

Package ‘StochFracPoisson’

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Title Stochastic Poisson Processes and Fractional Counting Models

Version 0.1.0

Description Implementation of advanced stochastic counting processes.

Main models include the Fractional Counting Process at Levy times (Garg et al. (2025) <[doi:10.1007/s10955-025-03515-9](https://doi.org/10.1007/s10955-025-03515-9)>), the Generalized Iterated Poisson Process (Soni & Pathak (2024) <[doi:10.1007/s10959-024-01362-0](https://doi.org/10.1007/s10959-024-01362-0)>), the Generalized Fractional Risk Process (Soni & Pathak (2024) <[doi:10.1007/s11009-024-10111-z](https://doi.org/10.1007/s11009-024-10111-z)>), and the Tempered Space-Time Fractional Negative Binomial Process (Garg et al. (2025) <[doi:10.1007/s11009-025-10179-1](https://doi.org/10.1007/s11009-025-10179-1)>).

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Suggests testthat (>= 3.0.0)

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alternative_gfrp	<i>Alternative Generalized Fractional Risk Process (AGFRP)</i>
------------------	--

Description

Computes the alternative GFRP with premium function $\hat{R}^\beta(t) = \nu + \eta(1 + \rho)\epsilon(W_\beta(t)) - \sum_{j=1}^{N^\beta(t)} X_j$

Usage

```
alternative_gfrp(
  nu,
  eta,
  rho,
  beta,
  lambda,
  t,
  epsilon = function(x) x,
  claim_dist = "gamma",
```

```

    claim_params = list(),
    n_sim = 1000
)

```

Arguments

nu	Initial capital (> 0)
eta	Mean claim size (> 0)
rho	Safety loading parameter (≥ 0)
beta	Fractional parameter ($0 < \beta < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
t	Time (> 0)
epsilon	Premium function (default = identity)
claim_dist	Distribution of claim sizes ("gamma", "exponential", "lognormal")
claim_params	Parameters for claim distribution
n_sim	Number of simulations (default = 1000)

Value

Simulated values of the alternative risk process at time t

Examples

```

alternative_gfrp(100, 10, 0.1, 0.7, c(1.0, 0.5), 1.0,
  function(x) x^0.5, "gamma", list(shape = 2, rate = 0.2))

```

bell_polynomial	<i>Classical Bell Polynomials</i>
-----------------	-----------------------------------

Description

Computes the classical Bell polynomials $B_n(x)$

Usage

```
bell_polynomial(n, x, n_terms = 50)
```

Arguments

n	Polynomial order (non-negative integer)
x	Argument
n_terms	Number of terms in series expansion (default = 50)

Value

Value of the Bell polynomial

Examples

```
bell_polynomial(5, 1.0)
```

check_lrd_gipp	<i>Check Long-Range Dependence of GIPP</i>
----------------	--

Description

Checks if the GIPP exhibits long-range dependence

Usage

```
check_lrd_gipp(beta, lambda, lambda_beta, s, t_max = 100)
```

Arguments

beta	Fractional parameter ($0 < \beta < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
s	Fixed time for correlation analysis
t_max	Maximum time for asymptotic analysis

Value

Logical indicating LRD property

Examples

```
check_lrd_gipp(0.7, 1.0, 0.5, 0.5, 100)
```

check_martingale_gfrp	<i>Check Martingale Property of GFRP</i>
-----------------------	--

Description

Checks if the GFRP is a martingale, submartingale, or supermartingale

Usage

```
check_martingale_gfrp(rho)
```

Arguments

rho	Safety loading parameter (≥ 0)
-----	---------------------------------------

Value

String indicating martingale property

Examples

```
check_martingale_gfrp(0.1)
```

compound_gfcp	<i>Compound Generalized Fractional Counting Process (CGFCP)</i>
---------------	---

Description

Computes the probability mass function of the CGFCP $C^\beta(t) = \sum_{j=1}^{N^\beta(t)} X_j$ From Soni and Pathak (2024)

Usage

```
compound_gfcp(  
  beta,  
  lambda,  
  t,  
  n,  
  jump_dist = "poisson",  
  jump_params = list(),  
  n_terms = 50  
)
```

Arguments

beta	Fractional parameter (0 < beta < 1)
lambda	Vector of rates lambda_1, ..., lambda_k for the GFCP
t	Time (> 0)
n	Vector of non-negative integers
jump_dist	Distribution of jump sizes ("poisson", "geometric")
jump_params	Parameters for jump distribution
n_terms	Number of terms in series expansion (default = 50)

Value

Vector of probabilities P(C(t) = n)

Examples

```
compound_gfcp(0.7, c(1.0, 0.5), 1.0, 0:10, "poisson", list(lambda = 2))
```

cov_gipp	<i>Covariance of GIPP</i>
----------	---------------------------

Description

Computes the covariance of the GIPP

Usage

```
cov_gipp(beta, lambda, lambda_beta, s, t)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
s	First time ($0 < s \leq t$)
t	Second time

Value

Covariance value

Examples

```
cov_gipp(0.7, 1.0, 0.5, 0.5, 1.0)
```

cov_inverse_stable	<i>Covariance of Inverse Stable Subordinator</i>
--------------------	--

Description

Computes the covariance of the inverse beta-stable subordinator Based on Leonenko et al. (2014)

Usage

```
cov_inverse_stable(s, t, beta)
```

Arguments

s	First time
t	Second time ($s \leq t$)
beta	Stability parameter ($0 < \text{beta} < 1$)

Value

Covariance value

Examples

```
cov_inverse_stable(0.5, 1, 0.8)
```

cov_tstfnbp	<i>Covariance of TSTFNBP</i>
-------------	------------------------------

Description

Computes the covariance of the TSTFNBP

Usage

```
cov_tstfnbp(beta, alpha, mu, lambda1, beta1, lambda, s, t)
```

Arguments

- beta Fractional parameter ($0 < \text{beta} < 1$)
- alpha Stability parameter ($0 < \text{alpha} < 1$)
- mu Tempering parameter (> 0)
- lambda1 Gamma subordinator rate ($> \text{mu} * \text{alpha}$)
- beta1 Gamma subordinator shape (> 0)
- lambda Poisson parameter (> 0)
- s First time ($0 < s \leq t$)
- t Second time

Value

Covariance value

Examples

```
cov_tstfnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 0.5, 1.0)
```

create_fcp	Create FCP Object
------------	-------------------

Description

Creates an object representing the Fractional Counting Process

Usage

```
create_fcp(mu, zeta, theta, lambda, t, n, vartheta = NULL, n_terms = 50)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
n	Vector of non-negative integers
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_terms	Number of terms in series expansion (default = 50)

Value

An object of class fcp containing parameters, probabilities, mean, and variance.

