

# Package ‘GameTheoryAllocation’

January 20, 2025

**Type** Package

**Title** Tools for Calculating Allocations in Game Theory

**Version** 1.0

**Date** 2016-07-20

**Author** Alejandro Saavedra-Nieves

**Maintainer** Alejandro Saavedra-Nieves <alejandro.saavedra.nieves@gmail.com>

**Depends** R(>= 2.15.0),e1071, lpSolveAPI

**Description** Many situations can be modeled as game theoretic situations. Some procedures are included in this package to calculate the most important allocations rules in Game Theory: Shapley value, Owen value or nucleolus, among other. First, we must define as an argument the value of the unions of the envolved agents with the characteristic function.

**License** GPL-2

**LazyLoad** yes

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2016-07-24 17:25:09

## Contents

GameTheoryAllocation-package . . . . .	2
coalitions . . . . .	3
EPM_allocation . . . . .	4
isinthecore . . . . .	5
nucleolus . . . . .	6
Owen_value . . . . .	7
Shapley_value . . . . .	8
<b>Index</b>	<b>9</b>

---

 GameTheoryAllocation-package

*Tools for Calculating Allocations in Game Theory*


---

## Description

Many situations can be modeled as game theoretic situations. Some procedures are included in this package to calculate the most important allocations rules in Game Theory: Shapley value, Owen value or nucleolus, among other. First, we must define as an argument the value of the unions of the involved agents with the characteristic function.

## Author(s)

A. Saavedra-Nieves

Maintainer: A. Saavedra-Nieves (alejandro.saavedra.nieves@gmail.com)

## References

Frisk, M., Gothe-Lundgren, M.,Jornsten, K., Ronnqvist, M. (2010). Cost allocation in collaborative forest transportation. European Journal of Operational Research, Vol. 205, pp. 448-458.

Gillies, D.B. (1953). Some theorems on n-person games. PhD thesis, Princeton University.

Owen, G. (1977). Values of games with a priori unions. Mathematical Economics and Game Theory: Essays in Honor of Oskar Morgenstern (Eds.: O. Moeschlin R. Hein). Springer, New York.

Shapley, L.S. (1953). A value por n-person games. In H. Kuhn y A. Tucker (eds), Contributions to the theory of games II, Vol. 28, Annals of Mathematics Studies. Princeton University Press.

Schmeidler, D. (1969). The nucleolus of a characteristic function game, SIAM Journal of Applied Mathematics, vol. 17, pp. 1163-1170.

## Examples

```
# Example 1
```

```
characteristic_function<-c(0,0.538, 0.761, 1.742, 0.554, 0.137, 0.293, 0.343)
isinthecore(characteristic_function,allocation=c(0.1,0.2,0.043),game="cost")
#[1] "The allocation is not in the core"
#NULL
```

```
isinthecore(characteristic_function,allocation=c(0.05,0.206,0.087),game="cost")
#[1] "The allocation is in the core"
#NULL
```

```
nucleolus(characteristic_function,game="cost")
#[1] "Nucleolus"
#      1      2 3
# 0.137 0.206 0
# Example 2
```

```

characteristic_function<-c(1,1,2,1,2,2,2)
Owen_value(characteristic_function,union=list(c(1,2),c(3)),game="cost")
#[1] "Owen Value"
#    1    2    3
# 0.25 0.25 1.5

```

---

coalitions

*coalitions*


---

## Description

This function gives all the coalitions in a binary mode and usual way.

## Usage

```
coalitions(n)
```

## Arguments

`n`                      Number of the involved players

## Value

A list with the following components:

`Binary`                A matrix where each row indicates a binary representation of the coalition in a binary mode. The second one, the usual way.

`Classic`               A vector with the associated representation of the coalitions.

