Discrete Fourier transform

$A = \{ 2\pi\text{-periodic function: } \mathbb{R} \to \mathbb{C} \}$, e.g. spectral density $f(\cdot)$

$\Downarrow$ discrete Fourier transform (FT)

$B = \{ \text{function: } \mathbb{Z} \to \mathbb{C} \}$, e.g. autocovariance function $R(\cdot)$

FT: for $f \in A$, integrable

$$f \xmapsto{FT} \hat{f}, \quad \hat{f}_h = \int_{-\pi}^{\pi} e^{-ih\lambda} f(\lambda) d\lambda \in B$$

Fourier-Bachtransform:

if $\sum_{h=-\infty}^{\infty} |\hat{f}_h| < \infty$, then

$$f(\lambda) = \frac{1}{2\pi} \sum_{h=-\infty}^{\infty} \hat{f}_h e^{-ih\lambda} \quad \text{(almost everywhere)}$$

convolution formulae: for $f, g \in A$

$$\widehat{f * g} = \hat{f} \cdot \hat{g}$$

$$\hat{f} * \hat{g} = \widehat{f \cdot g} \cdot 2\pi$$