Split Plot Designs

- Large and small units
- Confounding main effects

Study in Dental Medicine

- Can measurement of electric resistance help in detecting tooth decay?
- 40 measurements on teeth with and without inflamed gums, with and without special treatment.
- 2^2 factorial with factor A (inflammation) and factor B (special treatment).

Correct anova table?

Source	df	MS	F
A	1		MS_A/MS_{res}
B	1		MS_B/MS_{res}
AB	1		MS_{AB}/MS_{res}
Residual	36		
Total	39		

Depends on design structure. How many subjects, how many teeth?

8 subjects, one tooth each

One treatment per person, 5 repeated measurements

Stratum	Source	df	F
Person	A	1	$MS_A/MS_{res-person}$
	B	1	$MS_B/MS_{res-person}$
	AB	1	$MS_{AB}/MS_{res-person}$
	Residual	4	
	Total	7	
Measurement	Residual	32	
	Total	39	

5 subjects, 4 teeth each

Each person has 2 inflamed and 2 not inflamed teeth. Each tooth was measured once with special treatment and once without special treatment.

Stratum	Source	df	F
Person	Person	4	
Tooth	A	1	$MS_A/MS_{res-tooth}$
	Residual	14	
	Total	15	
Measurement	B	1	$MS_B/MS_{res-meas}$
	AB	1	$MS_{AB}/MS_{res-meas}$
	Residual	18	
	Total	20	
	Total	39	

Special properties of this design

- Replication on three stages: persons, teeth and measurements.
- One factor varies between teeth, the other between measurements.
- main plot= tooth, sub-plot = measurement

Split-plot design

- A first factor needs to be applied to large plots, called main plots.
- Main plots are split into smaller plots, called subplots. Theses are assigned to different levels of a second factor.
- Two different levels for comparing factor levels: effects of the first factor must be examined relative to main plot variation, effects of the second factor must be examined relative to subplot variation.

Rice experiment

4 irrigation methods I1-I4 on main plots, 3 fertilizer mixtures X,Y, Z on sub-plots, 2 complete replicates.

Layout:

Block I				Block II			
z	x	y	z	x	y	z	x
x	z	z	y	z	x	x	y
y	y	x	x	y	z	y	z
I4	I2	I3	I1	I2	I1	I4	I3

Irrigation is confounded with main plots.

Model

$$Y_{ijk} = \mu + b_i + Irr_j + \epsilon_{ij} + Fert_k + (Irr : Fert)_{jk} + \delta_{ijk}$$

$$i = 1, \ldots, I; j = 1, \ldots, J; k = 1, \ldots, n.$$

b_i : ith block effect

Irr_j : jth effect of irrigation

ϵ_{ij} : main plot error

$Fert_k$: kth effect of fertilizer

$(Irr : Fert)_{ij}$: jkth interaction

δ_{ijk} : sub-plot error

Skeleton Anova

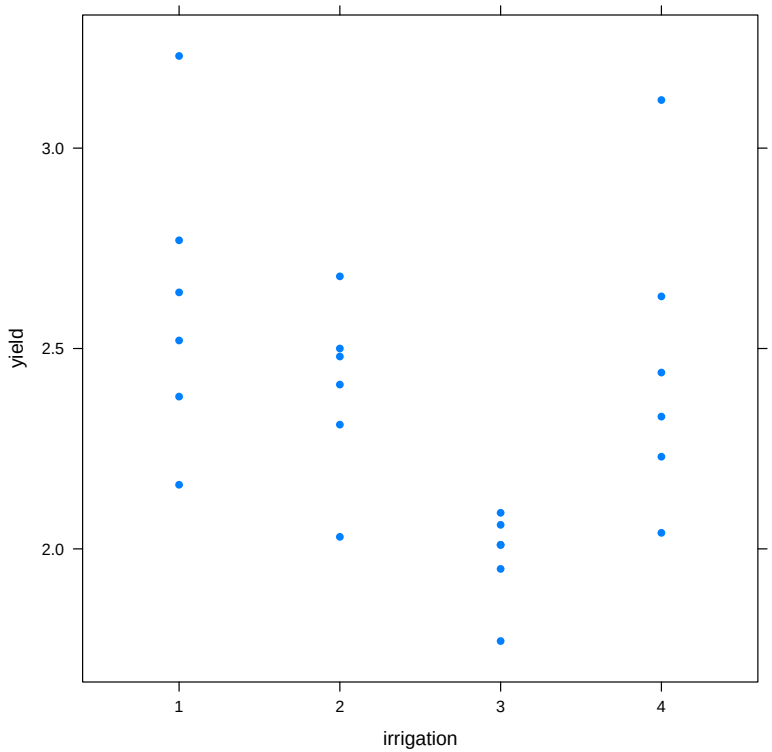
Stratum	Source	df	F
Blocks	Blocks	1	
Main plots	Irr	3	$MS_{Irr}/MS_{res-main}$
	Residual	3	
	Total	7	
Sub-plots	Fert	2	$MS_{Fert}/MS_{res-sub}$
	Irr:Fert	6	$MS_{Irr:Fert}/MS_{res-sub}$
	Residual	8	
	Total	16	
	Total	23	

