

# Package ‘PerRegMod’

November 14, 2024

**Type** Package

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**Title** Fitting Periodic Coefficients Linear Regression Models

**Version** 4.4.1

**Description** Provides tools for fitting periodic coefficients regression models to data where periodicity plays a crucial role. It allows users to model and analyze relationships between variables that exhibit cyclical or seasonal patterns, offering functions for estimating parameters and testing the periodicity of coefficients in linear regression models. For simple periodic coefficient regression model see Regui et al. (2024) <[doi:10.1080/03610918.2024.2314662](https://doi.org/10.1080/03610918.2024.2314662)>.

**License** GPL

**Encoding** UTF-8

**Imports** expm, readxl, sn

**URL** <https://doi.org/10.1080/03610918.2024.2314662>

**NeedsCompilation** no

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**Repository** CRAN

**Date/Publication** 2024-11-14 16:00:14 UTC

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A_x_B	<i>A Kronecker product B</i>
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**Description**

A\_x\_B() function gives A Kronecker product B

**Usage**

A\_x\_B(A,B)

**Arguments**

A	A matrix.
B	A matrix.

**Value**

A\_x\_B(A, B) returns the matrix A Kronecker product B,  $A \otimes B$

**Examples**

```
A=matrix(rep(1,6),3,2)
B=matrix(seq(1,8),2,4 )
A_x_B(A,B)
```

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check_periodicity	<i>Checking the periodicity of parameters in the regression model</i>
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**Description**

check\_periodicity() function allows to detect the periodicity of parameters in the regression model using [pseudo\\_gaussian\\_test](#). See *Regui et al. (2024)* for periodic simple regression model.  $T^{(n)} =$

$$\left( \Delta_1^{\circ(n)'} , \Delta_2^{\circ(n)'} , \Delta_3^{\circ(n)'} \right) \begin{pmatrix} \Gamma_1^{\circ} & \Gamma_{12}^{\circ} & \mathbf{0} \\ \Gamma_{12}^{\circ} & \Gamma_{22}^{\circ} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Gamma_{33}^{\circ} \end{pmatrix}^{-1} \begin{pmatrix} \Delta_1^{\circ(n)} \\ \Delta_2^{\circ(n)} \\ \Delta_3^{\circ(n)} \end{pmatrix}, \text{ where } \Delta_1^{\circ(n)} = n^{\frac{-1}{2}} \sum_{r=0}^{m-1} \begin{pmatrix} \hat{\phi}(Z_{1+S_r}) - \hat{\phi}(Z_{S+S_r}) \\ \vdots \\ \hat{\phi}(Z_{S-1+S_r}) - \hat{\phi}(Z_{S+S_r}) \end{pmatrix}$$

$$\Delta_2^{\circ(n)} = \frac{n^{\frac{-1}{2}}}{2\hat{\sigma}} \sum_{r=0}^{m-1} \begin{pmatrix} \hat{\psi}(Z_{1+S_r}) - \hat{\psi}(Z_{S+S_r}) \\ \vdots \\ \hat{\psi}(Z_{S-1+S_r}) - \hat{\psi}(Z_{S+S_r}) \end{pmatrix},$$

$$\Delta_3^{\circ(n)} = n^{-\frac{1}{2}} \sum_{r=0}^{m-1} \begin{pmatrix} \hat{\phi}(Z_{1+S_r}) \mathbf{K}_1^{(n)} \mathbf{X}_{1+S_r} - \hat{\phi}(Z_{S+S_r}) \mathbf{K}_S^{(n)} \mathbf{X}_{S+S_r} \\ \vdots \\ \hat{\phi}(Z_{S-1+S_r}) \mathbf{K}_{S-1}^{(n)} \mathbf{X}_{S-1+S_r} - \hat{\phi}(Z_{S+S_r}) \mathbf{K}_S^{(n)} \mathbf{X}_{S+S_r} \end{pmatrix}, \Gamma_{11}^{\circ} = \frac{\hat{I}_n}{S} \Sigma,$$

