1 Introductory examples

Main feature of time series data: observations are interpreted as realization from **dependent** random variables $X_1, \dots, X_n$. This is in contrast to the classical setting where we typically assume that $X_1, \dots, X_n$ are independent and identically distributed (i.i.d.).

Real data examples of time series are shown in Figure 1 and 2.

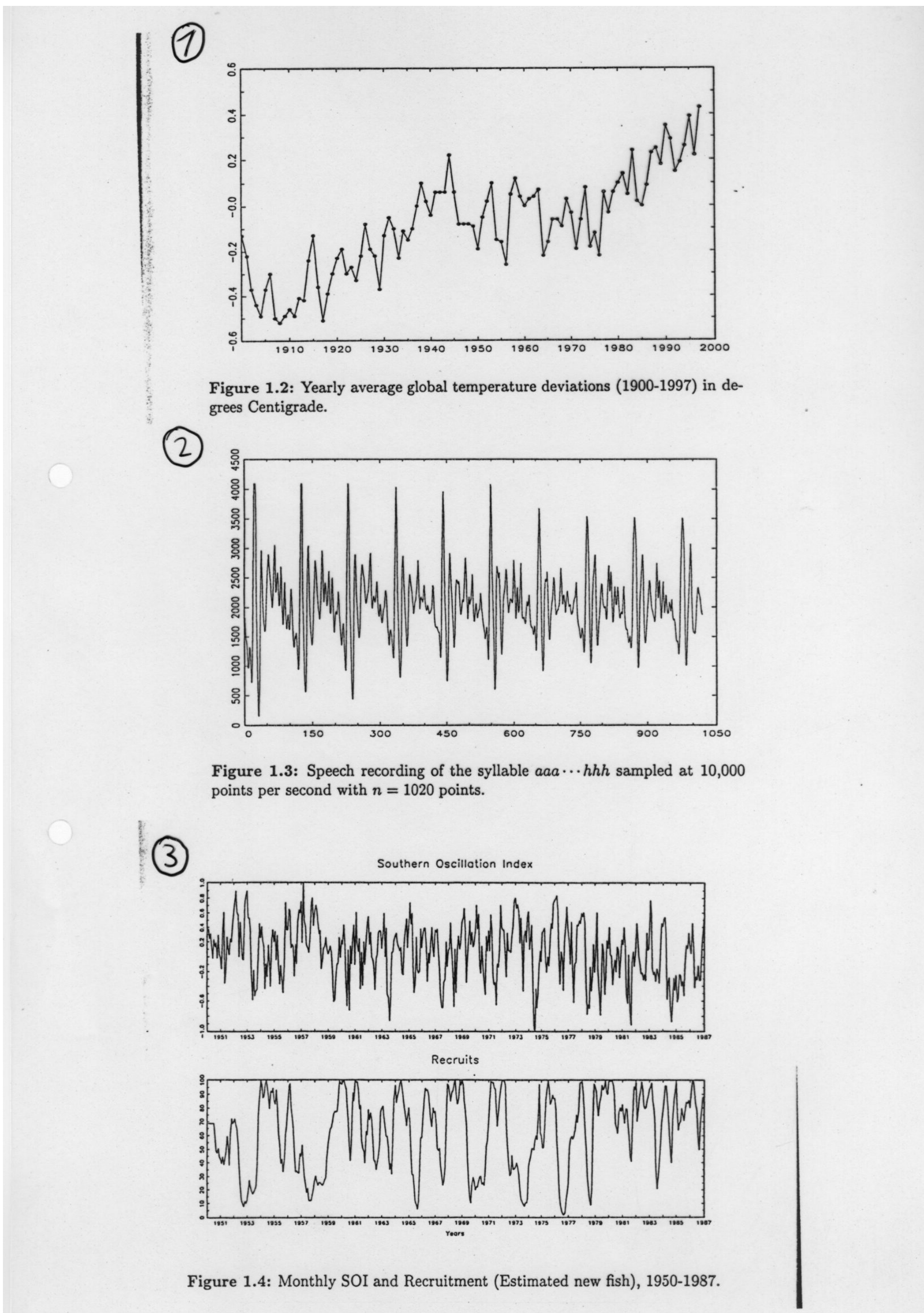

Figure 1: Example 3 is a bivariate time series.

