Current status data with competing risks: Nonparametric estimation

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Joint work with Piet Groeneboom and Jon Wellner

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- People are tested for HIV at follow-up times $\Rightarrow$ interval censoring:
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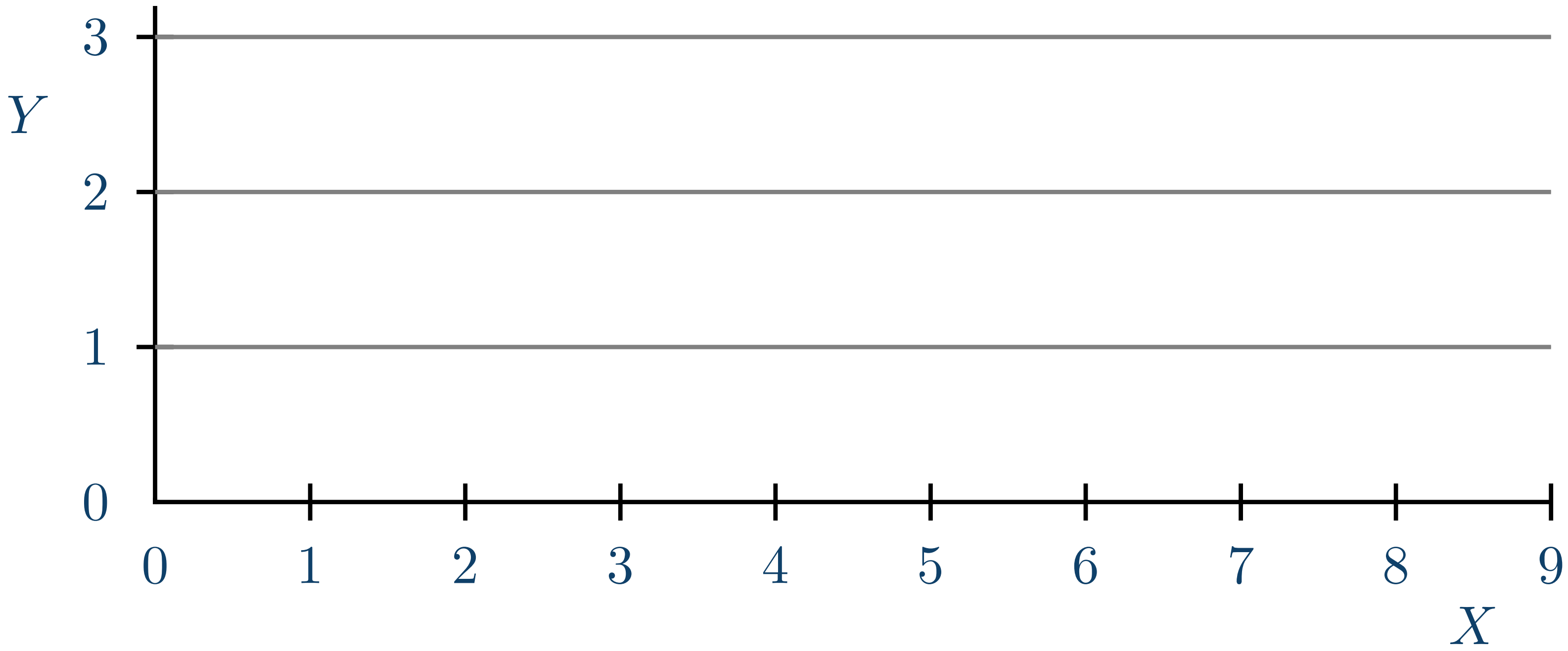
  - Y is sometimes observed
- As a first step, we consider only one observation time per person: current status censoring