$$\Gamma_{22}^{\circ} = \frac{\hat{I}_n}{4S\hat{\sigma}^2} \Sigma, \Gamma_{12}^{\circ} = \frac{\hat{N}_n}{2S\hat{\sigma}} \Sigma, \text{ and } \Gamma_{33}^{\circ} = \frac{\hat{I}_n}{S} \Sigma \otimes \mathbf{I}_{p \times p} \text{ with } \hat{I}_n = \frac{1}{nT} \sum_{s=1}^S \sum_{r=0}^{m-1} \hat{\phi}^2 \left( \frac{\hat{Z}_{s+S_r}}{\hat{\sigma}_s} \right),$$

$$\hat{N}_n = \frac{1}{nT} \sum_{s=1}^S \sum_{r=0}^{m-1} \hat{\phi}^2 \left( \frac{\hat{Z}_{s+S_r}}{\hat{\sigma}_s} \right) \frac{\hat{Z}_{s+S_r}}{\hat{\sigma}_s},$$

$$\Sigma = \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \dots & 1 & 2 \end{bmatrix}, Z_{s+S_r} = \frac{y_{s+S_r} - \hat{\mu}_s - \sum_{j=1}^p \hat{\beta}_s^j x_{s+S_r}^j}{\hat{\sigma}_s}, \mathbf{X}_{s+S_r} = (x_{s+S_r}^1, \dots, x_{s+S_r}^p)',$$

$$\mathbf{K}_s^{(n)} = \begin{bmatrix} \overline{(x_s^1)^2} & \overline{x_s^i x_s^j} \\ & \ddots \\ \overline{x_s^j x_s^i} & \overline{(x_s^p)^2} \end{bmatrix}^{-\frac{1}{2}},$$

$$\overline{x_s^i x_s^j} = \frac{1}{m} \sum_{r=0}^{m-1} x_{s+S_r}^i x_{s+S_r}^j, \overline{(x_s^i)^2} = \frac{1}{m} \sum_{r=0}^{m-1} (x_{s+S_r}^i)^2, \hat{\psi}(x) = x\hat{\phi}(x) - 1, \text{ and}$$

$$\hat{\phi}(x) = \frac{1}{b_n^2} \frac{\sum_{s=1}^S \sum_{r=0}^{m-1} (x - Z_{s+S_r}) \exp\left(-\frac{(x - Z_{s+S_r})^2}{2b_n^2}\right)}{\sum_{s=1}^S \sum_{r=0}^{m-1} \exp\left(-\frac{(x - Z_{s+S_r})^2}{2b_n^2}\right)} \text{ with } b_n \rightarrow 0.$$

### Usage

check\_periodicity(x,y,s)

### Arguments

- x A list of independent variables with dimension  $p$ .
- y A response variable.
- s A period of the regression model.

### Value

check\_periodicity()

returns the value of observed statistic,  $T^{(n)}$ , degrees of freedom,  $(S-1) \times (p+2)$ , and p-value

### References

Regui, S., Akharif, A., & Mellouk, A. (2024). "Locally optimal tests against periodic linear regression in short panels." *Communications in Statistics-Simulation and Computation*, 1–15. doi:10.1080/03610918.2024.2314662

**Examples**

```

library(expm)
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
check_periodicity(x,y,s)

```

DELTA

*Calculating the component of vector DELTA***Description**

DELTA() function gives the value of the component of vector DELTA  $\Delta$ . See *Regui et al. (2024)* for

periodic simple regression model.  $\Delta = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$ , where  $\Delta_1$  is a vector of dimension  $S$  with com-

ponent  $\frac{n-\frac{1}{2}}{\sigma_s} \sum_{r=0}^{m-1} \hat{\phi}(Z_{s+Sr,t})$ ,  $\Delta_2$  is a vector of dimension  $pS$  with component  $\frac{n-\frac{1}{2}}{\sigma_s} \sum_{r=0}^{m-1} \hat{\phi}(Z_{s+Sr}) K_s^{(n)} \mathbf{X}_{s+Sr}$ ,

$\Delta_3$  is a vector of dimension  $S$  with component  $\frac{n-\frac{1}{2}}{2\sigma_s^2} \sum_{r=0}^{m-1} Z_{s+Sr} \hat{\phi}(Z_{s+Sr}) - 1$ .

