Package ‘mggd’

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Title Multivariate Generalised Gaussian Distribution; Kullback-Leibler Divergence

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Description Distance between multivariate generalised Gaussian distributions, as presented by N. Bouhlel and A. Dziri (2019) <doi:10.1109/LSP.2019.2915000>. Manipulation of multivariate generalised Gaussian distributions (methods presented by Gomez, Gomez-Villegas and Marin (1998) <doi:10.1080/03610929808832115> and Pascal, Bombrun, Tourneret and Berthoumieu (2013) <doi:10.1109/TSP.2013.2282909>).

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Imports MASS, rgl, lifecycle, data.table

License GPL (>= 3)

URL https://forgemia.inra.fr/imhorphen/mggd

BugReports https://forgemia.inra.fr/imhorphen/mggd/-/issues

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Contents

`mggd-package` *Tools for Multivariate Generalized Gaussian Distributions*

Description

This package provides tools for multivariate generalized Gaussian distributions (MGGD):

- Calculation of distances/divergences between multivariate generalized Gaussian distributions:
  - Kullback-Leibler divergence: `kldggd`
- Tools for MGGD:
  - Probability density: `dmggd`
  - Estimation of the parameters: `estparmggd`
  - Simulation from a MGGD: `rmggd`
  - Plot of the density of a MGGD with 2 variables: `plotmggd`, `contourmggd`

Author(s)

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References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

F. Pascal, L. Bombrun, J.Y. Tourneret, Y. Berthoumieu. Parameter Estimation For Multivariate Generalized Gaussian Distribution. IEEE Trans. Signal Processing, vol. 61 no. 23, p. 5960-5971, Dec. 2013. doi:10.1109/TSP.2013.2282909 #' @keywords internal

See Also

Useful links:

- `https://forgemia.inra.fr/imhorphen/mggd`
- Report bugs at `https://forgemia.inra.fr/imhorphen/mggd/-/issues`

contourmggd *Contour Plot of the Bivariate Generalised Gaussian Density*

Description

Draws the contour plot of the probability density of the generalised Gaussian distribution with 2 variables with mean vector `mu`, dispersion matrix `Sigma` and shape parameter `beta`.

Usage

```
contourmggd(mu, Sigma, beta,
                    xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                    ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]),
                    zlim = NULL, npt = 30, nx = npt, ny = npt,
                    main = "Multivariate generalised Gaussian density",
                    sub = NULL, nlevels = 10,
                    levels = pretty(zlim, nlevels), tol = 1e-6, ...)
```

Arguments

`mu`	length 2 numeric vector.
`Sigma`	symmetric, positive-definite square matrix of order 2. The dispersion matrix.
`beta`	positive real number. The shape of the first distribution.
`xlim`, `ylim`	x-and y- limits.
`zlim`	z- limits. If NULL, it is the range of the values of the density on the x and y values within `xlim` and `ylim`.
`npt`	number of points for the discretisation.
`nx`, `ny`	number of points for the discretisation among the x- and y- axes.
`main`, `sub`	main and sub title, as for `title`.
`nlevels`, `levels`	arguments to be passed to the `contour` function.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. see `dmggd`.
`...`	additional arguments to `plot.window`, `title`, `Axis` and `box`, typically graphical parameters such as `cex.axis`.

Value

Returns invisibly the probability density function.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

plotmggd: plot of a bivariate generalised Gaussian density.

dmggd: Probability density of a multivariate generalised Gaussian distribution.

Examples

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
beta <- 0.74
contourmggd(mu, Sigma, beta)
```

---

dmggd — *Density of a Multivariate Generalized Gaussian Distribution*

Description

Density of the multivariate (p variables) generalized Gaussian distribution (MGGD) with mean vector `mu`, dispersion matrix `Sigma` and shape parameter `beta`.

Usage