Graphical representation

Example, $K = 3$



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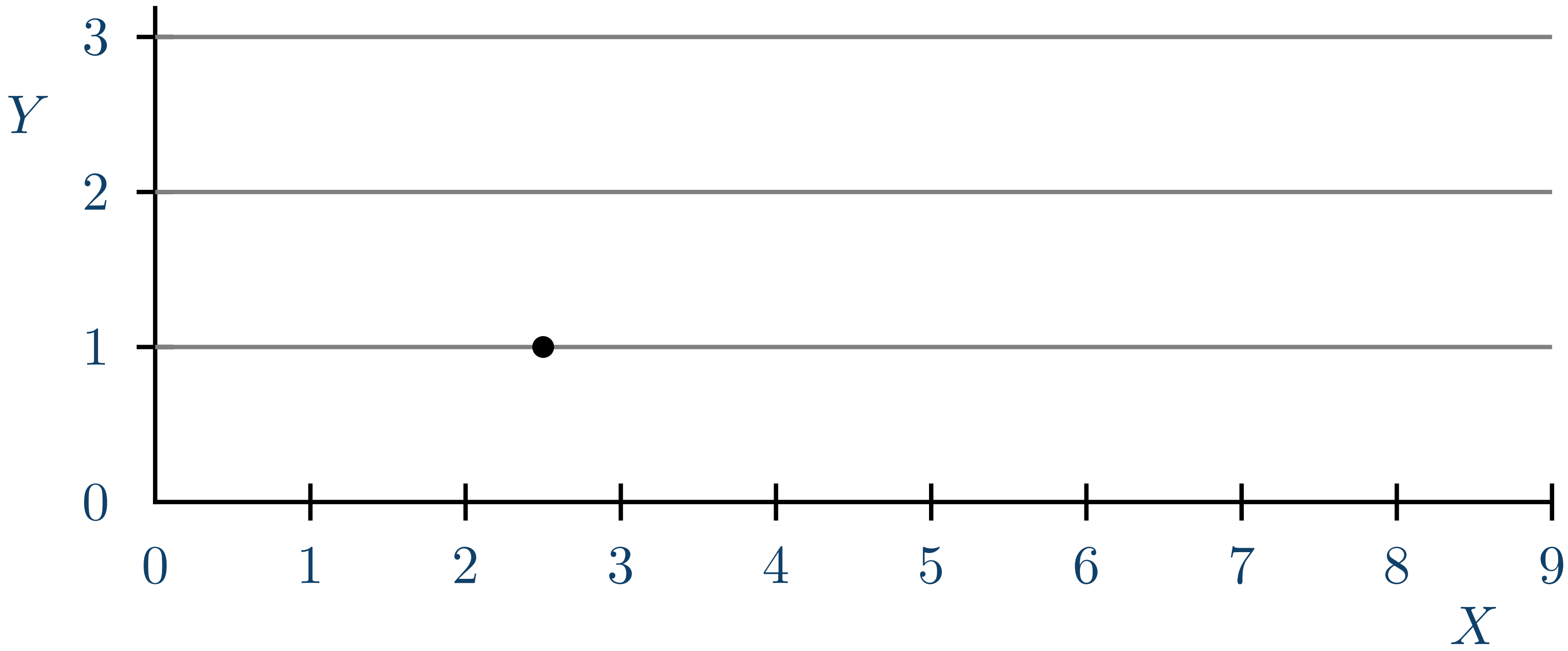

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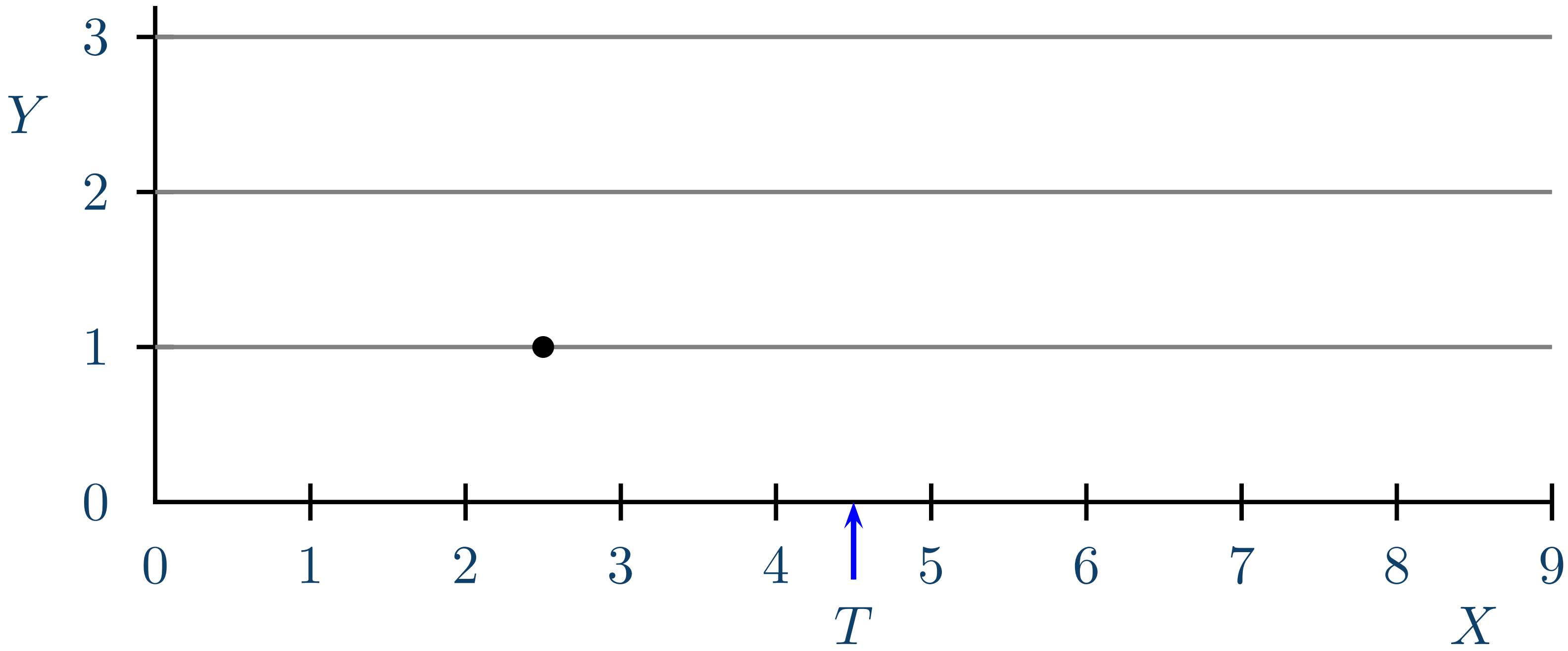