**Usage**

```
DELTA(x,phi,s,e,sigma)
```

**Arguments**

x	A list of independent variables with dimension $p$ .
phi	<a href="#">phi_n</a> .
s	A period of the regression model.
e	The residuals vector.
sigma	<a href="#">sd_estimation_for_each_s</a> .

**Value**

DELTA() returns the values of  $\Delta$ . See *Regui et al. (2024)* for simple periodic coefficients regression model.

## References

Regui, S., Akharif, A., & Mellouk, A. (2024). "Locally optimal tests against periodic linear regression in short panels." *Communications in Statistics-Simulation and Computation*, 1–15. doi:10.1080/03610918.2024.2314662

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estimate\_para\_adaptive\_method

*Adaptive estimator for periodic coefficients regression model*

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## Description

estimate\_para\_adaptive\_method() function gives the adaptive estimation of parameters of a periodic coefficients regression model.

## Usage

```
estimate_para_adaptive_method(n, s, y, x)
```

## Arguments

n	The length of vector $y$ .
s	A period of the regression model.
y	A response variable.
x	A list of independent variables with dimension $p$ .

## Value

beta_ad	Parameters to be estimated.
---------	-----------------------------

## Examples

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
model=lm(y~x1+x2+x3+x4)
z=model$residuals
estimate_para_adaptive_method(n,s,y,x)
```

**Description**

GAMMA() function gives the value of the component of matrix GAMMA  $\Gamma$ . See *Regui et al.*

(2024) for periodic simple regression model.  $\Gamma = \frac{1}{S} \begin{bmatrix} (\Gamma_{11})_{S \times S} & \mathbf{0} & \Gamma_{13} \\ \mathbf{0} & (\Gamma_{22})_{pS \times pS} & \mathbf{0} \\ \Gamma_{13} & \mathbf{0} & (\Gamma_{33})_{S \times S} \end{bmatrix}$ ,

where  $\Gamma_{11} = \hat{I}_n \text{diag}(\frac{1}{\hat{\sigma}_1^2}, \dots, \frac{1}{\hat{\sigma}_S^2})$ ,  $\Gamma_{13} = \frac{\hat{N}_n}{2} \text{diag}(\frac{1}{\hat{\sigma}_1^2}, \dots, \frac{1}{\hat{\sigma}_S^2})$ ,  $\Gamma_{22} = \hat{I}_n \text{diag}(\frac{1}{\hat{\sigma}_1^2}, \dots, \frac{1}{\hat{\sigma}_S^2}) \otimes \mathbf{I}_p$ ,

$\Gamma_{33} = \frac{\hat{J}_n}{4} \text{diag}(\frac{1}{\hat{\sigma}_1^4}, \dots, \frac{1}{\hat{\sigma}_S^4})$ ,  $\hat{I}_n = \frac{1}{nT} \sum_{s=1}^S \sum_{r=0}^{m-1} \hat{\phi}^2\left(\frac{\hat{Z}_{s+Sr}}{\hat{\sigma}_s}\right)$ ,  $\hat{N}_n = \frac{1}{nT} \sum_{s=1}^S \sum_{r=0}^{m-1} \hat{\phi}^2\left(\frac{\hat{Z}_{s+Sr}}{\hat{\sigma}_s}\right) \frac{\hat{Z}_{s+Sr}}{\hat{\sigma}_s}$ ,

$\hat{J}_n = \frac{1}{nT} \sum_{s=1}^S \sum_{r=0}^{m-1} \hat{\phi}^2\left(\frac{\hat{Z}_{s+Sr}}{\hat{\sigma}_s}\right) \left(\frac{\hat{Z}_{s+Sr}}{\hat{\sigma}_s}\right)^2 - 1$ , and

$$\hat{\phi}(x) = \frac{1}{b_n^2} \frac{\sum_{s=1}^S \sum_{r=0}^{m-1} (x - Z_{s+Sr}) \exp\left(-\frac{(x - Z_{s+Sr})^2}{2b_n^2}\right)}{\sum_{s=1}^S \sum_{r=0}^{m-1} \exp\left(-\frac{(x - Z_{s+Sr})^2}{2b_n^2}\right)} \text{ with } b_n \rightarrow 0.$$