Examples

```
fcp_obj <- create_fcp(0.8, 1.0, 0.5, 1.0, 1.0, 0:10)
```

create_gipp	Create GIPP Object
-------------	--------------------

Description

Creates an object representing the GIPP

Usage

```
create_gipp(beta, lambda, lambda_beta, t, k, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)
k	Vector of non-negative integers
n_terms	Number of terms in series expansion (default = 50)

Value

An object of class gipp containing parameters, probabilities, mean, and variance.

Examples

```
gipp_obj <- create_gipp(0.7, 1.0, 0.5, 1.0, 0:10)
```

create_tcfcp	<i>Create TCFCP Object</i>
--------------	----------------------------

Description

Creates an object representing the Time-Changed Fractional Counting Process

Usage

```
create_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  n,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL,
  n_terms = 50
)
```

Arguments

mu	Parameter mu ($0 < \text{mu} \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \text{theta} \leq 1$)
lambda	Parameter lambda (> 0)

t	Time (> 0)
n	Vector of non-negative integers
subordinator_type	Type of subordinator
subordinator_params	List of parameters for the subordinator
vartheta	Parameter ϑ ($\geq \mu \cdot \zeta$, default = ζ)
n_terms	Number of terms in series expansion (default = 50)

Value

An object of class `tcfc` containing parameters, probabilities, mean, and variance.

Examples

```
tcfc_obj <- create_tcfc(0.8, 1.0, 0.5, 1.0, 1.0, 0:10, "gamma", list(shape = 2, rate = 1))
```

dgamma_subordinator *PDF of Gamma Subordinator*

Description

Computes the probability density function of the gamma subordinator

Usage

```
dgamma_subordinator(x, t, shape, rate)
```

Arguments

x	Value
t	Time
shape	Shape parameter (β_1)
rate	Rate parameter (λ_1)

Value

PDF value

Examples

```
dgamma_subordinator(1, 1, 2, 1)
```

dtempered_mittag_leffler_levy

PDF of Tempered Mittag-Leffler Lévy Process

Description

Computes the probability density function of the tempered Mittag-Leffler Lévy process Based on Kumar et al. (2019b)

Usage

```
dtempered_mittag_leffler_levy(x, t, alpha, mu, lambda1, beta1, n_terms = 50)
```

Arguments

x	Value
t	Time
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate (> 0)
beta1	Gamma subordinator shape (> 0)
n_terms	Number of terms in series expansion (default = 50)

Value

PDF value

Examples

```
dtempered_mittag_leffler_levy(1, 1, 0.8, 0.5, 2, 1)
```

egf_fractional_bell

Exponential Generating Function of Fractional Bell Polynomials

Description

Computes the exponential generating function $\sum_{m=0}^{\infty} (u^m/m!) B_m^{\beta}(x) = L_{\beta}(x(e^u - 1))$

Usage

```
egf_fractional_bell(beta, x, u, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
x	Argument
u	Generating function variable
n_terms	Number of terms in series expansion (default = 50)

Value

Value of the exponential generating function

Examples

```
egf_fractional_bell(0.7, 1.0, 0.5)
```

fde_cgfcf

*Fractional Differential Equation for CGFCP***Description**

Computes the fractional differential equation governing the CGFCP

Usage

```
fde_cgfcf(
  beta,
  lambda,
  t,
  n,
  jump_dist = "poisson",
  jump_params = list(),
  h = 1e-06
)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
t	Time (> 0)
n	Value at which to evaluate the derivative
jump_dist	Distribution of jump sizes
jump_params	Parameters for jump distribution
h	Step size for numerical differentiation (default = $1e-6$)

Value

Value of the fractional derivative

Examples

```
fde_cgfcg(0.7, c(1.0, 0.5), 1.0, 5, "poisson", list(lambda = 2))
```

```
first_passage_density_gipp
```

First Passage Time Density of GIPP

Description

Computes the density of the first passage time for GIPP

Usage

```
first_passage_density_gipp(beta, lambda, lambda_beta, t, k, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)
k	Threshold level (positive integer)
n_terms	Number of terms in series expansion (default = 50)

Value

Density value at t

Examples

```
first_passage_density_gipp(0.7, 1.0, 0.5, 1.0, 3)
```

first_passage_time_fcp

First Passage Time Distribution of FCP

Description

Computes the survival function of the first passage time $P(T_w > t)$ where $T_w = \inf\{t \geq 0: N(t) \geq w\}$

Usage

```
first_passage_time_fcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  w,
  vartheta = NULL,
  n_terms = 50
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
w	Threshold level (positive integer)
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_terms	Number of terms in series expansion (default = 50)

Value

Survival probability $P(T_w > t)$

Examples

```
first_passage_time_fcp(0.8, 1.0, 0.5, 1.0, 1.0, 3)
```

first_passage_time_gipp

First Passage Time Distribution of GIPP

Description

Computes the survival function of the first passage time for GIPP

Usage

```
first_passage_time_gipp(beta, lambda, lambda_beta, t, k, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)
k	Threshold level (positive integer)
n_terms	Number of terms in series expansion (default = 50)

Value

Survival probability $P(T_k > t)$

Examples

```
first_passage_time_gipp(0.7, 1.0, 0.5, 1.0, 3)
```

first_passage_time_tcfcp

First Passage Time Distribution of TCFCP

Description

Computes the survival function of the first passage time for TCFCP

Usage

```

first_passage_time_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  w,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL,
  n_terms = 50
)

```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
w	Threshold level (positive integer)
subordinator_type	Type of subordinator
subordinator_params	List of parameters for the subordinator
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_terms	Number of terms in series expansion (default = 50)

Value

Survival probability $P(T_w > t)$

Examples

```

first_passage_time_tcfcp(0.8, 1.0, 0.5, 1.0, 1.0, 3, "gamma", list(shape = 2, rate = 1))

```

first_waiting_time_fcp

First Waiting Time Distribution of FCP

Description

Computes the first waiting time distribution of the FCP

Usage

```
first_waiting_time_fcp(mu, zeta, theta, lambda, tau, vartheta = NULL)
```