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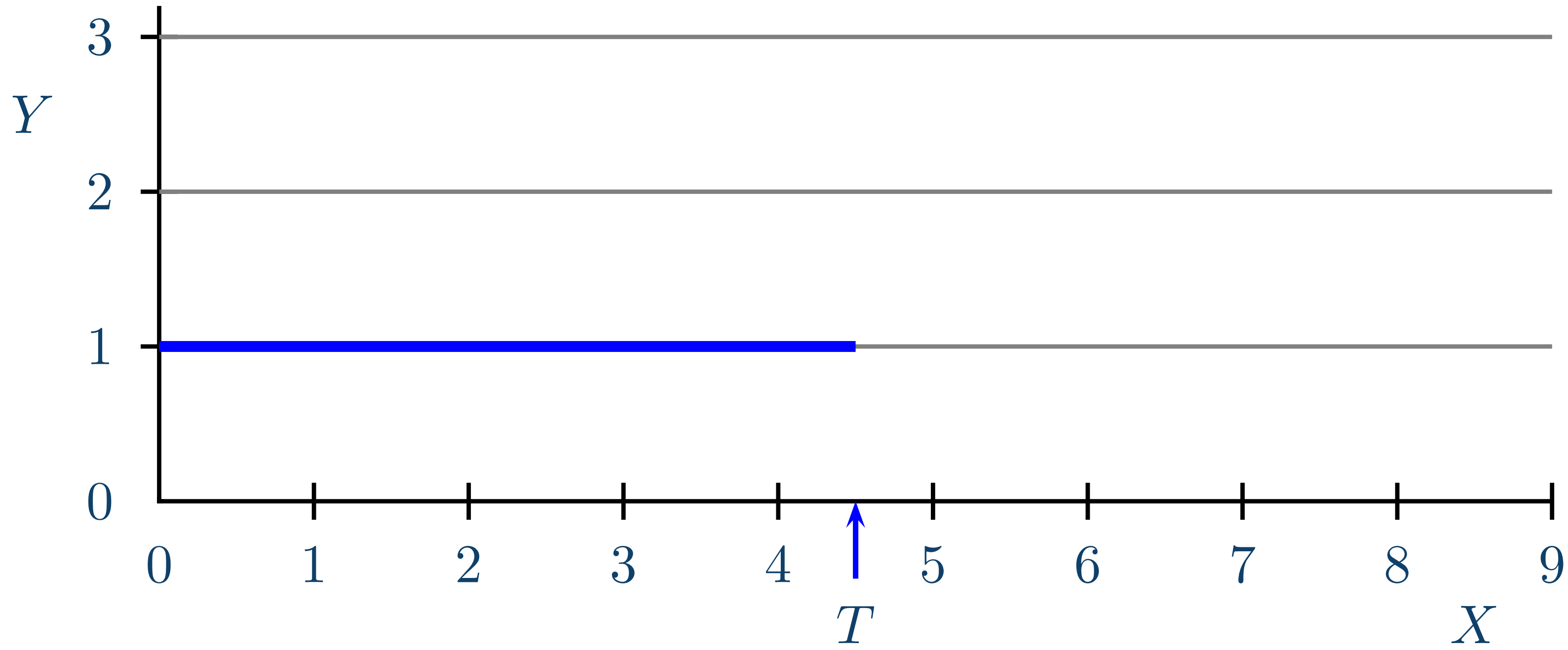