**Usage**

GAMMA(x, phi, s, z, sigma)

**Arguments**

x	A list of independent variables with dimension $p$ .
phi	<a href="#">phi_n</a> .
s	A period of the regression model.
z	The residuals vector.
sigma	<a href="#">sd_estimation_for_each_s</a> .

**Value**

GAMMA() returns the matrix  $\Gamma$ . See *Regui et al.* (2024) for simple periodic coefficients regression model.

**References**

Regui, S., Akharif, A., & Mellouk, A. (2024). "Locally optimal tests against periodic linear regression in short panels." *Communications in Statistics-Simulation and Computation*, 1–15. doi:10.1080/03610918.2024.2314662

lm\_per

Fitting periodic coefficients regression model by using LSE

**Description**

lm\_per() function gives the least squares estimation of parameters, intercept  $\mu_s$ , slope  $\beta_s$ , and standard deviation  $\sigma_s$ , of a periodic coefficients regression model using [LSE\\_Reg\\_per](#) and [sd\\_estimation\\_for\\_each\\_s](#)

functions.  $\hat{\vartheta} = (X'X)^{-1} X'Y$  where  $X = \begin{bmatrix} \mathbf{X}_1^1 & 0 & \dots & 0 & \mathbf{X}_1^p & 0 & \dots & 0 \\ 0 & \mathbf{X}_2^1 & \dots & 0 & 0 & \mathbf{X}_2^p & \dots & 0 \\ \mathbf{I}_S \otimes \mathbf{1}_m & 0 & 0 & \ddots & \vdots & \dots & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \mathbf{X}_S^1 & 0 & 0 & 0 & 0 & \mathbf{X}_S^p \end{bmatrix}$ ,

$\mathbf{X}_s^j = (x_s^j, \dots, x_{s+(m-1)S}^j)'$ ,  $Y = (\mathbf{Y}'_1, \dots, \mathbf{Y}'_S)'$ ,  $\mathbf{Y}_s = (y_s, \dots, y_{(m-1)S+s})'$ ,  $\epsilon = (\epsilon'_1, \dots, \epsilon'_S)'$ ,  $\epsilon_s = (\epsilon_s, \dots, \epsilon_{(m-1)S+s})'$ ,  $\mathbf{1}_m$  is a vector of ones of dimension  $m$ ,  $\mathbf{I}_S$  is the identity matrix of dimension  $S$ ,  $\otimes$  denotes the Kronecker product, and  $\vartheta = (\boldsymbol{\mu}', \boldsymbol{\beta}')'$  with  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_S)'$  and  $\boldsymbol{\beta} = (\beta_1^1, \dots, \beta_S^1; \dots; \beta_1^p, \dots, \beta_S^p)'$ .

**Usage**

```
lm_per(x, y, s)
```

**Arguments**

x A list of independent variables with dimension  $p$ .  
y A response variable.  
s A period of the regression model.

**Value**

Residuals the residuals, that is response minus fitted values  
Coefficients a named vector of coefficients  
Root mean square error The root mean square error

**Examples**

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
lm_per(x,y,s)
```

---

lm_per_AE	<i>Fitting periodic coefficients regression model by using Adaptive estimation method</i>
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---

### Description

lm\_per\_AE() function gives the adaptive estimation of parameters, intercept  $\mu_s$ , slope  $\beta_s$ , and standard deviation  $\sigma_s$ , of a periodic coefficients regression model.  $\hat{\theta}_{AE} = \hat{\theta}_{LSE} + \frac{1}{\sqrt{n}} \Gamma^{-1} \Delta$ .

