Exercise Series 5

1. Simulation of Confidence Intervals

a) Simulate 500 samples of 10 normally distributed random variables, with expectation value $\mu = 0$ and standard deviation $\sigma = 2$. Store your simulation data in a matrix `stichprob` (one line per sample).
Hint: You need the functions `matrix` and `rnorm`.

b) The goal is, for every sample, to calculate the 95% confidence interval for μ based on the t-test. This information is contained in the object `conf.int` in the output of the function `t.test()`.

- Hence first you need to calculate the t-test for each line of your matrix `stichprob` and look at the resulting object with `str()`.
  Hint: First try this on one line (sample) of your matirx:
  `t.a <- t.test(stichprob[1,])` and `str(t.a)`
- Now define a function that returns the confidence interval for an input vector `x`.
  Hint:
  ```
  f.conf <- function(x){
  t.a <- t.test(x)
  t.conf <- t.a$conf.int
  return(t.conf)
  }
  ```
- Apply your function to each line of your matrix `stichprob`. Use the command `apply(stichprob,...,...)` and store the result in `t.conf`
- To find out how often the true parameter $\mu = 0$ is contained in the confidence interval, use the following operators $==, <=, >=, <, >$ and $!=$ for *comparisons* and $\&, |$ and ! for *logical links*.
  Note (and use the fact) that the logical values `T` (for TRUE) and `F` (for FALSE) are automatically interpreted as numbers 1 and 0 in functions such as `sum()`.
  Which result would you expect from theory?

c) Write a function `f.konfidenz` (using the exercise part b) above), that takes the number of samples and the desired confidence level as input. The output should be the percentage of cases in which $\mu = 0$ lies within the confidence interval.
Example:
```
> f.konfidenz(500,0.95)
[1] 0.948
```

For those interested: The confidence interval based on the t-test is defined as follows:

$$\overline{X} \pm t_{0.975,n-1} \cdot \hat{\sigma}/\sqrt{n}\,,$$

where $\overline{X}$ *is the arithmetic mean and* $\hat{\sigma}$ *the empirical standard deviation of the sample* $X_1, \ldots, X_n$*, and* $t_{0.975,n-1}$ *the 97.5%-quantile of the t-distribution with* $n-1$ *degrees of freedom.*

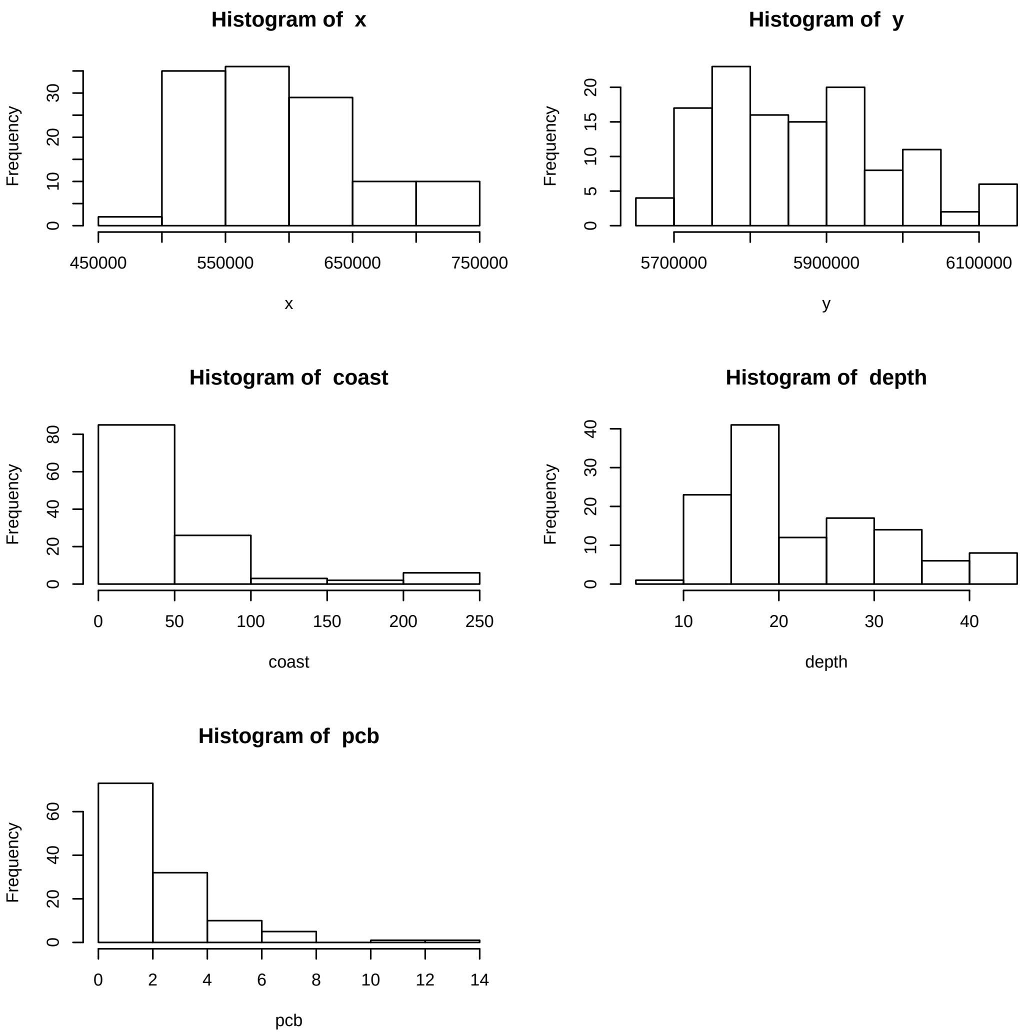
2. Functions for Graphical Output

a) Create a function that displays each variable in a data.frame as a histogram. Add sensible titles and axis labels.

Develop your plotting function for the 5 non-factor variables of the data.frame `d.pcb[,2:6]`.

```
d.pcb <- read.table("http://stat.ethz.ch/~stahel/courses/R/pcb.txt",
        header=TRUE).
```

The function should take the name of a dataframe as the input variable. First find out how many variables there are to plot. Then automatically arrange the histograms on one page using the `par(mfrow=...)` command, and label the axes and add titles.
The output should look similar to:



(*) As an extension to question (a), try to include an if-condition in your function that checks for factor-variables and adds barplots for those to the histograms.

b) Now apply your function to a new data set. Use `d.sport` to check whether your function works for any data set.

```
d.sport <- read.table("http://stat.ethz.ch/Teaching/Datasets/NDK/sport.dat",
        header=TRUE).
```