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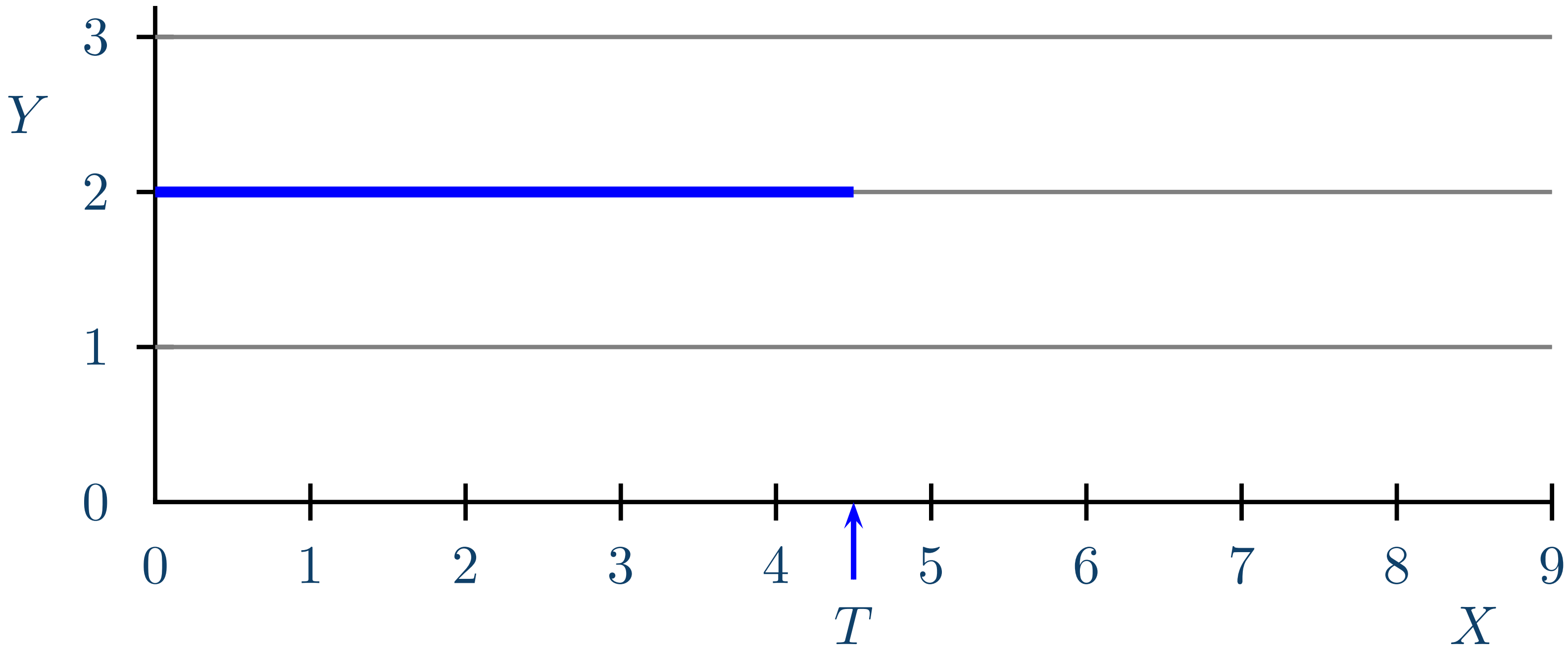

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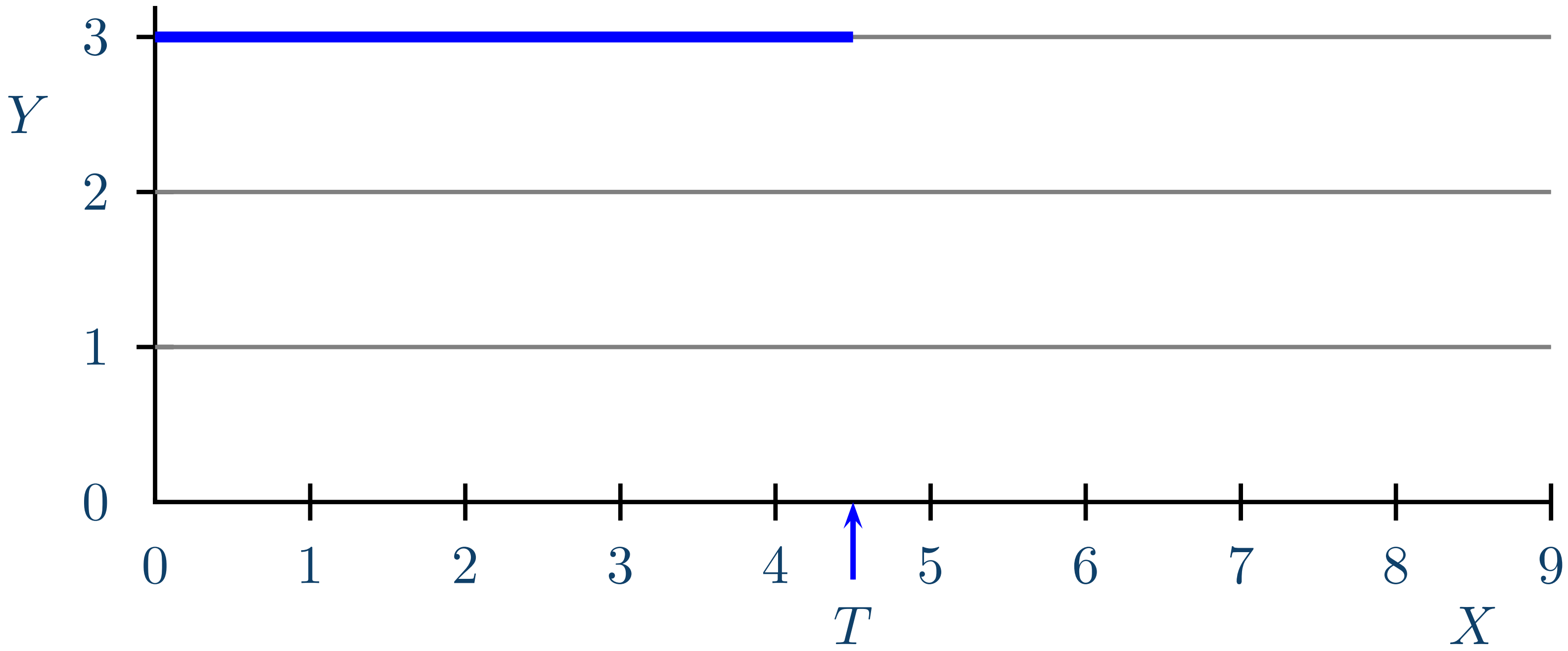