Arguments

<code>mu</code>	Parameter μ ($0 < \mu \leq 1$)
<code>zeta</code>	Parameter ζ (> 0)
<code>theta</code>	Parameter θ ($0 < \theta \leq 1$)
<code>lambda</code>	Parameter λ (> 0)
<code>tau</code>	Time (> 0)
<code>vartheta</code>	Parameter ϑ ($\geq \mu \cdot \zeta$, default = ζ)

Value

First waiting time density at τ

Examples

```
first_waiting_time_fcp(0.8, 1.0, 0.5, 1.0, 0.5)
```

```
first_waiting_time_tcfcp
```

First Waiting Time Distribution of TCFCP

Description

Computes the first waiting time distribution of the TCFCP

Usage

```
first_waiting_time_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  tau,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL,
  n_terms = 50
)
```

Arguments

<code>mu</code>	Parameter μ ($0 < \mu \leq 1$)
<code>zeta</code>	Parameter ζ (> 0)
<code>theta</code>	Parameter θ ($0 < \theta \leq 1$)
<code>lambda</code>	Parameter λ (> 0)
<code>tau</code>	Time (> 0)
<code>subordinator_type</code>	Type of subordinator
<code>subordinator_params</code>	List of parameters for the subordinator
<code>vartheta</code>	Parameter ϑ ($\geq \mu \cdot \zeta$, default = ζ)
<code>n_terms</code>	Number of terms in series expansion (default = 50)

Value

First waiting time density at τ

Examples

```
first_waiting_time_tcfcp(0.8, 1.0, 0.5, 1.0, 0.5, "gamma", list(shape = 2, rate = 1))
```

<code>fractional_bell_polynomial</code>
<i>Fractional Bell Polynomials</i>

Description

Computes the fractional Bell polynomials $B_n^\beta(x)$ Uses log-space computations to prevent numerical overflow for large orders. From Soni and Pathak (2024)

Usage

```
fractional_bell_polynomial(n, beta, x, n_terms = 50)
```

Arguments

<code>n</code>	Polynomial order (non-negative integer)
<code>beta</code>	Fractional parameter ($0 < \beta < 1$)
<code>x</code>	Argument
<code>n_terms</code>	Number of terms in series expansion (default = 50)

Value

Value of the fractional Bell polynomial

Examples

```
fractional_bell_polynomial(5, 0.7, 1.0)
```

```
fractional_counting_process
```

Fractional Counting Process (FCP)

Description

Computes the probability mass function of the Fractional Counting Process based on the three-parameter Mittag-Leffler function (Prabhakar function) From Garg et al. (2025)

Usage

```
fractional_counting_process(  
  mu,  
  zeta,  
  theta,  
  lambda,  
  t,  
  n,  
  vartheta = NULL,  
  n_terms = 50  
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
n	Vector of non-negative integers
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_terms	Number of terms in series expansion (default = 50)

Value

Vector of probabilities $P(N(t) = n)$

Examples

```
fractional_counting_process(0.8, 1.0, 0.5, 1.0, 1.0, 0:10)
```

fractional_stirling2	<i>Fractional Stirling Numbers of the Second Kind</i>
----------------------	---

Description

Computes the fractional Stirling numbers of the second kind $S_{\beta}(m, k)$ Uses log-space computation to prevent numerical overflow. From Soni and Pathak (2024)

Usage

fractional_stirling2(m, k, beta)

Arguments

- m Upper index (non-negative integer)
- k Lower index (non-negative integer)
- beta Fractional parameter (0 < beta < 1)

Value

Value of the fractional Stirling number

Examples

fractional_stirling2(5, 3, 0.7)

gamma_subordinator	<i>Gamma Subordinator</i>
--------------------	---------------------------

Description

Simulates a gamma subordinator $G(t)$ with shape parameter $\text{beta1} \cdot t$ and rate lambda1

Usage

gamma_subordinator(t, shape, rate, n = 1)

Arguments

- t Time
- shape Shape parameter (beta1)
- rate Rate parameter (lambda1)
- n Number of simulations

Value

Simulated values of the gamma subordinator

Examples

```
gamma_subordinator(1, 2, 1, 10)
```

generalized_fractional_risk_process
<i>Generalized Fractional Risk Process (GFRP)</i>

Description

Computes the Generalized Fractional Risk Process for insurance applications $R^\beta(t) = \nu + \eta(1 + \rho) \sum_{i=1}^k i \lambda_i W_\beta(t) - \sum_{j=1}^{N^\beta(t)} X_j$ From Soni and Pathak (2024)

Usage

```
generalized_fractional_risk_process(  
  nu,  
  eta,  
  rho,  
  beta,  
  lambda,  
  t,  
  claim_dist = "gamma",  
  claim_params = list(),  
  n_sim = 1000  
)
```

Arguments

nu	Initial capital (> 0)
eta	Mean claim size (> 0)
rho	Safety loading parameter (>= 0)
beta	Fractional parameter (0 < beta < 1)
lambda	Vector of rates lambda_1, ..., lambda_k for the GFCP
t	Time (> 0)
claim_dist	Distribution of claim sizes ("gamma", "exponential", "lognormal")
claim_params	Parameters for claim distribution
n_sim	Number of simulations (default = 1000)

Value

Simulated values of the risk process at time t

Examples

```
generalized_fractional_risk_process(100, 10, 0.1, 0.7, c(1.0, 0.5), 1.0,
                                   "gamma", list(shape = 2, rate = 0.2))
```

generalized_fractional_stirling2
<i>Generalized Fractional Stirling Numbers of the Second Kind</i>

Description

Computes the generalized fractional Stirling numbers $S_{\mu,\vartheta}(p,i)$ From Laskin (2024)

Usage

```
generalized_fractional_stirling2(p, i, mu, vartheta)
```

Arguments

p	Upper index (non-negative integer)
i	Lower index (non-negative integer)
mu	Parameter mu ($0 < \mu \leq 1$)
vartheta	Parameter vartheta (> 0)

Value

Value of the generalized fractional Stirling number

Examples

```
generalized_fractional_stirling2(5, 3, 0.8, 1.0)
```

generalized_iterated_poisson
<i>Generalized Iterated Poisson Process (GIPP)</i>

Description

Computes the probability mass function of the Generalized Iterated Poisson Process $Q(t) = N(N_beta(t), \lambda)$ - composition of HPP with TFPP. From Soni and Pathak (2024)

Usage

```
generalized_iterated_poisson(beta, lambda, lambda_beta, t, k, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)
k	Vector of non-negative integers
n_terms	Number of terms in series expansion (default = 50)

Value

Vector of probabilities $P(Q(t) = k)$

Examples

```
generalized_iterated_poisson(0.7, 1.0, 0.5, 1.0, 0:10)
```

gfcf

Generalized Fractional Counting Process (GFCP)

Description

Computes the probability mass function of the GFCP (with jumps of size 1, ..., k) using PGF inversion via FFT.