## Author(s)

A. Saavedra-Nieves

## Examples

```

coalitions(3)
#$Binary
#      [,1] [,2] [,3]
#[1,]  0   0   0
#[2,]  1   0   0
#[3,]  0   1   0
#[4,]  0   0   1
#[5,]  1   1   0
#[6,]  1   0   1
#[7,]  0   1   1
#[8,]  1   1   1
#
#$Classic
#[1] "0"           "'{ 1 }'"      "'{ 2 }'"      "'{ 3 }'"      "'{ 1,2 }'"
#[6] "'{ 1,3 }'"    "'{ 2,3 }'"    "'{ 1,2,3 }'"

```

---

EPM\_allocation                      *Equal Profit Method allocation*

---

### Description

This function gives the Equal Profit Method allocation described in Frisk et al. (2010).

### Usage

```
EPM_allocation(characteristic_function, r = NA, info = NA, game = c("profit", "cost"))
```

### Arguments

characteristic_function	A vector with the characteristic function $v(S)$ (or $c(S)$ , if we work with a cost game), according to the order of coalitions shown in coalitions function.
r	Incremental step for calculating the EPM-allocation. If no solution is found, we increase iteratively in r units the allowed excess to get an epsilon-core allocation. Procedure stops when a solution is found.
info	For knowing information about the resolution, make info=1
game	Characters to indicate if the game is a cost or profit game. The possibles values are "cost" or "profit".

### Value

A vector with the allocation that EPM method proposes.

### Author(s)

A. Saavedra-Nieves

### References

Frisk, M., Gothe-Lundgren, M.,Jornsten, K., Ronnqvist, M. (2010). Cost allocation in collaborative forest transportation. European Journal of Operational Research, Vol. 205, pp. 448-458.

### Examples

```
characteristic_function<-c(0,0.538, 0.761, 1.742, 0.554, 0.137, 0.293, 0.343)
EPM_allocation(characteristic_function,r=0.01,info=1,game="cost")
#[1] "EPM_allocation"
#[1] "The cost game has a non-empty core"
#   1   2   3
# 0.05 0.206 0.087
```

---

isinthecore	<i>Isinthecore (core allocations)</i>
-------------	---------------------------------------

---

### Description

This function checks if the allocation belongs to the core of the TU game.

### Usage

```
isinthecore(characteristic_function, allocation, game = c("profit", "cost"))
```

### Arguments

characteristic_function	A vector with the characteristic function $v(S)$ (or $c(S)$ , if we work with a cost game), according to the order of coalitions shown in coalitions function.
allocation	A vector with the allocation where each component indicates the part of each agent.
game	Characters to indicate if the game is a cost or profit game. The possibles values are "cost" or "profit".

### Author(s)

A. Saavedra-Nieves

### References

Gillies, D.B. (1953). Some theorems on n-person games. PhD thesis, Princeton University.

### Examples

```
characteristic_function<-c(0,0.538, 0.761, 1.742, 0.554, 0.137, 0.293, 0.343)
isinthecore(characteristic_function,allocation=c(0.1,0.2,0.043),game="cost")
#[1] "The allocation is not in the core"
#NULL

isinthecore(characteristic_function,allocation=c(0.05,0.206,0.087),game="cost")
#[1] "The allocation is in the core"
#NULL
```

---

nucleolus

*Nucleolus*

---

### Description

This function gives the nucleolus described in Schmeidler (1969).

### Usage

```
nucleolus(characteristic_function, game = c("profit", "cost"))
```

### Arguments

`characteristic_function`

A vector with the characteristic function  $v(S)$  (or  $c(S)$ , if we work with a cost game), according to the order of coalitions shown in `coalitions` function.

`game`

Characters to indicate if the game is a cost or profit game. The possible values are "cost" or "profit".

### Value

A vector with the allocation that nucleolus proposes.

### Author(s)

A. Saavedra-Nieves

### References

Schmeidler, D. (1969). The nucleolus of a characteristic function game, *SIAM Journal of Applied Mathematics*, vol. 17, pp. 1163-1170.

