Solution Exercise 6

1. **a)** We have:

$$\begin{aligned} n &= 4 \\ b &= 6 \\ k &= 2 \\ r &= \frac{kb}{n} = \frac{12}{4} = 3. \\ \lambda &= \frac{r(k-1)}{n-1} = 1 \end{aligned}$$

We find the BIBD: (Note that $\lambda = 1$ implies that any combination of 2 factors can appear just once).

	1	2	3	4
1	x	x		
2	x		x	
3	x			x
4		x	x	
5		x		x
6			x	x

b) We have:

$$\begin{aligned} n &= 7 \\ b &= 7 \\ k &= 3 \\ r &= \frac{kb}{n} = \frac{21}{7} = 3. \\ \lambda &= \frac{r(k-1)}{n-1} = 1 \end{aligned}$$

We find the BIBD. (Note that $\lambda = 1$ implies that any combination of 2 factors can appear just once).

	1	2	3	4	5	6	7
1	x	x	x				
2	x			x	x		
3	x					x	x
4		x		x		x	
5		x			x		x
6			x	x			x
7			x		x	x	

2. We have the following model:

Stratum	Source	df	F
Main plots	Treatment	1	$MS_{TR}/MSres-main$
	Residual	19	
	Total	20	
Sub-plots	Time	1	$MS_{Time}/MSres-sub$
	TR:Time	1	$MS_{TR:Time}/MSres-sub$
	Residual	19	$MS_{TR:Time}/MSres-sub$
	Total	21	
	Total	41	

With the R -function

```
Sh.fit <- aov(Y Time*Treatment+Error(Subject/Time),data=Sh)
summary(Sh.fit)
```

we obtain:

```
Error: Subject
          Df Sum Sq Mean Sq F value  Pr(>F)
Treatment  1  847.5  847.48  3.6266 0.07212 .
Residuals 19 4440.0  233.68
---
Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1

Error: Within
               Df Sum Sq Mean Sq F value    Pr(>F)
Time            1 542.88  542.88  15.142 0.0009823 ***
Time:Treatment  1 407.41  407.41  11.363 0.0032085 **
Residuals      19 681.21   35.85
---
Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
```

`Time` and interaction `Time:Treatment` are significant. A plot also shows that the new treatment improves response values after surgery, whereas the rates are unchanged with a standard operation. The new operation is therefore superior to the standard treatment.

3. Let

$$A = packing$$
$$B = pizza$$

a) This is a split plot design with persons as main plots and the ratings of different packings as subplots.

Strata	Source	df	MS	F
Person	B	2	MS_B	$\text{MS}_B/\text{MS}_{res-main}$
	Residual	87	$\text{MS}_{res-main}$	
Subplots	A	5	MS_A	$\text{MS}_A/\text{MS}_{res-sub}$
	AB	10	MS_{AB}	$\text{MS}_{AB}/\text{MS}_{res-sub}$
	Residual	435	$\text{MS}_{res-sub}$	
	Total	539		

b) This is a factorial design.

Source	df	MS	F
A	5	MS_A	MS_A/MS_{res}
B	2	MS_B	MS_B/MS_{res}
AB	10	MS_{AB}	MS_{AB}/MS_{res}
Residual	72	MS_{res}	
Total	89		

c) This is a complete block design with persons as blocks.

Source	df	MS	F
Blocks	89	MS_{blocks}	
A	5	MS_A	MS_A/MS_{res}
B	2	MS_B	MS_B/MS_{res}
AB	10	MS_{AB}	MS_{AB}/MS_{res}
Residual	1513	MS_{res}	
Total	1619		

4. Using R and the function `lm` we obtain:

```
d.st <- lm(formula=Pu T1+Pr1,data=d)
d.st$coefficients

(Intercept)          T1          Pr1
      84.10       -0.85         0.25
```

This can be interpreted as follows:

$$\hat{y} = 84.10 - 0.85 \cdot T + 0.25 \cdot P \ ,$$

By letting $\hat{y}$ constant we obtain an equation for the contour lines, i.e. contour lines satisfy the equation

$$P = \frac{0.85}{0.25} \cdot T + constant = m_0 T + c \ .$$

The direction of steepest ascent is then:

$$-\frac{1}{m_0} = -\frac{5}{17} \ .$$