

Biochemical Experiment

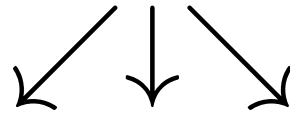
Serum levels after four medical treatments. Only four people can be treated per day, one for each medication.

Treat.	Day							
	1	2	3	4	5	6	7	8
I	4.4	5.3	5.3	1.8	3.7	6.5	5.4	5.2
II	2.8	3.3	7.0	2.6	5.9	5.4	6.9	6.8
III	4.8	1.9	4.3	3.1	6.2	5.7	6.2	7.9
IV	6.8	8.7	7.2	4.8	5.1	6.7	9.3	7.9

Block Design

Subjects

Randomisation



	Block 1	Block 2	...	Block J
Group 1	×	×		×
Group 2	×	×		×
Group 3	×	×		×
⋮	⋮	⋮	⋮	⋮
Group I	×	×		×

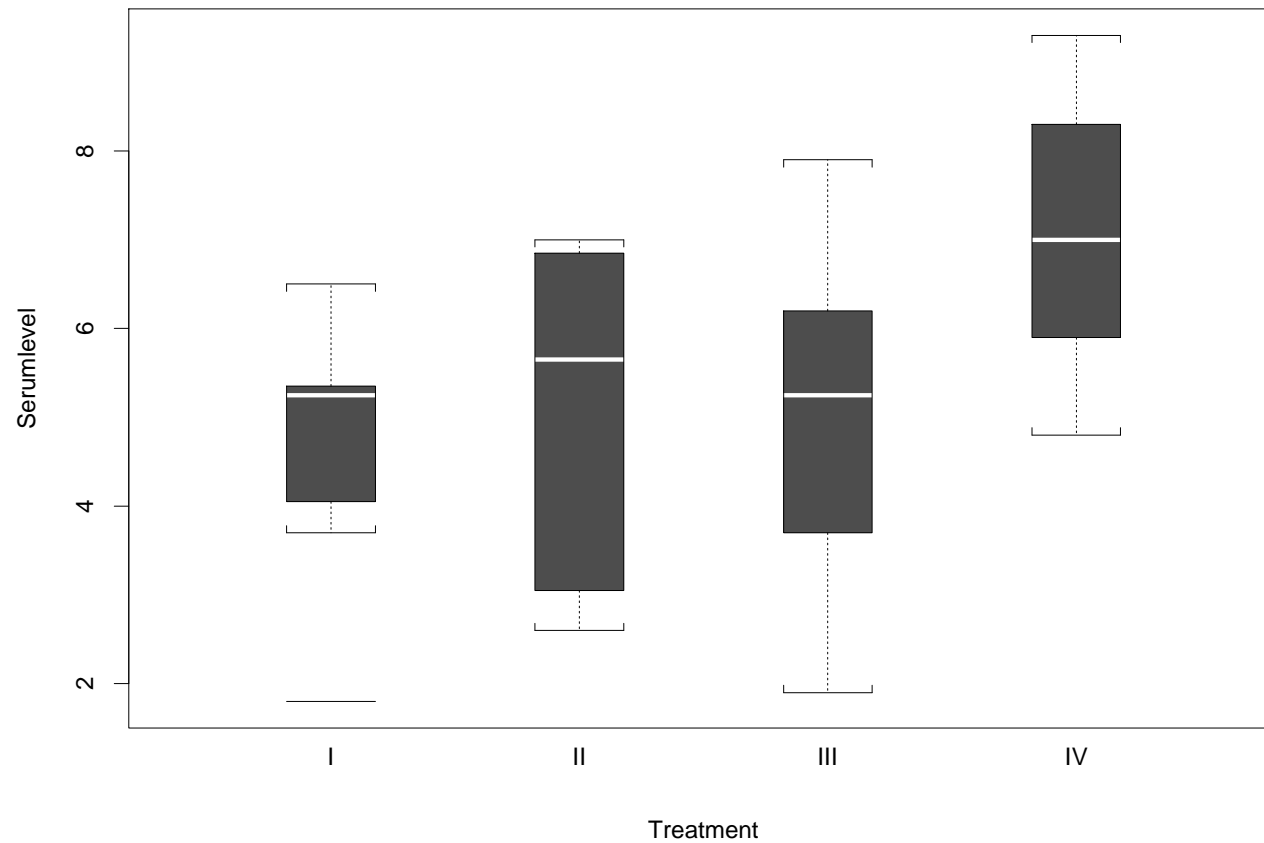
Block Randomisation

R: `sample(rep(1:8,4)), sample(4)`

Subjects

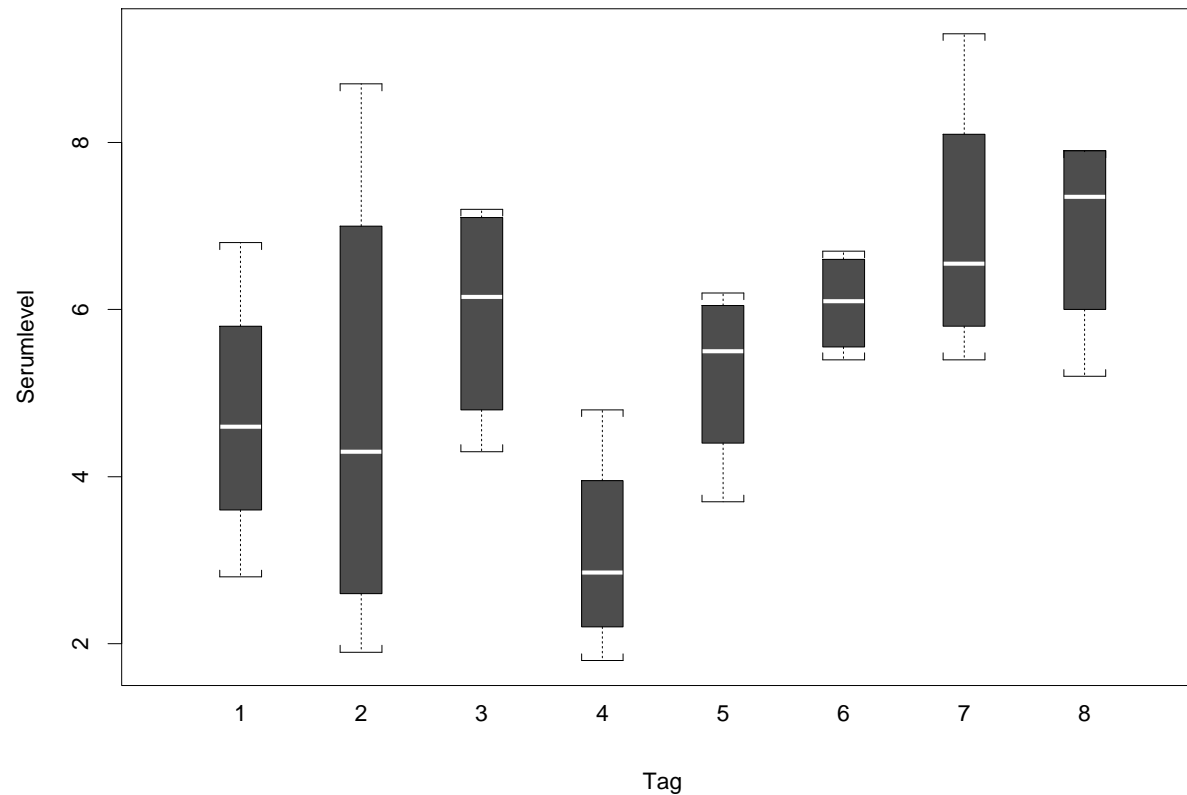
	Day							
Treatment	1	2	3	4	5	6	7	8
I	13	3	26	23	4	28	20	21
II	24	18	6	10	9	25	32	1
III	19	7	8	22	27	30	16	14
IV	2	11	15	12	31	17	29	5

Serum levels by Treatment



Mean: 4.7 5.09 5.01 7.06

Serum levels by Day



Mean: 4.7 4.8 5.95 3.08 5.23 6.07 6.95 6.95

Randomized Complete Block Design

- Each treatment in each block equally often.
- Model:

$$y_{ij} = \mu + A_i + b_j + \epsilon_{ij} \quad (2)$$

b_j : Effect of block j

- **Fixed-Effects Model:**

$$\sum A_i = 0, \sum b_j = 0, \epsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$$

Mixed Model:

$$\sum A_i = 0, b_j \sim \mathcal{N}(0, \sigma_b^2), \epsilon_{ij} \sim \mathcal{N}(0, \sigma_e^2)$$

all b_j and ϵ_{ij} independent.

Random-Effects Model: all factors are random

Block effects: fixed or random?

fixed: a few levels, interest in levels themselves

random: levels are chosen from a population, interest in variability, blocks for reduction of variability

$$SS_{tot} = SS_{treat} + SS_{blocks} + SS_{res}$$

Source	SS	df	MS	F
Blocks	47.3	$J - 1 = 7$	6.75	
Treatments	27.9	$I - 1 = 3$	9.29	...
Residual	35.3	$(I - 1)(J - 1) = 21$	1.68	
Total	110.4	$N - 1 = 31$		

F Tests

Fixed-effects Model:

$$H_0 : A_i = 0 \quad \forall i, \quad F = \frac{MS_{treat}}{MS_{res}} \sim F_{I-1, (I-1)(J-1)}$$

$$H_0 : b_j = 0 \quad \forall j, \quad F = \frac{MS_{blocks}}{MS_{res}} \sim F_{J-1, (I-1)(J-1)}$$

Mixed Model:

$$H_0 : A_i = 0 \quad \forall i, \quad F = \frac{MS_{treat}}{MS_{res}} \text{ as above}$$

$$H_0 : \sigma_b^2 = 0 \quad F = \frac{MS_{blocks}}{MS_{res}} \text{ usually not tested}$$

$MS_{blocks} \gg MS_{res}$: Blocking good

$MS_{blocks} \leq MS_{res}$: Blocking not necessary

Example:

$F_A = 5.53$ Medication significant

$MS_{blocks} = 6.75 > MS_{res} = 1.68$ Blocking good