



Marcel Dettling

Institute for Data Analysis and Process Design

Zurich University of Applied Sciences

marcel.dettling@zhaw.ch

http://stat.ethz.ch/~dettling

ETH Zürich, May 13, 2013





Exponential Smoothing Revisited

Basic notion: $X_t = \mu_t + E_t$

- μ_t is the conditional expectation, which we try to estimate from the data. The estimate a_t is called level of the series.
- E_t is a completely random innovation term.

Estimation of the level: two notions exist...

- Weighted updating: $a_t = \alpha x_t + (1 \alpha)a_{t-1}$
- Exponential smoothing: $a_t = \sum_{i=0}^{\infty} \alpha (1-\alpha)^i x_{t-i}$

Practice & Forecasting: see blackboard...



Zurich University

Exponential Smoothing: Example





Zurich University

Exponential Smoothing: Example

> fit <- HoltWinters(cmpl, beta=F, gamma=F)</pre>

Holt-Winters exponential smoothing without trend and without seasonal component.

Smoothing parameters:

- alpha: 0.1429622
- beta : FALSE
- gamma: FALSE

Coefficients: [,1] a 17.70343



Zurich University

Exponential Smoothing: Example



Holt-Winters filtering

Zurich University of Applied Sciences

Applied Time Series Analysis SS 2013 – Week 12



Holt-Winters Method

Purpose:

- is for time series with deterministic trend and/or seasonality
- is still a heuristic, model-free approach
- again based on weighted averaging

Is based on these 3 formulae:

$$a_{t} = \alpha(x_{t} - s_{t-p}) + (1 - \alpha)(a_{t-1} + b_{t-1})$$

$$b_{t} = \beta(a_{t} - a_{t-1}) + (1 - \beta)b_{t-1}$$

$$s_{t} = \gamma(x_{t} - a_{t}) + (1 - \gamma)s_{t-p}$$

→ See the blackboard for the derivation...



Zurich University

Holt-Winters: Example



Sales of Australian White Wine



Zurich University

Holt-Winters: Example





Zurich University

Holt-Winters: R-Code and Output

> HoltWinters(x = log(aww))

Holt-Winters exponential smoothing with trend and additive seasonal component.

```
Smoothing parameters:
 alpha: 0.4148028; beta : 0; gamma: 0.4741967
Coefficients:
```

a 5.62591329; b 0.01148402

```
sl -0.01230437; s2 0.01344762; s3 0.06000025
```

```
s4 0.20894897; s5 0.45515787; s6 -0.37315236
```

```
s7 -0.09709593; s8 -0.25718994; s9 -0.17107682
```

```
s10 -0.29304652; s11 -0.26986816; s12 -0.01984965
```



Zurich University

Holt-Winters: Fitted Values & Predictions



Holt-Winters filtering



Zurich University

Holt-Winters: In-Sample Analysis



Holt-Winters-Fit



Zurich University

Holt-Winters: Predictions on Original Scale





Exercise

Data:

- \rightarrow use the Australian white wine sales data...
- \rightarrow ... or any other dataset you like

Goal:

- Find a good model describing these data
- Evaluate which model yields the best predictions
- Generate a 29-month forecast from this model

Method:

→ Remove the last 29 observations and mimic oos-forecasting