### Usage

```
lm_per_AE(x, y, s)
```

### Arguments

x	A list of independent variables with dimension $p$ .
y	A response variable.
s	A period of the regression model.

### Value

Residuals	the residuals, that is response minus fitted values
Coefficients	a named vector of coefficients
Root mean square error	The root mean square error

### Examples

```
set.seed(6)
n=200
s=2
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
lm_per_AE(x,y,s)
```



---

`LSE_Reg_per`*Least squares estimator for periodic coefficients regression model*

---

**Description**

`LSE_Reg_per()` function gives the least squares estimation of parameters of a periodic coefficients regression model.

**Usage**

```
LSE_Reg_per(x, y, s)
```

**Arguments**

<code>x</code>	A list of independent variables with dimension $p$ .
<code>y</code>	A response variable.
<code>s</code>	A period of the regression model.

**Value**

<code>beta</code>	Parameters to be estimated.
<code>X</code>	Matrix of predictors.
<code>Y</code>	The response vector.

**Examples**

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
LSE_Reg_per(x,y,s)
```

---

phi\_n *Calculating the value of  $\phi$  function*

---

**Description**

phi\_n() function gives the value of  $\hat{\phi}(x) = \frac{1}{b_n^2} \frac{\sum_{s=1}^S \sum_{r=0}^{m-1} (x - Z_{s+Sr}) \exp\left(-\frac{(x - Z_{s+Sr})^2}{2b_n^2}\right)}{\sum_{s=1}^S \sum_{r=0}^{m-1} \exp\left(-\frac{(x - Z_{s+Sr})^2}{2b_n^2}\right)}$  with  $b_n = 0.2$ .

**Usage**

phi\_n(x)

**Arguments**

x                    A numeric value.

**Value**

returns the value of  $\hat{\phi}(x)$

---

pseudo\_gaussian\_test *Detecting periodicity of parameters in the regression model*

---

**Description**

pseudo\_gaussian\_test() function gives the value of the statistic test,  $T^{(n)}$ , for detecting periodicity of parameters in the regression model. See [check\\_periodicity](#) function.

**Usage**

pseudo\_gaussian\_test(x, z, s)

**Arguments**

x                    A list of independent variables with dimension  $p$ .  
z                    The residuals vector.  
s                    A period of the regression model.

**Value**

returns the value of the statistic test,  $T^{(n)}$ .

---

 sd\_estimation\_for\_each\_s

*Estimating periodic variances in a periodic coefficients regression model*

---

### Description

sd\_estimation\_for\_each\_s() function gives the estimation of variances,  $\hat{\sigma}_s^2 = \frac{1}{m-p-1} \sum_{r=0}^{m-1} \hat{\varepsilon}_{s+Sr}^2$  for all  $s = 1, \dots, S$ , in a periodic coefficients regression model.

### Usage

```
sd_estimation_for_each_s(x, y, s, beta_hat)
```

### Arguments

x	A list of independent variables with dimension $p$ .
y	A response variable.
s	A period of the regression model.
beta_hat	The least squares estimation using <a href="#">LSE_Reg_per</a> .

### Value

returns the value of  $\hat{\sigma}_s^2$ .

### Examples

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
beta_hat=LSE_Reg_per(x,y,s)$beta
sd_estimation_for_each_s(x,y,s,beta_hat)
```

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