Usage

```
gfcf(beta, lambda, t, n)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$
t	Time (> 0)
n	Vector of non-negative integers

Value

Vector of probabilities $P(N(t) = n)$

Examples

```
gfcf(0.7, c(1.0, 0.5), 1.0, 0:10)
```

incomplete_beta	<i>Incomplete Beta Function</i>
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Description

Computes the incomplete beta function $B(a, b; x) = \int_0^x t^{a-1} (1-t)^{b-1} dt$

Usage

```
incomplete_beta(a, b, x)
```

Arguments

a	Parameter a > 0
b	Parameter b > 0
x	Upper limit of integration (0 <= x <= 1)

Value

Value of the incomplete beta function

Examples

```
incomplete_beta(0.5, 0.5, 0.5)
```

inverse_stable_subordinator	<i>Inverse Stable Subordinator</i>
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Description

Simulates an inverse beta-stable subordinator $E_{\text{beta}}(t) = \inf\{r \geq 0: S_{\text{beta}}(r) > t\}$ Using the exact relationship $E_{\text{beta}}(t) = t^{\text{beta}} * S_{\text{beta}}(1)^{(-\text{beta})}$.

Usage

```
inverse_stable_subordinator(t, beta, n = 1)
```

Arguments

t	Time
beta	Stability parameter (0 < beta < 1)
n	Number of simulations

Value

Simulated values of the inverse stable subordinator

Examples

```
inverse_stable_subordinator(1, 0.8, 10)
```

invert_pgf

Numerical Inversion of Probability Generating Function

Description

Computes the probability mass function (PMF) of a discrete random variable from its PGF using Fast Fourier Transform (FFT).

Usage

```
invert_pgf(pgf_func, max_n, r = 0.85, M = NULL)
```

Arguments

pgf_func	A function that evaluates the PGF at complex arguments
max_n	Maximum value of the random variable to compute the probability for
r	Radius of the contour circle ($0 < r < 1$, default = 0.85)
M	Number of points on the circle (must be a power of 2, default = NULL)

Value

Vector of probabilities $P(X = 0), \dots, P(X = \text{max_n})$

lt_fcp

Laplace Transform of FCP

Description

Computes the Laplace transform of the FCP (expectation of $\exp(-s * N(t))$)

Usage

```
lt_fcp(mu, zeta, theta, lambda, t, s, vartheta = NULL)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
s	Laplace parameter
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)

Value

Laplace transform value

Examples

```
lt_fcp(0.8, 1.0, 0.5, 1.0, 1.0, 0.5)
```

lt_tcfcp

Laplace Transform of TCFCP

Description

Computes the Laplace transform of the TCFCP (expectation of $\exp(-s * Z(t))$)

Usage

```
lt_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  s,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL,
  n_terms = 50
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
s	Laplace parameter
subordinator_type	Type of subordinator
subordinator_params	List of parameters for the subordinator
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_terms	Number of terms in series expansion (default = 50)

Value

Laplace transform value

Examples

```
lt_tcfcp(0.8, 1.0, 0.5, 1.0, 1.0, 0.5, "gamma", list(shape = 2, rate = 1))
```

lt_tempered_mittag_leffler_levy

Laplace Transform of Tempered Mittag-Leffler Lévy Process

Description

Computes the Laplace transform $E(\exp(-u * M(t)))$

Usage

```
lt_tempered_mittag_leffler_levy(u, t, alpha, mu, lambda1, beta1)
```

Arguments

u	Laplace transform parameter
t	Time
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate (> 0)
beta1	Gamma subordinator shape (> 0)

Value

Laplace transform value

Examples

```
lt_tempered_mittag_leffler_levy(1, 1, 0.8, 0.5, 2, 1)
```

lt_tstfnbp

Laplace Transform of TSTFNBP

Description

Computes the Laplace transform of the TSTFNBP (expectation of $\exp(-u * Q(t))$)

Usage

```
lt_tstfnbp(beta, alpha, mu, lambda1, beta1, lambda, t, u, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
alpha	Stability parameter ($0 < \text{alpha} < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate ($> \text{mu} * \text{alpha}$)
beta1	Gamma subordinator shape (> 0)
lambda	Poisson parameter (> 0)
t	Time (> 0)
u	Laplace parameter
n_terms	Number of terms in series expansion (default = 50)

Value

Laplace transform value

Examples

```
lt_tstfnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 1.0, 0.5)
```

mean_cgfcf

Mean of CGFCF

Description

Computes the mean of the CGFCF

Usage

```
mean_cgfcf(beta, lambda, t, jump_dist = "poisson", jump_params = list())
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCF
t	Time (> 0)
jump_dist	Distribution of jump sizes ("poisson", "geometric")
jump_params	Parameters for jump distribution

Value

Mean value

Examples

```
mean_cgfcf(0.7, c(1.0, 0.5), 1.0, "poisson", list(lambda = 2))
```

mean_fcp

Mean of Fractional Counting Process

Description

Computes the mean of the FCP

Usage

```
mean_fcp(mu, zeta, theta, lambda, t, vartheta = NULL)
```

Arguments

mu	Parameter μ ($0 < \mu \leq 1$)
zeta	Parameter ζ (> 0)
theta	Parameter θ ($0 < \theta \leq 1$)
lambda	Parameter λ (> 0)
t	Time (> 0)
vartheta	Parameter ϑ ($\geq \mu \cdot \zeta$, default = ζ)

Value

Mean value

Examples

```
mean_fcp(0.8, 1.0, 0.5, 1.0, 1.0)
```

mean_gfrp	<i>Mean of GFRP</i>
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Description

Computes the mean of the GFRP

Usage

```
mean_gfrp(nu, eta, rho, beta, lambda, t)
```

Arguments

- nu Initial capital (> 0)
- eta Mean claim size (> 0)
- rho Safety loading parameter (≥ 0)
- beta Fractional parameter ($0 < \beta < 1$)
- lambda Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
- t Time (> 0)

Value

Mean value

Examples

