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Numerical Accuracy in a Statistics Package:

What Precision is Needed ***When*** *?*

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Overview

1. Graphics: Precision In Axis Labelling `pretty` etc.
   ($\longrightarrow$ need Offset)
2. Statistical Package as Calculator:
   $\longrightarrow$ expect "full" precision, but . . . .
3. Probability Computations:
   (a) Speed vs. Accuracy : May need both
   (b) Tails: P vs. $1 - P$ — only one is precise!

1 Extreme Axis Extents

(⟶ 2 Examples)

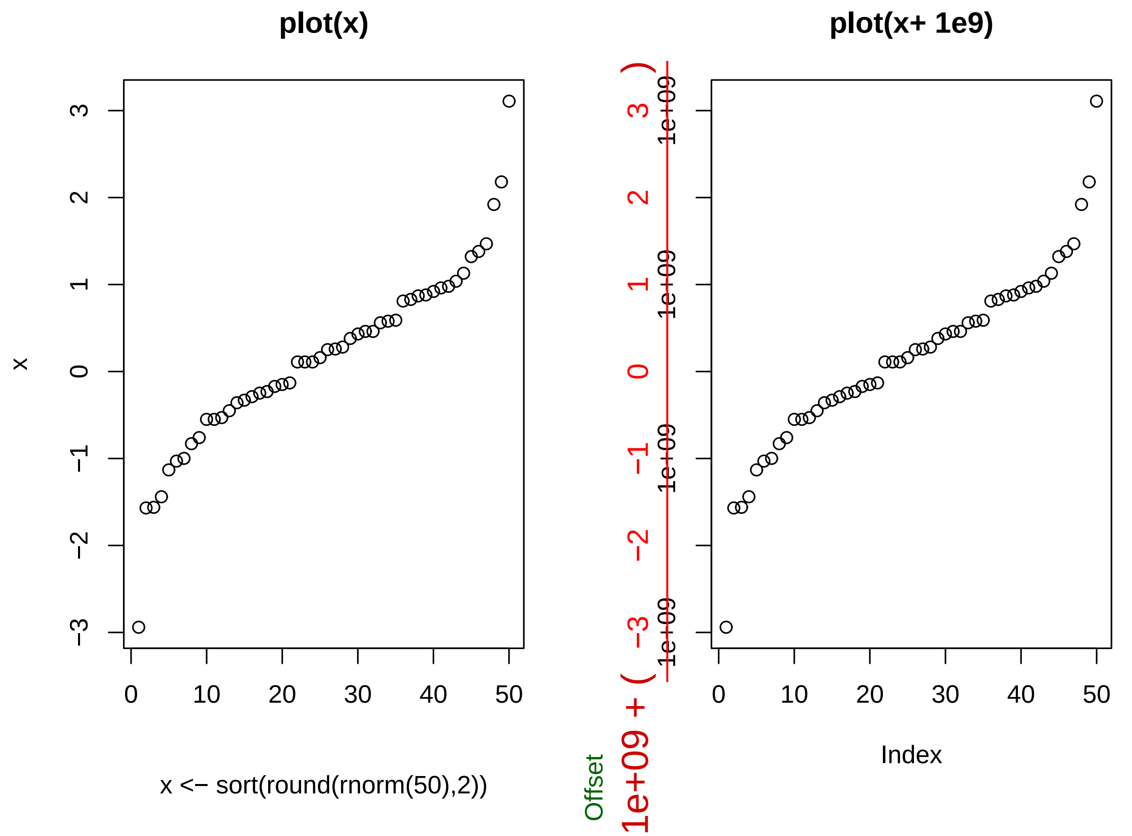
1. For extremely *small* ranges: need an OFFSET.

   (major: *design* of plot components).

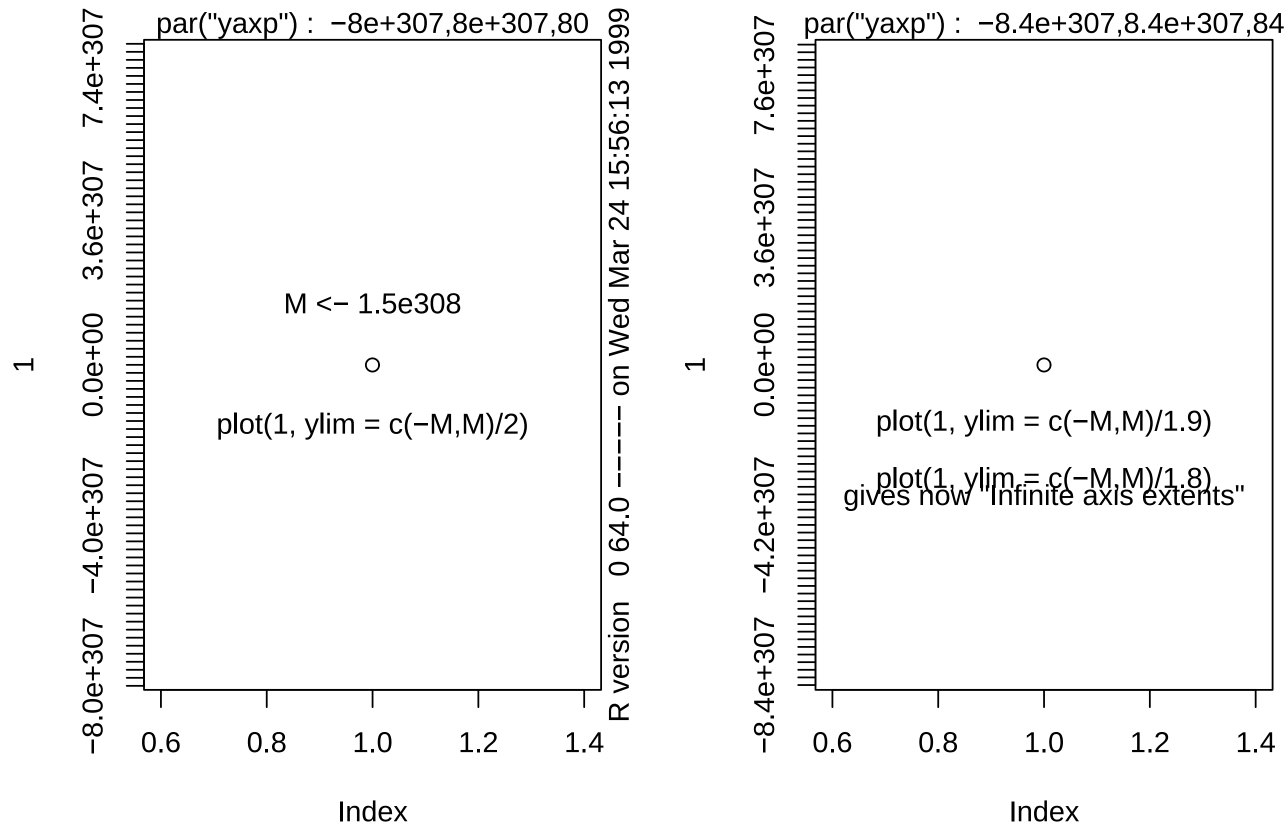
2. For extremely *large* ranges: need careful tickmark/`pretty` calculations