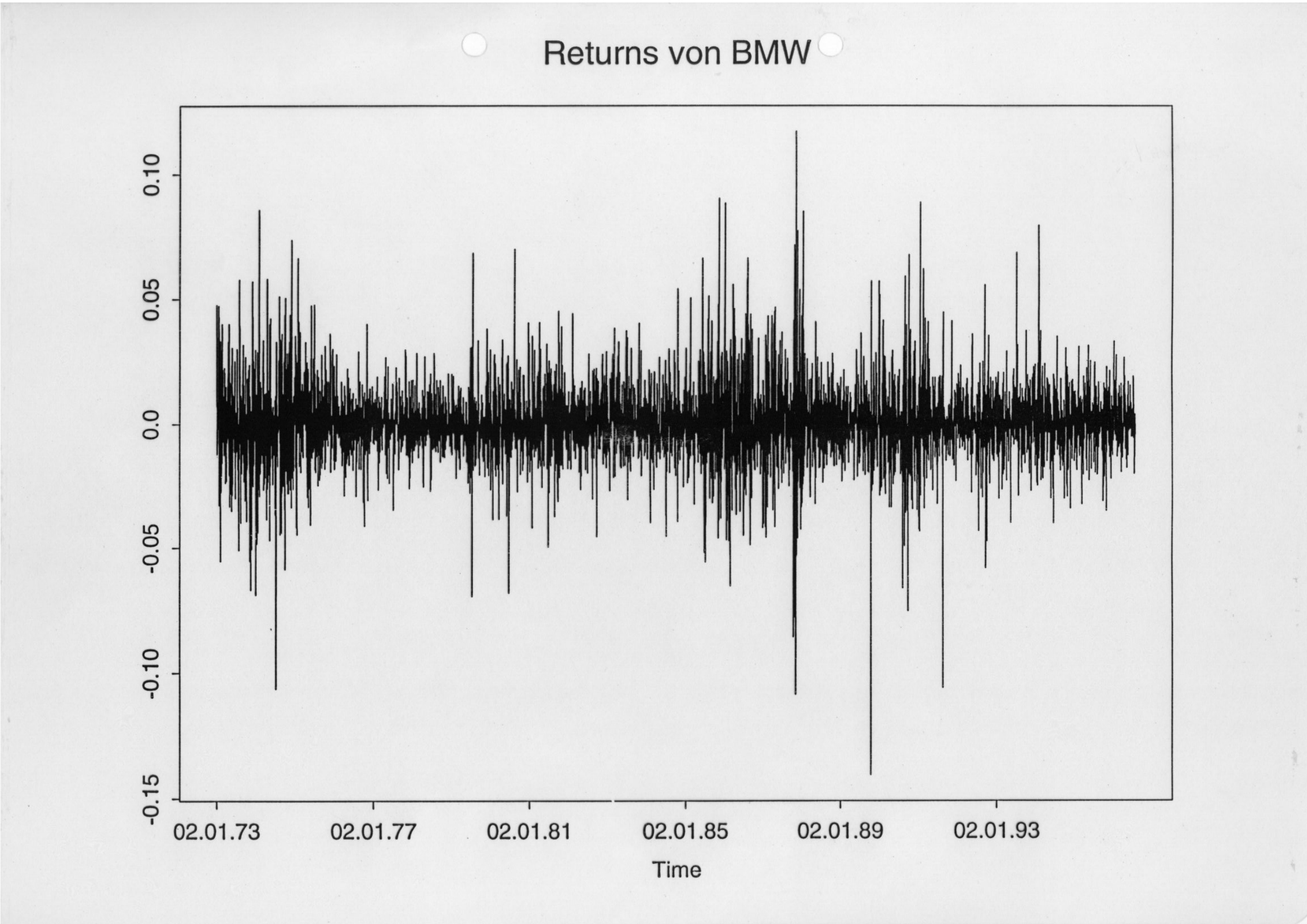

Figure 2: Daily log-returns from BMW share price.

1.1 Stochastic processes

As indicated above, we consider the following setting. The time series data is

$$x_t, \ t = 1, \ldots, n,$$

where each $x_t \in \mathbb{R}^p$. For univariate time series, $p = 1$. We assume that these values are from **one finite realization** of a stochastic process in discrete time

$$\{X_t : \Omega \to \mathbb{R}^p; \ t \in \mathbb{Z}\}.$$

From a probability point of view: we want to infer all finite-dimensional distributions

$$F_{i,j}(x_i, x_{i+1}, \ldots, x_j) = \mathbb{P}[X_t \le x_t; \ i \le t \le j]$$

for all $i < j$. If we would know $F_{i,j}(\cdot)$ for all $i < j$, we would know the distribution of the stochastic process $(X_t)_{t \in \mathbb{Z}}$. This fact is based on the following extension theorem.

Theorem 1 (Kolmogorov) *There exists a unique probability measure on $\Omega = \mathbb{R}^{\mathbb{Z}}$ such that*

$$\mathbb{P}[\underbrace{\omega_t}_{\textit{coordinates of } (\omega_t)_{t\in\mathbb{Z}}} \le x_t; \ i \le t \le j] = F_{i,j}(x_i, \ldots, x_j).$$