```
mean_gfrp(100, 10, 0.1, 0.7, c(1.0, 0.5), 1.0)
```

`mean_gipp`*Mean of GIPP*

Description

Computes the mean of the GIPP

Usage

```
mean_gipp(beta, lambda, lambda_beta, t)
```

Arguments

<code>beta</code>	Fractional parameter ($0 < \text{beta} < 1$)
<code>lambda</code>	Parameter of outer Poisson process (> 0)
<code>lambda_beta</code>	Parameter of inner fractional Poisson process (> 0)
<code>t</code>	Time (> 0)

Value

Mean value

Examples

```
mean_gipp(0.7, 1.0, 0.5, 1.0)
```

`mean_inverse_stable`*Mean of Inverse Stable Subordinator*

Description

Computes the mean of the inverse beta-stable subordinator

Usage

```
mean_inverse_stable(t, beta)
```

Arguments

<code>t</code>	Time
<code>beta</code>	Stability parameter ($0 < \text{beta} < 1$)

Value

Mean value

Examples

```
mean_inverse_stable(1, 0.8)
```

mean_tcfcp	<i>Mean of Time-Changed FCP</i>
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Description

Computes the mean of the TCFCP

Usage

```
mean_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
subordinator_type	Type of subordinator
subordinator_params	List of parameters for the subordinator
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)

Value

Mean value

Examples

```
mean_tcfcp(0.8, 1.0, 0.5, 1.0, 1.0, "gamma", list(shape = 2, rate = 1))
```

mean_tstfnbp	<i>Mean of TSTFNBP</i>
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Description

Computes the mean of the TSTFNBP

Usage

```
mean_tstfnbp(beta, alpha, mu, lambda1, beta1, lambda, t)
```

Arguments

beta	Fractional parameter ($0 < \beta < 1$)
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate ($> \mu * \alpha$)
beta1	Gamma subordinator shape (> 0)
lambda	Poisson parameter (> 0)
t	Time (> 0)

Value

Mean value

Examples

```
mean_tstfnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 1.0)
```

mgamma_subordinator	<i>Moment of Gamma Subordinator</i>
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Description

Computes the q-th moment of the gamma subordinator

Usage

```
mgamma_subordinator(q, t, shape, rate)
```

Arguments

q	Order of moment
t	Time
shape	Shape parameter (beta1)
rate	Rate parameter (lambda1)

Value

q-th moment value

Examples

mgamma_subordinator(2, 1, 2, 1)

mgf_fcp	<i>Moment Generating Function of FCP</i>
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Description

Computes the moment generating function of the Fractional Counting Process

Usage

mgf_fcp(mu, zeta, theta, lambda, t, s, vartheta = NULL)

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
s	MGF argument
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)

Value

MGF value

Examples

mgf_fcp(0.8, 1.0, 0.5, 1.0, 1.0, 0.5)

mgf_gipp	<i>Moment Generating Function of GIPP</i>
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Description

Computes the moment generating function of the GIPP

Usage

```
mgf_gipp(beta, lambda, lambda_beta, t, u)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)
u	MGF argument

Value

MGF value

Examples

```
mgf_gipp(0.7, 1.0, 0.5, 1.0, 0.5)
```

mittag_leffler	<i>Mittag-Leffler Function</i>
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Description

Computes the Mittag-Leffler function $E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} z^k / \Gamma(\alpha k + \beta)$ Supports complex arguments and uses log-space computation for numerical stability.

Usage

```
mittag_leffler(alpha, beta = 1, z, n_terms = 100, tolerance = 1e-10)
```

Arguments

alpha	Parameter $\alpha > 0$
beta	Parameter $\beta > 0$ (default = 1)
z	Complex argument (can be a vector)
n_terms	Number of terms in series expansion (default = 100)
tolerance	Convergence tolerance (default = 1e-10)

Value

Value of the Mittag-Leffler function

Examples

```
mittag_leffler(0.8, 1, -1)
mittag_leffler(1, 1, 1) # Should equal exp(1)
```

```
mittag_leffler_prabhakar
```

Prabhakar (Three-Parameter) Mittag-Leffler Function

Description

Computes the generalized three-parameter Mittag-Leffler function $E_{\alpha,\beta}^{\gamma}(z) = \sum_{k=0}^{\infty} (\gamma)_k z^k / (k! \Gamma(\alpha k + \beta))$. Supports complex arguments and uses log-space computation for numerical stability.

Usage

```
mittag_leffler_prabhakar(
    alpha,
    beta,
    gamma,
    z,
    n_terms = 100,
    tolerance = 1e-10
)
```

Arguments

alpha	Parameter alpha > 0
beta	Parameter beta > 0
gamma	Parameter gamma > 0
z	Complex argument (can be a vector)
n_terms	Number of terms in series expansion (default = 100)
tolerance	Convergence tolerance (default = 1e-10)

Value

Value of the Prabhakar Mittag-Leffler function

Examples

```
mittag_leffler_prabhakar(0.8, 1, 1, -1)
```

net_profit_condition_gfrp

Net Profit Condition for GFRP

Description

Checks if the net profit condition is satisfied

Usage

```
net_profit_condition_gfrp(eta, rho, beta, lambda)
```

Arguments

eta	Mean claim size (> 0)
rho	Safety loading parameter (≥ 0)
beta	Fractional parameter ($0 < \beta < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP

Value

Logical indicating if net profit condition is satisfied

Examples

```
net_profit_condition_gfrp(10, 0.1, 0.7, c(1.0, 0.5))
```

partial_ordinary_bell *Partial Ordinary Bell Polynomials*

Description

Computes the partial ordinary Bell polynomials $\hat{B}_{j,i}(a)$ Used in the asymptotic expansion of tempered Mittag-Leffler moments.

Usage

```
partial_ordinary_bell(j, i, a)
```

Arguments

j	Index (non-negative integer)
i	Index (non-negative integer)
a	Coefficients vector

Value

Value of the partial ordinary Bell polynomial

Examples

```
partial_ordinary_bell(3, 2, c(0.5, 0.3, 0.2))
```

pgamma_subordinator	<i>CDF of Gamma Subordinator</i>
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Description

Computes the cumulative distribution function of the gamma subordinator

Usage