   (minor).

Small axis range ⟶ Offset



Large axis range ⟶ be more careful!



2 Statistical Calculator

Our calculator (usually) has 53 bits (double) precision ($\approx$ 16.0 digits:
$53 \times log_{10}(2) \approx= 53 \times .30103 = 15.95$)

Certainly we expect about 52–53 bit precision in multiplication `x * y`,

then why not also for $\exp(x), \log(x)$?

then why not also for $\Gamma(x)$ (=`gamma(x)`) ?

then why not also for $B(a, b)$ (=`beta(a,b)`) ?

then why not also for “incomplete Beta” $I_x(a, b)$ = `pbeta(x, a,b)` ?
(and this is the basis for t– and F-distribution)

Why not (always) *Full* Precision?

— because it costs :

- longer (Taylor) series expansions
- higher degree of rational approximations
- In quite a few cases:

  no published algorithms for high precision, many at least not (yet) available
- extreme case; "Extended precision" for intermediate results

Proposition — Future:

S like

```
options(NumPrecision = 1e-7)
```

would be the *minimal* precision p that "basic" functions provide, i.e.,

$$\textit{precision} \;\; p \geq \frac{\left| f(x) - \hat{f}(x) \right|}{\max(\epsilon, |f(x)|)} \quad \forall f \quad \forall x$$

$=$ relative error, unless true value $f(x)$ is close to zero ($< \epsilon$).

$\epsilon =$0.1 or $= 10^{-7}$ (("arbitrary"; different conventions)))

Precision Loss ⟶ `warning(...)`

What should happen when the relative error of some basic computation cannot be garantueed to be less than `.Options$NumPrecision` or is even *known* to be larger?

`warning()`s should be (internally collected) and once per (toplevel?) call be reported, similarly to

```
> sqrt(-5:5)
Warning: NaNs produced in function "sqrt"
[1]   NaN   NaN   NaN   NaN   NaN 0.000 1.000 1.414 1.732 2.0
```

3 Probability Distribution Computations

3.1 Speed vs. Accuracy

Speed: For simulations, speed of inverse CDF may be crucial for Random Number Generation; in some cases, no (easily programmable) faster technique.

Accuracy `p`*`<distrib>`*`(..)` maybe needed in further mathematical statistical formulae; three digit precision maybe unacceptable.

Extreme value computations *common* in some fields (reliability; insurance).

Need also precision for (*both*) extreme tails.

3.2 P vs. $1 - P$ — in tails, only one is precise!

Especially important for asymmetric distributions.

E.g. (see also Knüsel's reports) $P[X > x = 190]$ for $X \sim \mathcal{P}(\lambda = 100)$ as

`1 - ppois(x=190, lam= 100)` gives 4.44e-16 in both S and R,

whereas the true value is 4.17e-16.

For $x = 195$ S and R give 0 (zero) because of full cancellation where the true value (via B.Brown's `dcdflib`) is 1.4795e-17.

$\longrightarrow$ ...

Proposal

Allow an extra parameter to all the `p<dist>` and `q<dist>` functions, e.g.

```
qpois <- function (p, lambda, lower.tail = TRUE)
```

such that `qpois(p,lam)` = `qpois(p,lam, lower.tail=TRUE)` $= P_\lambda[X \le p]$,

whereas `qpois(p,lam, lower.tail=FALSE)` $= P_\lambda[X > p]$
(usually *not* computed via $1 - P_\lambda[X \le p]$!!)

References

Higham, N. J. (1996). *Accuracy and Stability of Numerical Algorithms*, Society for Industrial and Applied Mathematics, Philadelphia.

Knüsel, L. (1998). Accuracy of statistical packages, `www.stat.uni-muenchen.de/~knuesel/`. *List of references, downloadable PDF files*

McCullough, B. D. (1998). Assessing the reliability of statistical software: Part I, *The American Statistician* **52**(4): 358–366. *Part II (1999) will discuss S-plus, SAS & SPSS*

NIST (1997). Statistical reference datasets (StRD), `www.nist.gov/itl/div898/strd/`. *mainly regression (incl. nonlin) & ANOVA; tech.report* not *downloadable*