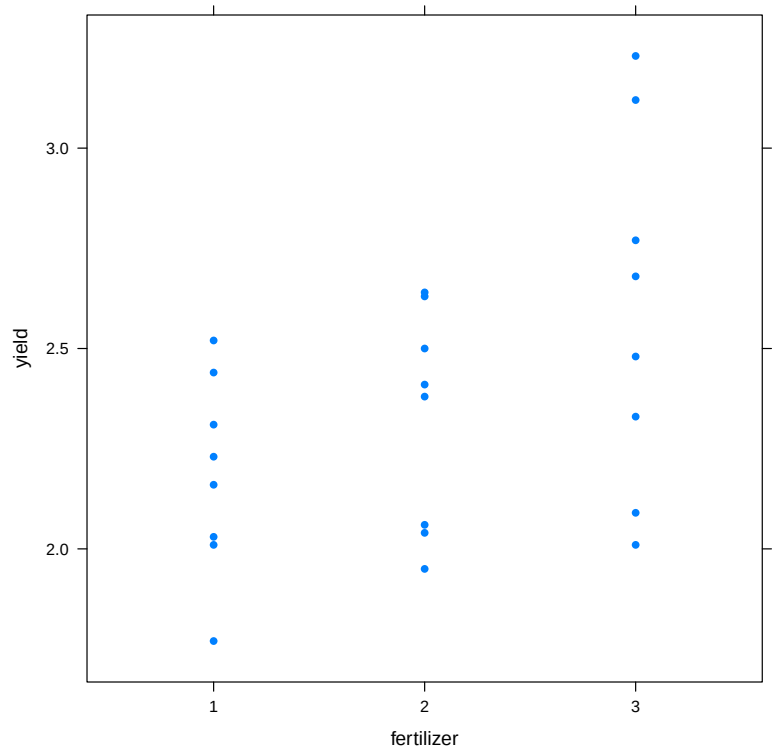
Data on crop yield (tonnes/hectare)

	Irrigation			
Block I	I1	I2	I3	I4
Fertilizer x	2.16	2.03	1.77	2.44
y	2.38	2.41	1.95	2.63
z	2.77	2.68	2.01	3.12

	Irrigation			
Block I	I1	I2	I3	I4
Fertilizer x	2.52	2.31	2.01	2.23
y	2.64	2.50	2.06	2.04
z	3.23	2.48	2.09	2.33

Graphical display





Anova Table

```
> mod2=aov(yield~irrigation*fertilizer+Error(block/irrigation))
> summary(mod2)

Error: block
          Df    Sum Sq   Mean Sq F value Pr(>F)
Residuals  1 0.0003375 0.0003375

Error: block:irrigation
           Df  Sum Sq Mean Sq F value Pr(>F)
irrigation  3 1.32971 0.44324  2.0424 0.2862
Residuals   3 0.65105 0.21702

Error: Within
                     Df  Sum Sq Mean Sq F value   Pr(>F)
fertilizer            2 0.67530 0.33765 16.6262 0.001414 **
irrigation:fertilizer  6 0.20110 0.03352  1.6504 0.250110
Residuals              8 0.16247 0.02031
```

Some Variations

- Repeated splitting for a third factor applied to split-split plot
- Confounding interactions of sub-plot factors in split-plot designs
- Other designs for main plots, e.g. Latin squares
- Strip-plot design