```
dmggd(x, mu, Sigma, beta, tol = 1e-6)
```

Arguments

`x`	length p numeric vector.
`mu`	length p numeric vector. The mean vector.
`Sigma`	symmetric, positive-definite square matrix of order p. The dispersion matrix.
`beta`	positive real number. The shape of the distribution.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

The density function of a multivariate generalized Gaussian distribution is given by:

$$f(\mathbf{x}|\boldsymbol{\mu}, \Sigma, \beta) = \frac{\Gamma\left(\frac{p}{2}\right)}{\pi^{\frac{p}{2}} \Gamma\left(\frac{p}{2\beta}\right) 2^{\frac{p}{2\beta}}} \frac{\beta}{|\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}\left((\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu})\right)^\beta}$$

When $p = 1$ (univariate case) it becomes:

$$f(x|\mu, \sigma, \beta) = \frac{\Gamma\left(\frac{1}{2}\right)}{\pi^{\frac{1}{2}} \Gamma\left(\frac{1}{2\beta}\right) 2^{\frac{1}{2\beta}}} \frac{\beta}{\sigma^{\frac{1}{2}}} \; e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{2\sigma}\right)^\beta} = \frac{\beta}{\Gamma\left(\frac{1}{2\beta}\right) 2^{\frac{1}{2\beta}} \sqrt{\sigma}} \; e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma}\right)^\beta}$$

Value

The value of the density.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

rmggd: random generation from a MGGD.

estparmggd: estimation of the parameters of a MGGD.

plotmggd, contourmggd: plot of the probability density of a bivariate generalised Gaussian distribution.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
dmggd(c(0, 1, 4), mu, Sigma, beta)
dmggd(c(1, 2, 3), mu, Sigma, beta)
```

`estparmggd` *Estimation of the Parameters of a Multivariate Generalized Gaussian Distribution*

Description

Estimation of the mean vector, dispersion matrix and shape parameter of a multivariate generalized Gaussian distribution (MGGD).

Usage

```
estparmggd(x, eps = 1e-6, display = FALSE, plot = display)
```

Arguments

`x`	numeric matrix or data frame.
`eps`	numeric. Precision for the estimation of the beta parameter.
`display`	logical. When `TRUE` the value of the `beta` parameter at each iteration is printed.
`plot`	logical. When `TRUE` the successive values of the `beta` parameter are plotted, allowing to visualise its convergence.

Details

The μ parameter is the mean vector of `x`.

The dispersion matrix Σ and shape parameter: β are computed using the method presented in Pascal et al., using an iterative algorithm.

The precision for the estimation of `beta` is given by the `eps` parameter.

Value

A list of 3 elements:

- `mu` the mean vector.
- `Sigma`: symmetric positive-definite matrix. The dispersion matrix.
- `beta` non-negative numeric value. The shape parameter.

with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

F. Pascal, L. Bombrun, J.Y. Tourneret, Y. Berthoumieu. Parameter Estimation For Multivariate Generalized Gaussian Distribution. IEEE Trans. Signal Processing, vol. 61 no. 23, p. 5960-5971, Dec. 2013. doi: 10.1109/TSP.2013.2282909

See Also

dmggd: probability density of a MGGD.

rmggd: random generation from a MGGD.

Examples

```
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
x <- rmggd(100, mu, Sigma, beta)

# Estimation of the parameters
estparmggd(x)
```

---

kldggd — *Kullback-Leibler Divergence between Centered Multivariate generalized Gaussian Distributions*

Description

Computes the Kullback- Leibler divergence between two random variables distributed according to multivariate generalized Gaussian distributions (MGGD) with zero means.

Usage

```
kldggd(Sigma1, beta1, Sigma2, beta2, eps = 1e-06)
```

Arguments

`Sigma1`	symmetric, positive-definite matrix. The dispersion matrix of the first distribution.
`beta1`	positive real number. The shape parameter of the first distribution.
`Sigma2`	symmetric, positive-definite matrix. The dispersion matrix of the second distribution.
`beta2`	positive real number. The shape parameter of the second distribution.
`eps`	numeric. Precision for the computation of the Lauricella function (see lauricella). Default: 1e-06.

Details

Given $\mathbf{X}_1$, a random vector of $\mathbb{R}^p$ ($p > 1$) distributed according to the MGGD with parameters $(\mathbf{0}, \Sigma_1, \beta_1)$ and $\mathbf{X}_2$, a random vector of $\mathbb{R}^p$ distributed according to the MGGD with parameters $(\mathbf{0}, \Sigma_2, \beta_2)$.