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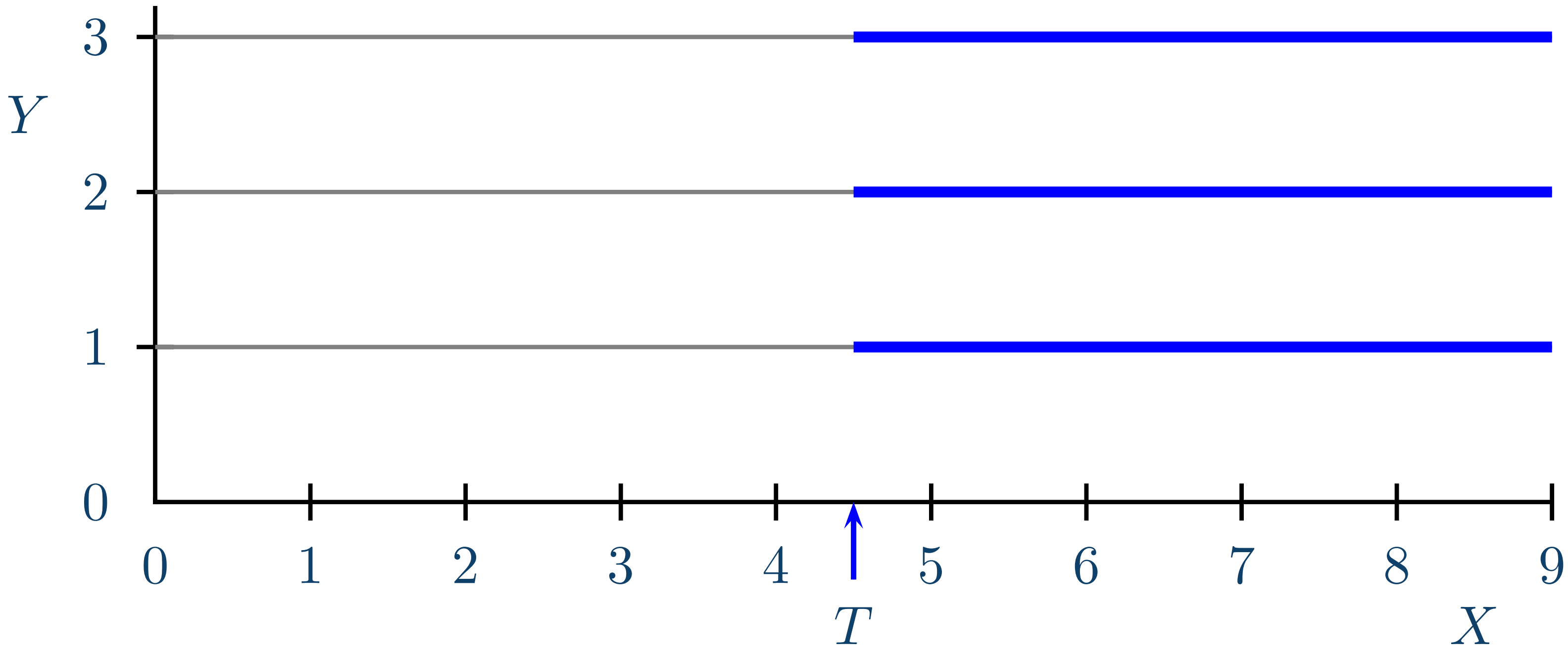

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- n i.i.d. observations of (T, Δ), where $\Delta = (\Delta_1, \ldots, \Delta_{K+1})$:
  - $\Delta_j = 1\{X \leq T, Y = j\}, \qquad j = 1, \ldots, K$
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- We assume that T is independent of (X, Y)

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- The sub-distribution functions are related to each other in the sense that $\sum_{j=1}^{K} F_{0j}(t) = P(X \leq t) \leq 1$.

Overview of previous work in this area

- Key papers:
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- Main results in these papers:
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- In this talk I focus on the MLE and a ‘naive’ estimator:
  - Consistency
  - Rate of convergence