```
pgamma_subordinator(x, t, shape, rate)
```

Arguments

x	Value
t	Time
shape	Shape parameter (beta1)
rate	Rate parameter (lambda1)

Value

CDF value

Examples

```
pgamma_subordinator(1, 1, 2, 1)
```

pgf_cgfcf

*Probability Generating Function of CGFCF***Description**

Computes the probability generating function of the CGFCF

Usage

```
pgf_cgfcf(
  beta,
  lambda,
  t,
  u,
  jump_dist = "poisson",
  jump_params = list(),
  n_terms = 50
)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCF
t	Time (> 0)
u	PGF argument ($ u \leq 1$)
jump_dist	Distribution of jump sizes
jump_params	Parameters for jump distribution
n_terms	Number of terms in series expansion (default = 50)

Value

PGF value

Examples

```
pgf_cgfcf(0.7, c(1.0, 0.5), 1.0, 0.5, "poisson", list(lambda = 2))
```

pgf_fcp

Probability Generating Function of FCP

Description

Computes the probability generating function of the Fractional Counting Process

Usage

```
pgf_fcp(mu, zeta, theta, lambda, t, s, vartheta = NULL)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
s	PGF argument ($ s \leq 1$)
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)

Value

PGF value

Examples

```
pgf_fcp(0.8, 1.0, 0.5, 1.0, 1.0, 0.5)
```

pgf_gipp

Probability Generating Function of GIPP

Description

Computes the probability generating function of the GIPP

Usage

```
pgf_gipp(beta, lambda, lambda_beta, t, u)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)
u	PGF argument ($ u \leq 1$)

Value

PGF value

Examples

```
pgf_gipp(0.7, 1.0, 0.5, 1.0, 0.5)
```

pgf_tstfnbp

Probability Generating Function of TSTFNBP

Description

Computes the probability generating function of the TSTFNBP

Usage

```
pgf_tstfnbp(beta, alpha, mu, lambda1, beta1, lambda, t, u, n_terms = 50)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
alpha	Stability parameter ($0 < \text{alpha} < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate ($> \mu * \text{alpha}$)
beta1	Gamma subordinator shape (> 0)
lambda	Poisson parameter (> 0)
t	Time (> 0)
u	PGF argument ($ u \leq 1$)
n_terms	Number of terms in series expansion (default = 50)

Value

PGF value

Examples

```
pgf_tstfnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 1.0, 0.5)
```

pochhammer	<i>Pochhammer Symbol</i>
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Description

Computes the Pochhammer symbol $(a)_n = \Gamma(a+n)/\Gamma(a)$ Uses log-gamma for numerical stability and preventing overflow.

Usage

```
pochhammer(a, n)
```

Arguments

a	Real number
n	Non-negative integer

Value

Value of the Pochhammer symbol

Examples

```
pochhammer(1, 5) # Should equal 120 = 5!
```

print.fcp	<i>S3 Print Method for FCP</i>
-----------	--------------------------------

Description

S3 Print Method for FCP

Usage

```
## S3 method for class 'fcp'
print(x, ...)
```

Arguments

x	FCP object
...	Additional arguments

Value

No return value, called for side effects.

print.gipp	<i>S3 Print Method for GIPP</i>
------------	---------------------------------

Description

S3 Print Method for GIPP

Usage

```
## S3 method for class 'gipp'  
print(x, ...)
```

Arguments

x	GIPP object
...	Additional arguments

Value

No return value, called for side effects.

print.tcfcp	<i>S3 Print Method for TCFCP</i>
-------------	----------------------------------

Description

S3 Print Method for TCFCP

Usage

```
## S3 method for class 'tcfcp'  
print(x, ...)
```

Arguments

x	TCFCP object
...	Additional arguments

Value

No return value, called for side effects.

ruin_probability_gfrp *Ruin Probability for GFRP*

Description

Estimates the ruin probability using coherent path simulation

Usage

```
ruin_probability_gfrp(
  nu,
  eta,
  rho,
  beta,
  lambda,
  t_max,
  claim_dist = "gamma",
  claim_params = list(),
  n_grid = 100,
  n_sim = 1000
)
```

Arguments

nu	Initial capital (> 0)
eta	Mean claim size (> 0)
rho	Safety loading parameter (≥ 0)
beta	Fractional parameter ($0 < \beta < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
t_max	Maximum time horizon
claim_dist	Distribution of claim sizes
claim_params	Parameters for claim distribution
n_grid	Number of grid points for path discretization (default = 100)
n_sim	Number of simulations (default = 1000)

Value

Estimated ruin probability

Examples

```
ruin_probability_gfrp(100, 10, 0.1, 0.7, c(1.0, 0.5), 10,
  "gamma", list(shape = 2, rate = 0.2), n_sim = 100)
```

simulate_cgfcfcp	<i>Simulate Sample Paths of CGFCF</i>
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Description

Simulates sample paths of the Compound Generalized Fractional Counting Process.

Usage

```
simulate_cgfcfcp(
  beta,
  lambda,
  t_max,
  jump_dist = "poisson",
  jump_params = list(),
  n_paths = 1,
  n_grid = 100
)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$
t_max	Maximum simulation time
jump_dist	Distribution of jump sizes ("poisson", "geometric")
jump_params	Parameters for jump distribution
n_paths	Number of paths to simulate (default = 1)
n_grid	Number of grid points for path discretization (default = 100)

Value

A list of paths, each path is a list containing times, states, and the subordinator path

Examples

```
simulate_cgfcfcp(0.7, c(1.0, 0.5), 10, "poisson", list(lambda = 2), n_paths = 2)
```

simulate_fcp	<i>Simulate Sample Paths of FCP</i>
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Description

Simulates sample paths of the Fractional Counting Process. Uses exact Chambers-Mallows-Stuck simulation for FPP ($\text{zeta} = \text{vartheta} = 1$) and numerical CDF inversion for general FCP.

