widehat{ABC} = \frac{1}{4n}((abc - bc) - (ac - (c)) - (ab - b) - (a - (1))$$

Algebraic signs for calculating effects

Treatment	I	A	B	AB	C	AC	BC	ABC
(1)	+	–	–	+	–	+	+	–
a	+	+	–	–	–	–	+	+
b	+	–	+	–	–	+	–	+
ab	+	+	+	+	–	–	–	–
c	+	–	–	+	+	–	–	+
ac	+	+	–	–	+	+	–	–
bc	+	–	+	–	+	–	+	–
abc	+	+	+	+	+	+	+	+

Blocking in Factorials

run	A	B	C	D
1	–	–	–	1
2	–	–	+	1
3	–	+	–	1
4	–	+	+	1
5	+	–	–	2
6	+	–	+	2
7	+	+	–	2
8	+	+	+	2

What is wrong with this design?

Example

```
> data
    y  A  B  C
1  13 -1 -1 -1
2  63 -1 -1  1
3  91 -1  1 -1
4 113 -1  1  1
5 119  1 -1 -1
6 125  1 -1  1
7 137  1  1 -1
8 139  1  1  1
```

Example continued

```
> mod1=aov(y~A*B*C)
> summary(mod1)
        Df  Sum of Sq  Mean Sq
     A   1       7200     7200
     B   1       3200     3200
     C   1        800      800
   A:B   1       1152     1152
   A:C   1        512      512
   B:C   1        128      128
 A:B:C   1         72       72

> mod1$coef
 (Intercept)  A  B  C A:B A:C B:C A:B:C
         100 30 20 10 -12  -8  -4     3
```

with blocking

```
> mod2=aov(y~D+A*B*C)
> summary(mod2)
      Df Sum of Sq Mean Sq
    D  1      7200    7200
    B  1      3200    3200
    C  1       800     800
  A:B  1      1152    1152
  A:C  1       512     512
  B:C  1       128     128
A:B:C  1        72      72
> mod2$coef
 (Intercept)  D  A  B  C A:B A:C B:C A:B:C
         100 30 NA 20 10 -12  -8  -4     3
```

A little bit better:

run	A	B	C	D
1	–	–	–	2
2	–	–	+	1
3	–	+	–	1
4	–	+	+	2
5	+	–	–	2
6	+	–	+	1
7	+	+	–	1
8	+	+	+	2

Blocks confounded with BC

```
> mod3=aov(y~D+A*B*C)
> summary(mod3)
      Df Sum of Sq Mean Sq
    D  1       128     128
    A  1      7200    7200
    B  1      3200    3200
    C  1       800     800
  A:B  1      1152    1152
  A:C  1       512     512
A:B:C  1        72      72
> mod3$coef
 (Intercept)  D  A  B  C A:B A:C B:C A:B:C
         100 -4 30 20 10 -12  -8  NA     3
```

Blocks confounded with ABC

run	A	B	C	D
1	–	–	–	1
2	–	–	+	2
3	–	+	–	2
4	–	+	+	1
5	+	–	–	2
6	+	–	+	1
7	+	+	–	1
8	+	+	+	2

Construction method

- Choose an interaction to be confounded with blocks
- The principal block consists of (1) and all treatments which have an even number of letters in common with the chosen interaction.
- 2^k design in 2^l blocks: choose l confounded interactions. The principal block consists of (1) and all treatments which have an even number of letters in common with the chosen interactions. For the other blocks multiply the principal block with a letter not included yet.

Partial confounding

2^3 design in 2 blocks: [(1),ab,ac,ab] and [a,b,c,abc]
Take four replicates to get sufficient precision,
confound a different interaction in each replicate.

I:	[(1),ab,ac,ab]	and	II:	[a,b,c,abc]	ABC confounded
III:	[(1),a,bc,abc]	and	IV:	[b,c,ab,ac]	BC confounded
V:	[(1),b,ac,abc]	and	VI:	[a,c,ab,bc]	AC confounded
VII:	[(1),c,ab,abc]	and	VIII:	[a,b,ac,bc]	AB confounded

Main effects are estimated from 8 blocks, interactions from 6 blocks.