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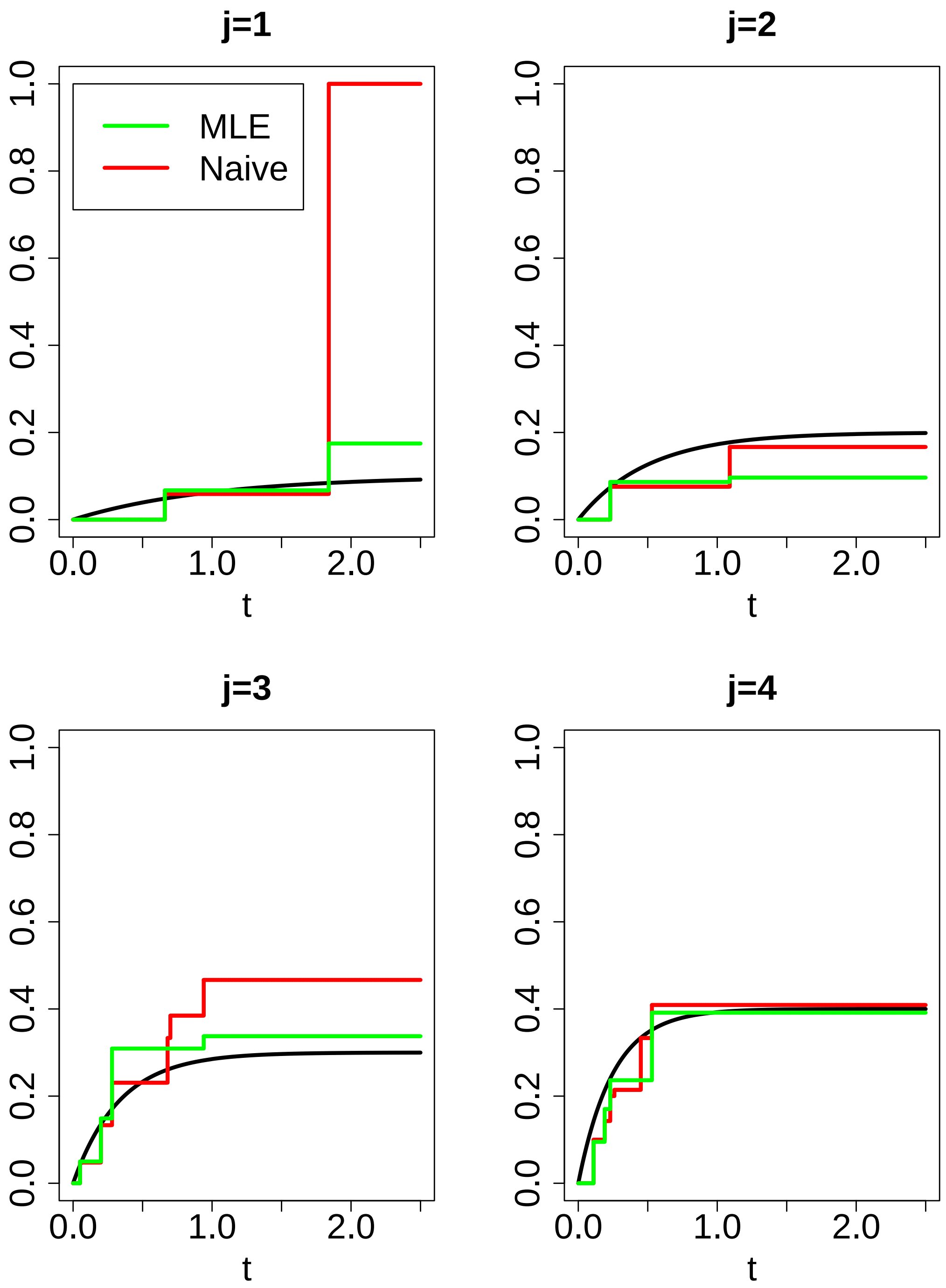
$F_{0j}(t) = \frac{j}{10}(1 - \exp(-jt))$