Usage

```
simulate_fcp(mu, zeta, theta, lambda, t_max, n_paths = 1, vartheta = NULL)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t_max	Maximum simulation time
n_paths	Number of paths to simulate (default = 1)
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)

Value

A list of paths, each path is a list containing times and states

Examples

```
simulate_fcp(0.8, 1.0, 0.5, 1.0, 10, n_paths = 2)
```

simulate_gfrp	<i>Simulate Sample Paths of GFRP</i>
---------------	--------------------------------------

Description

Simulates sample paths of the Generalized Fractional Risk Process.

Usage

```
simulate_gfrp(
  nu,
  eta,
  rho,
  beta,
  lambda,
  t_max,
  claim_dist = "gamma",
  claim_params = list(),
  n_paths = 1,
  n_grid = 100
)
```

Arguments

nu	Initial capital (> 0)
eta	Mean claim size (> 0)
rho	Safety loading parameter (≥ 0)
beta	Fractional parameter ($0 < \beta < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
t_max	Maximum simulation time
claim_dist	Distribution of claim sizes ("gamma", "exponential", "lognormal")
claim_params	Parameters for claim distribution
n_paths	Number of paths to simulate (default = 1)
n_grid	Number of grid points for path discretization (default = 100)

Value

A list of paths, each path is a list containing times, states (capital), subordinator (W_β), and claims (accumulated claims)

Examples

```
simulate_gfrp(100, 10, 0.1, 0.7, c(1.0, 0.5), 10, "gamma", list(shape = 2, rate = 0.2), n_paths = 2)
```

 simulate_gipp

Simulate Sample Paths of GIPP

Description

Simulates sample paths of the Generalized Iterated Poisson Process. $Q(t) = N(N_\beta(t), \lambda)$.

Usage

```
simulate_gipp(beta, lambda, lambda_beta, t_max, n_paths = 1, n_grid = 100)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t_max	Maximum simulation time
n_paths	Number of paths to simulate (default = 1)
n_grid	Number of grid points for path discretization (default = 100)

Value

A list of paths, each path is a list containing times, states, and inner_process (TFPP path)

Examples

```
simulate_gipp(0.7, 1.0, 0.5, 10, n_paths = 2)
```

simulate_tcfcp	<i>Simulate Sample Paths of TCFCP</i>
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Description

Simulates sample paths of the Time-Changed Fractional Counting Process. First simulates the Lévy subordinator path on a time grid, then time-changes an independent FPP/FCP path.

Usage

```
simulate_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  t_max,
  subordinator_type = "gamma",
  subordinator_params = list(),
  n_paths = 1,
  vartheta = NULL,
  n_grid = 100
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t_max	Maximum simulation time
subordinator_type	Type of subordinator ("gamma", "tempered_mittag_leffler")
subordinator_params	List of parameters for the subordinator
n_paths	Number of paths to simulate (default = 1)
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_grid	Number of grid points for path discretization (default = 100)

Value

A list of paths, each path is a list containing times, states, and the subordinator path

Examples

```
simulate_tcfcp(0.8, 1.0, 0.5, 1.0, 10, "gamma", list(shape = 2, rate = 1), n_paths = 2)
```

simulate_tstfnbp	<i>Simulate Sample Paths of TSTFNB</i>
------------------	--

Description

Simulates sample paths of the Tempered Space-Time Fractional Negative Binomial Process. First simulates the TMLLP subordinator path, then time-changes an independent FPP path.

Usage

```
simulate_tstfnbp(
  beta,
  alpha,
  mu,
  lambda1,
  beta1,
  lambda,
  t_max,
  n_paths = 1,
  n_grid = 100
)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
alpha	Stability parameter ($0 < \text{alpha} < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate ($> \text{mu} * \text{alpha}$)
beta1	Gamma subordinator shape (> 0)
lambda	Poisson parameter (> 0)
t_max	Maximum simulation time
n_paths	Number of paths to simulate (default = 1)
n_grid	Number of grid points for path discretization (default = 100)

Value

A list of paths, each path is a list containing times, states, and the subordinator path

Examples

```
simulate_tstfnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 10, n_paths = 2)
```

stirling2

Classical Stirling Numbers of the Second Kind

Description

Computes the classical Stirling numbers of the second kind $S(m, k)$

Usage

```
stirling2(m, k)
```

Arguments

m	Upper index (non-negative integer)
k	Lower index (non-negative integer)

Value

Value of the Stirling number

Examples

```
stirling2(5, 3)
```

tempered_mittag_leffler_levy

Tempered Mittag-Leffler Lévy Process

Description

Simulates the tempered Mittag-Leffler Lévy process $M_{\alpha,\mu,\lambda_1,\beta_1}(t) = S_{\alpha,\mu}(G_{\lambda_1,\beta_1}(t))$

Usage

```
tempered_mittag_leffler_levy(t, alpha, mu, lambda1, beta1, n = 1)
```

Arguments

t	Time
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate (> 0)
beta1	Gamma subordinator shape (> 0)
n	Number of simulations

Value

Simulated values of the tempered Mittag-Leffler Lévy process

Examples

```
tempered_mittag_leffler_levy(1, 0.8, 0.5, 2, 1, 10)
```

tempered_mittag_leffler_moment

Fractional Moment of Tempered Mittag-Leffler Lévy Process

Description

Computes the q -th order moment of the tempered Mittag-Leffler Lévy process based on asymptotic results from Kumar et al. (2019b)

Usage

```
tempered_mittag_leffler_moment(q, alpha, mu, lambda1, beta1, t)
```

Arguments

q	Order of moment (> 0)
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate (> 0)
beta1	Gamma subordinator shape (> 0)
t	Time (> 0)

Value

Asymptotic value of the q-th moment

Examples

```
tempered_mittag_leffler_moment(2, 0.8, 0.5, 2, 1, 1)
```

```
tempered_stable_subordinator
```

Tempered Stable Subordinator

Description

Simulates a tempered alpha-stable subordinator. Uses exact rejection-sampling via tilting of stable random variables.

