Incomplete block designs

- Small block size, larger number of treatments
- Non-orthogonal designs

Test of 7 different Tyres

		Cars						
		1	2	3	4	5	6	7
Tyres	1	x	x	x	x			
	2	x	x			x	x	
	3	x	x		x	x		
	4			x	x		x	x
	5			x	x	x		x
	6		x	x			x	x
	7	x				x	x	x

Blocks	Treatments			
1	1	2	3	7
2	1	2	3	6
3	1	4	5	6
4	1	3	4	5
5	2	3	5	7
6	2	4	6	7
7	4	5	6	7

Balanced incomplete block design

- n treatments, block size k, ($k < n$)
- Any two treatments occur together the same number of times (λ times)

First Solution: $\binom{n}{k}$ blocks, a different combination of treatments in each block.

$n = 7, k = 4 : \binom{7}{4} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$ cars

Search for smaller designs

Necessary conditions for a BIBD

b blocks, each treatment occurs r times

(1) $$nr = bk$$

(2) $$r(k-1) = \lambda(n-1)$$

(1) number of observations
(2) number of treatment pairs for a fixed treatment

Design is called symmetric if $n = b$.

Construction of BIBD

- Problem: Given k and n, how large are r,b, and λ?
- Conditions (1) and (2) are necessary but not sufficient.
- Several methods of construction exist.
- There are tables of BIBD with small sizes (Cochran & Cox 1992).
- Partially balanced block designs (PBIB) if some treatment comparisons are less important.

Analysis of BIBD

- Statistical model:

$$Y_{ij} = \mu + T_i + \beta_j + \epsilon_{ij}$$

  where T_i is the treatment effect, β_j the block effect.

- Block and treatment factor are not orthogonal, because not all combinations appear.
- Calculate first block sum of squares, then adjusted treatment sum of squares.