$T \sim \text{Exp}(2)$

$n = 100$



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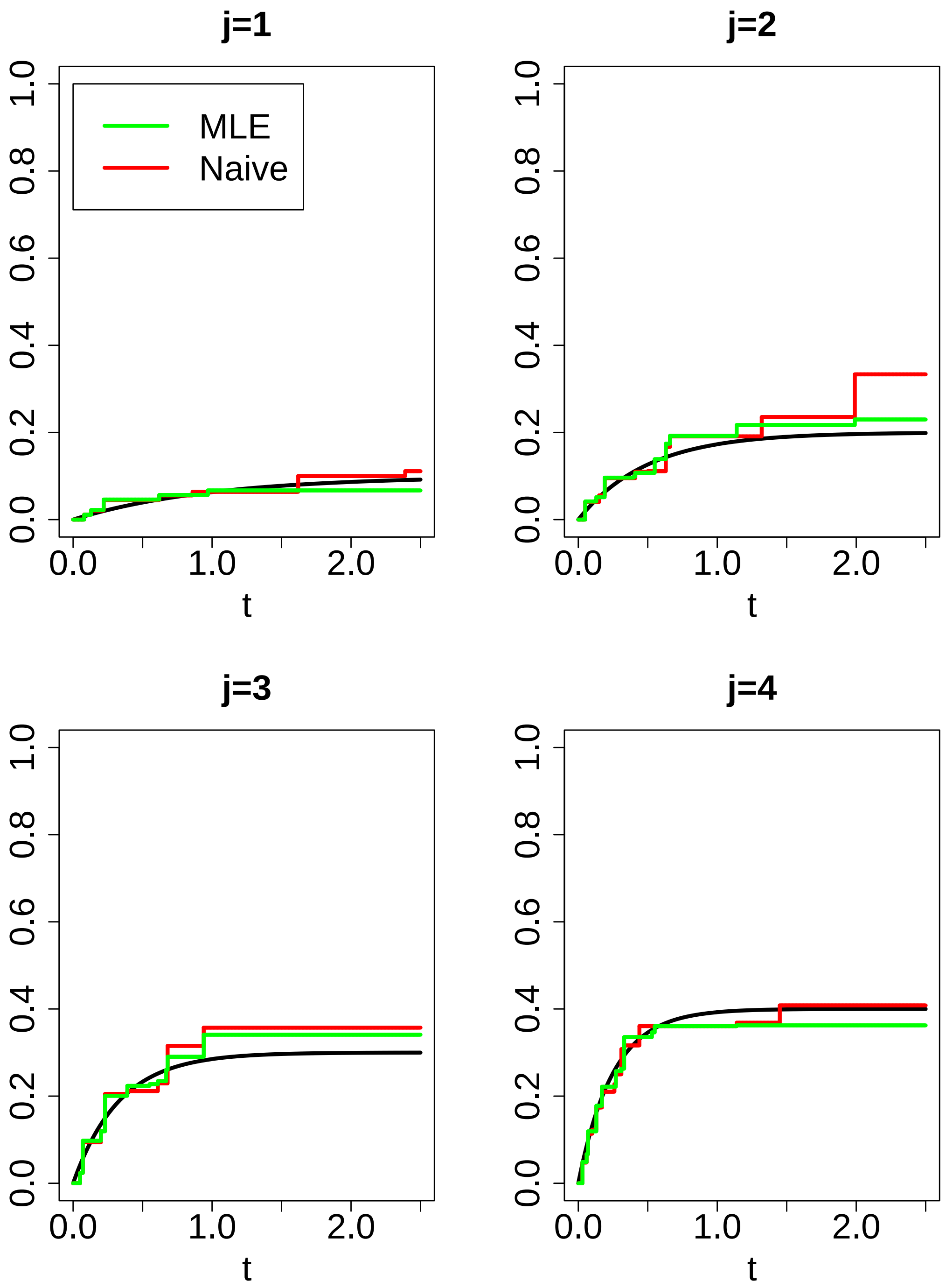

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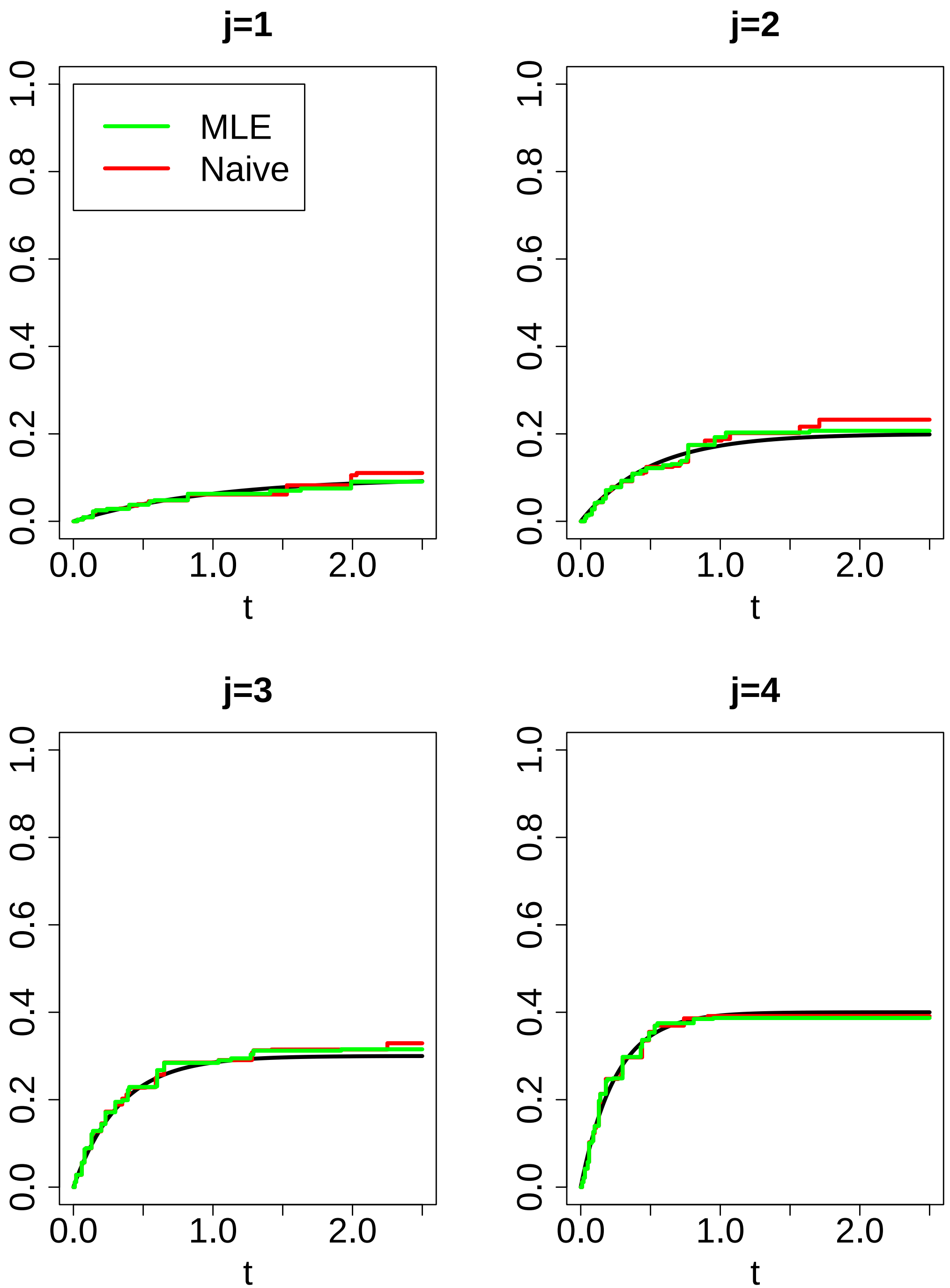