Usage

```
tempered_stable_subordinator(t, alpha, mu, n = 1)
```

Arguments

t	Time
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
n	Number of simulations

Value

Simulated values of the tempered stable subordinator

Examples

```
tempered_stable_subordinator(1, 0.8, 0.5, 10)
```

tempered_st_fnbp	<i>Tempered Space-Time Fractional Negative Binomial Process (TSTFNBP)</i>
------------------	---

Description

Computes the probability mass function of the TSTFNBP $Q_{\alpha,\mu,\lambda_1,\beta_1}^\beta(t,\lambda) = N_\beta(M_{\alpha,\mu,\lambda_1,\beta_1}(t),\lambda)$
From Garg et al. (2025)

Usage

tempered_st_fnbp(beta, alpha, mu, lambda1, beta1, lambda, t, n, n_terms = 50)

Arguments

- beta Fractional parameter (0 < beta < 1)
- alpha Stability parameter (0 < alpha < 1)
- mu Tempering parameter (> 0)
- lambda1 Gamma subordinator rate (> mu*alpha)
- beta1 Gamma subordinator shape (> 0)
- lambda Poisson parameter (> 0)
- t Time (> 0)
- n Vector of non-negative integers
- n_terms Number of terms in series expansion (default = 50)

Value

Vector of probabilities P(Q(t) = n)

Examples

tempered_st_fnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 1.0, 0:10)

time_changed_fcp	<i>Time-Changed Fractional Counting Process (TCFCP)</i>
------------------	---

Description

Computes the probability mass function of the Time-Changed Fractional Counting Process obtained by subordinating the FCP with a Lévy subordinator. From Garg et al. (2025)

Usage

```
time_changed_fcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  n,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL,
  n_terms = 50
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
n	Vector of non-negative integers
subordinator_type	Type of subordinator ("gamma", "tempered_mittag_leffler")
subordinator_params	List of parameters for the subordinator
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)
n_terms	Number of terms in series expansion (default = 50)

Value

Vector of probabilities $P(Z(t) = n)$

Examples

```
time_changed_fcp(0.8, 1.0, 0.5, 1.0, 1.0, 0:10,
  "gamma", list(shape = 2, rate = 1))
```

var_cgfcfcp	<i>Variance of CGFCP</i>
-------------	--------------------------

Description

Computes the variance of the CGFCP

Usage

```
var_cgfcfcp(beta, lambda, t, jump_dist = "poisson", jump_params = list())
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
t	Time (> 0)
jump_dist	Distribution of jump sizes ("poisson", "geometric")
jump_params	Parameters for jump distribution

Value

Variance value

Examples

```
var_cgfcfcp(0.7, c(1.0, 0.5), 1.0, "poisson", list(lambda = 2))
```

var_fcp	<i>Variance of Fractional Counting Process</i>
---------	--

Description

Computes the variance of the FCP

Usage

```
var_fcp(mu, zeta, theta, lambda, t, vartheta = NULL)
```

Arguments

mu	Parameter μ ($0 < \mu \leq 1$)
zeta	Parameter ζ (> 0)
theta	Parameter θ ($0 < \theta \leq 1$)
lambda	Parameter λ (> 0)
t	Time (> 0)
vartheta	Parameter ϑ ($\geq \mu * \zeta$, default = ζ)

Value

Variance value

Examples

```
var_fcp(0.8, 1.0, 0.5, 1.0, 1.0)
```

var_gfrp	<i>Variance of GFRP</i>
----------	-------------------------

Description

Computes the variance of the GFRP

Usage

```
var_gfrp(  
  nu,  
  eta,  
  rho,  
  beta,  
  lambda,  
  t,  
  claim_dist = "gamma",  
  claim_params = list()  
)
```

Arguments

nu	Initial capital (> 0)
eta	Mean claim size (> 0)
rho	Safety loading parameter (≥ 0)
beta	Fractional parameter ($0 < \beta < 1$)
lambda	Vector of rates $\lambda_1, \dots, \lambda_k$ for the GFCP
t	Time (> 0)
claim_dist	Distribution of claim sizes
claim_params	Parameters for claim distribution

Value

Variance value

Examples

```
var_gfrp(100, 10, 0.1, 0.7, c(1.0, 0.5), 1.0, "gamma", list(shape = 2, rate = 0.2))
```

var_gipp	<i>Variance of GIPP</i>
----------	-------------------------

Description

Computes the variance of the GIPP

Usage

```
var_gipp(beta, lambda, lambda_beta, t)
```

Arguments

beta	Fractional parameter ($0 < \text{beta} < 1$)
lambda	Parameter of outer Poisson process (> 0)
lambda_beta	Parameter of inner fractional Poisson process (> 0)
t	Time (> 0)

Value

Variance value

Examples

```
var_gipp(0.7, 1.0, 0.5, 1.0)
```

var_tcfcp	<i>Variance of Time-Changed FCP</i>
-----------	-------------------------------------

Description

Computes the variance of the TCFCP using the law of total variance.

Usage

```
var_tcfcp(
  mu,
  zeta,
  theta,
  lambda,
  t,
  subordinator_type = "gamma",
  subordinator_params = list(),
  vartheta = NULL
)
```

Arguments

mu	Parameter mu ($0 < \mu \leq 1$)
zeta	Parameter zeta (> 0)
theta	Parameter theta ($0 < \theta \leq 1$)
lambda	Parameter lambda (> 0)
t	Time (> 0)
subordinator_type	Type of subordinator
subordinator_params	List of parameters for the subordinator
vartheta	Parameter vartheta ($\geq \mu * \text{zeta}$, default = zeta)

Value

Variance value

Examples

```
var_tcfcp(0.8, 1.0, 0.5, 1.0, 1.0, "gamma", list(shape = 2, rate = 1))
```

var_tstfnbp	<i>Variance of TSTFNBP</i>
-------------	----------------------------

Description

Computes the variance of the TSTFNBP

Usage

```
var_tstfnbp(beta, alpha, mu, lambda1, beta1, lambda, t)
```

Arguments

beta	Fractional parameter ($0 < \beta < 1$)
alpha	Stability parameter ($0 < \alpha < 1$)
mu	Tempering parameter (> 0)
lambda1	Gamma subordinator rate ($> \mu * \alpha$)
beta1	Gamma subordinator shape (> 0)
lambda	Poisson parameter (> 0)
t	Time (> 0)

Value

Variance value

Examples

```
var_tstfnbp(0.7, 0.8, 0.5, 2, 1, 1.0, 1.0)
```

wright_function	<i>Generalized Wright Function</i>
-----------------	------------------------------------

Description

Computes the generalized Wright function $\psi_{p,q}[(a_i, \alpha_i)_{1,p}; (b_j, \beta_j)_{1,q}](z)$ Uses log-space computation for numerical stability.

Usage

```
wright_function(a, alpha, b, beta, z, n_terms = 100, tolerance = 1e-10)
```

Arguments

a	Vector of a_i parameters
alpha	Vector of alpha_i parameters
b	Vector of b_j parameters
beta	Vector of beta_j parameters
z	Complex argument (can be a vector)
n_terms	Number of terms in series expansion (default = 100)
tolerance	Convergence tolerance (default = 1e-10)

Value

Value of the generalized Wright function

Examples

```
wright_function(c(1), c(1), c(1), c(1), 1)
```

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