### Examples

```
characteristic_function<-c(0,0.538, 0.761, 1.742, 0.554, 0.137, 0.293, 0.343)
nucleolus(characteristic_function,game="cost")
#[1] "Nucleolus"
#      1      2 3
# 0.137 0.206 0
```

---

Owen_value	<i>Owen_value (Owen value)</i>
------------	--------------------------------

---

**Description**

This function gives the Owen value described in Owen (1977).

**Usage**

```
Owen_value(characteristic_function, union, game = c("profit", "cost"))
```

**Arguments**

characteristic_function	A vector with the characteristic function $v(S)$ (or $c(S)$ , if we work with a cost game), according to the order of coalitions shown in coalitions function.
union	A list with a partition of the players set. Each element of this list is a union a priori, following the proposal of Owen.
game	Characters to indicate if the game is a cost or profit game. The possible values are "cost" or "profit".

**Value**

A vector with the allocation that Owen value (Owen, 1977) proposes.

**Author(s)**

A. Saavedra-Nieves

**References**

Owen, G. (1977). Values of games with a priori unions. *Mathematical Economics and Game Theory: Essays in Honor of Oskar Morgenstern* (Eds.: O. Moeschlin R. Hein). Springer, New York.

**Examples**

```
characteristic_function<-c(1,1,2,1,2,2,2)
Owen_value(characteristic_function,union=list(c(1,2),c(3)),game="cost")
#[1] "Owen Value"
#   1   2   3
# 0.25 0.25 1.5

Shapley_value(characteristic_function,game="cost")
#[1] "Shapley Value"
#       1       2       3
# 0.3333333 0.3333333 1.3333333
```

---

Shapley\_value                      *Shapley\_value (Shapley Value)*

---

### Description

This function gives the Shapley value introduced in Shapley (1953).

### Usage

```
Shapley_value(characteristic_function, game = c("profit", "cost"))
```

### Arguments

`characteristic_function`  
 A vector with the characteristic function  $v(S)$  (or  $c(S)$ , if we work with a cost game), according to the order of coalitions shown in `coalitions` function.

`game`  
 Characters to indicate if the game is a cost or profit game. The possible values are "cost" or "profit".

### Value

A vector with the allocation that Shapley value (Shapley, 1953) proposes.

### Author(s)

A. Saavedra-Nieves

### References

Shapley, L.S. (1953). A value for  $n$ -person games. In H. Kuhn y A. Tucker (eds), Contributions to the theory of games II, Vol. 28, Annals of Mathematics Studies. Princeton University Press.

### Examples

```
characteristic_function<-c(1,1,2,1,2,2,2)

Shapley_value(characteristic_function,game="cost")
#[1] "Shapley Value"
#      1      2      3
# 0.333333 0.333333 1.333333
```



# Index

- \* **EPM\_allocation**
    - EPM\_allocation, 4
  - \* **EPM**
    - EPM\_allocation, 4
  - \* **GameTheory**
    - GameTheoryAllocation-package, 2
  - \* **coalition**
    - coalitions, 3
  - \* **coa**
    - coalitions, 3
  - \* **core\_allocations**
    - isinthecore, 5
  - \* **core**
    - isinthecore, 5
  - \* **nucleolusgt**
    - nucleolus, 6
  - \* **nucleolus**
    - nucleolus, 6
  - \* **owen\_value**
    - Owen\_value, 7
  - \* **owen**
    - Owen\_value, 7
  - \* **shapley\_value**
    - Shapley\_value, 8
  - \* **shapley**
    - Shapley\_value, 8
- coalitions, 3
- EPM\_allocation, 4
- GameTheoryAllocation  
(GameTheoryAllocation-package),  
2
- GameTheoryAllocation-package, 2
- isinthecore, 5
- nucleolus, 6
- Owen\_value, 7
- Shapley\_value, 8