The Kullback-Leibler divergence between X_1 and X_2 is given by:

$$KL(\mathbf{X}_1||\mathbf{X}_2) = \ln\left(\frac{\beta_1|\Sigma_1|^{-1/2}\Gamma\left(\frac{p}{2\beta_2}\right)}{\beta_2|\Sigma_2|^{-1/2}\Gamma\left(\frac{p}{2\beta_1}\right)}\right) + \frac{p}{2}\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)\ln 2 - \frac{p}{2\beta_2} + 2^{\frac{\beta_2}{\beta_1}-1}\frac{\Gamma\left(\frac{\beta_2}{\beta_1} + \frac{p}{\beta_1}\right)}{\Gamma\left(\frac{p}{2\beta_1}\right)}\lambda_p^{\beta_2}$$

$$\times F_D^{(p-1)}\left(-\beta_1; \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{p-1}; \frac{p}{2}; 1 - \frac{\lambda_{p-1}}{\lambda_p}, \dots, 1 - \frac{\lambda_1}{\lambda_p}\right)$$

where $\lambda_1 < ... < \lambda_{p-1} < \lambda_p$ are the eigenvalues of the matrix $\Sigma_1\Sigma_2^{-1}$
and $F_D^{(p-1)}$ is the Lauricella D-hypergeometric Function.

This computation uses the `lauricella` function.

When $p = 1$ (univariate case): let X_1, a random variable distributed according to the generalized Gaussian distribution with parameters $(0, \sigma_1, \beta_1)$ and X_2, a random variable distributed according to the generalized Gaussian distribution with parameters $(0, \sigma_2, \beta_2)$.

$$KL(X_1||X_2) = \ln\left(\frac{\frac{\beta_1}{\sqrt{\sigma_1}}\Gamma\left(\frac{1}{2\beta_2}\right)}{\frac{\beta_2}{\sqrt{\sigma_2}}\Gamma\left(\frac{1}{2\beta_1}\right)}\right) + \frac{1}{2}\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)\ln 2 - \frac{1}{2\beta_2} + 2^{\frac{\beta_2}{\beta_1}-1}\frac{\Gamma\left(\frac{\beta_2}{\beta_1} + \frac{1}{\beta_1}\right)}{\Gamma\left(\frac{1}{2\beta_1}\right)}\left(\frac{\sigma_1}{\sigma_2}\right)^{\beta_2}$$

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the result of the Lauricella function; 0 if the distributions are univariate) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

See Also

dmggd: probability density of a MGGD.

Examples

```
beta1 <- 0.74
beta2 <- 0.55
Sigma1 <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.2, 0.3, 0.5, 0.1, 0.2, 0.1, 0.7), nrow = 3)

# Kullback-Leibler divergence
kl12 <- kldggd(Sigma1, beta1, Sigma2, beta2)
```

```
kl21 <- kldggd(Sigma2, beta2, Sigma1, beta1)
print(kl12)
print(kl21)

# Distance (symmetrized Kullback-Leibler divergence)
kldist <- as.numeric(kl12) + as.numeric(kl21)
print(kldist)
```

`lauricella` *Lauricella D-Hypergeometric Function*

Description

Computes the Lauricella D-hypergeometric Function function.

Usage

```
lauricella(a, b, g, x, eps = 1e-06)
```

Arguments

a	numeric.
b	numeric vector.
g	numeric.
x	numeric vector. x must have the same length as b.
eps	numeric. Precision for the nested sums (default 1e-06).

Details

If n is the length of the b and x vectors, the Lauricella D-hypergeometric Function function is given by:

$$F_D^{(n)}\left(a, b_1, ..., b_n, g; x_1, ..., x_n\right) = \sum_{m_1 \geq 0} ... \sum_{m_n \geq 0} \frac{(a)_{m_1+...+m_n}(b_1)_{m_1} ... (b_n)_{m_n}}{(g)_{m_1+...+m_n}} \frac{x_1^{m_1}}{m_1!} ... \frac{x_n^{m_n}}{m_n!}$$

where $(x)_p$ is the Pochhammer symbol (see pochhammer).