MLE and naive estimator

- Notation:
  - $\Delta_+ = \sum_{j=1}^K \Delta_j, \qquad F_+(t) = \sum_{j=1}^K F_j(t)$
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Asymptotic properties of the estimators

- For all $j = 1, \ldots, K$, $\widetilde{F}_{nj}$ maximizes

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- For the MLE, we derive:
    - Consistency
    - Rate of convergence

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- Under some regularity conditions, there exists an $\epsilon > 0$ such that

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Proof: Follows from global consistency

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- Global rate of convergence:

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- Result is intuitively clear
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- Approach:
  - Characterize the MLE in terms of necessary and sufficient conditions
  - Assume that the distance between the estimator and the truth is larger than $Mn^{-1/3}$. Then derive that the characterization is violated with high probability.

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Local rate for the sum of the MLE

- Under some regularity conditions, we proved

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$$\sup_{t\in[-M,M]}\left|\widehat{F}_{n+}(t_0+n^{-1/3}t)-F_{0+}(t_0)\right|=O_p(n^{-1/3})$$

- However, this is not enough to control

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- Solution: rate result on a fixed neighborhood of t_0.

Rate for the sum of the MLE on a fixed neighborhood

- Define

$$v_n(t) = \begin{cases} n^{-1/3} & \text{for } t \le n^{-1/3}, \\ n^{-1/6}\,|t|^{1/2} & \text{for } t > n^{-1/3}. \end{cases}$$

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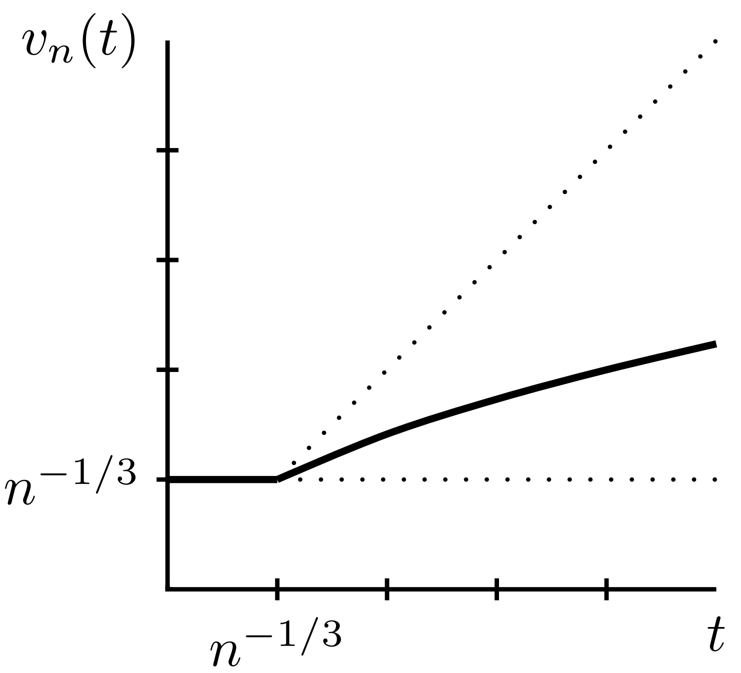

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uniformly in $t \in [t_0 - \epsilon, t_0 + \epsilon]$.

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  uniformly in $t \in [t_0 - \epsilon, t_0 + \epsilon]$.
- This result also holds for current status data without competing risks (take $K = 1$)
- It implies the usual result on a shrinking neighborhood

Proof of local rate of the MLE

- Characterization: for all $t \geq \tau_{nj}$

$$\int_{[\tau_{nj},t)} \left\{ \delta_j - \widehat{F}_{nj}(u) + \{\delta_+ - \widehat{F}_{n+}(u)\} \frac{F_{0j}(t)}{1 - F_{0+}(t)} + R \right\} d\mathbb{P}_n \geq 0$$

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- The local rate then follows as for the naive estimator

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- Two estimators: MLE and naive estimator
- Results:
  - Global and local consistency
  - Global and local rate of convergence: $n^{-1/3}$
  - New rate of convergence result for $\widehat{F}_{n+}$ on a fixed neighborhood
  - Both the MLE and the naive estimator converge locally at the optimal rate

Current and future work

- Current work: limiting distribution:

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