If $|x_i| < 1, i = 1, \dots, n$, this sum converges. Otherwise there is an error.

The eps argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

Sometimes, the convergence is too slow and the required precision cannot be reached. If this happens, the `attr(, "epsilon")` attribute is the precision that was really reached.

Value

A numeric value: the value of the Lauricella function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel, A. Dziri, Kullback-Leibler Divergence Between Multivariate Generalized Gaussian Distributions. IEEE Signal Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions. IEEE Signal Processing Letters, vol. 30, pp. 1672-1676, October 2023. doi:10.1109/LSP.2023.3324594

`lnpochhammer` *Logarithm of the Pochhammer Symbol*

Description

Computes the logarithm of the Pochhammer symbol.

Usage

```
lnpochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

So, if $n > 0$:

$$log\left((x)_n\right) = log(x) + log(x+1) + ... + log(x+n-1)$$

If $n = 0$, $log\left((x)_n\right) = log(1) = 0$

Value

Numeric value. The logarithm of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

pochhammer()

Examples

```
lnpochhammer(2, 0)
lnpochhammer(2, 1)
lnpochhammer(2, 3)
```

---

plotmggd — *Plot of the Bivariate Generalised Gaussian Density*

Description

Plots the probability density of the generalised Gaussian distribution with 2 variables with mean vector `mu`, dispersion matrix `Sigma` and shape parameter `beta`.

Usage

```
plotmggd(mu, Sigma, beta, xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                  ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]), n = 101,
                  xvals = NULL, yvals = NULL, xlab = "x", ylab = "y",
                  zlab = "f(x,y)", col = "gray", tol = 1e-6, ...)
```

Arguments

`mu`	length 2 numeric vector.
`Sigma`	symmetric, positive-definite square matrix of order 2. The dispersion matrix.
`beta`	positive real number. The shape of the distribution.
`xlim, ylim`	x-and y- limits.
`n`	A one or two element vector giving the number of steps in the x and y grid, passed to `plot3d.function`.
`xvals, yvals`	The values at which to evaluate `x` and `y`. If used, `xlim` and/or `ylim` are ignored.
`xlab, ylab, zlab`	The axis labels.
`col`	The color to use for the plot. See `plot3d.function`.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. see `dmggd`.
`...`	Additional arguments to pass to `plot3d.function`.

Value

Returns invisibly the probability density function.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

contourmggd: contour plot of a bivariate generalised Gaussian density.

dmggd: Probability density of a multivariate generalised Gaussian distribution.

Examples

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
beta <- 0.74
plotmggd(mu, Sigma, beta)
```

---

pochhammer *Pochhammer Symbol*

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

x	numeric.
n	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

---

rmggd *Simulate from a Multivariate Generalized Gaussian Distribution*

Description

Produces one or more samples from a multivariate (p variables) generalized Gaussian distribution (MGGD).

Usage

```
rmggd(n = 1 , mu, Sigma, beta, tol = 1e-6)
```

Arguments

n	integer. Number of observations.
mu	length p numeric vector. The mean vector.
Sigma	symmetric, positive-definite square matrix of order p. The dispersion matrix.
beta	positive real number. The shape of the distribution.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

A sample from a centered MGGD with dispersion matrix Σ and shape parameter β can be generated using:

$$X = \tau\, \Sigma^{1/2}\, U$$

where U is a random vector uniformly distributed on the unit sphere and τ is such that $\tau^{2\beta}$ is generated from a distribution Gamma with shape parameter $\frac{p}{2\beta}$ and scale parameter 2.

This property is used to generate a sample from a MGGD.

Value

A matrix with p columns and n rows.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

E. Gomez, M. Gomez-Villegas, H. Marin. A Multivariate Generalization of the Power Exponential Family of Distribution. Commun. Statist. 1998, Theory Methods, col. 27, no. 23, p 589-600. doi:10.1080/03610929808832115

See Also

dmggd: probability density of a MGGD..

estparmggd: estimation of the parameters of a MGGD.

Examples

```
mu <- c(0, 0, 0)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
beta <- 0.74
rmggd(100, mu, Sigma, beta)
```

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