

Dealing with missing values – part 1

Applied Multivariate Statistics – Spring 2012



Overview

- Bad news: Data Processing Inequality
- Types of missing values: MCAR, MAR, MNAR
- Methods for dealing with missing values:
 - Case-wise deletion
 - Single Imputation
 - (- Multiple Imputation in Part 2)

Information Theory 101

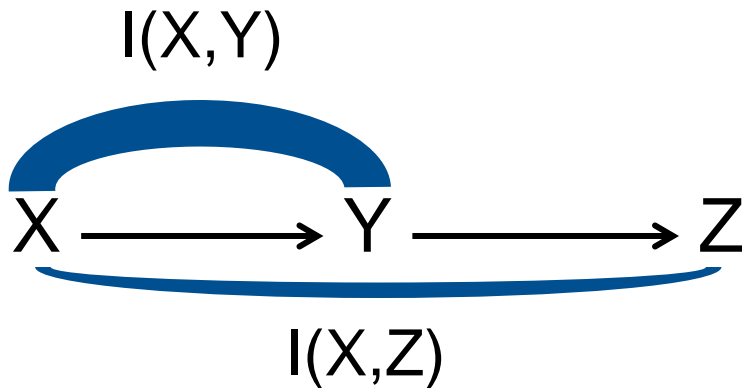
- Entropy: Amount of uncertainty

$$H(X) = - \sum_{x \in X} p(x) \log(p(x))$$

- Mutual Information btw. X and Y
 - What do you learn about X , if you know Y ?
 - Decrease in entropy of X , if Y is known

$$I(X, Y) = H(X) - H(X|Y)$$

Information Theory 101: Data Processing Inequality



For a Markov Chain: $I(X, Z) \leq I(X, Y)$

Postprocessing can never add information

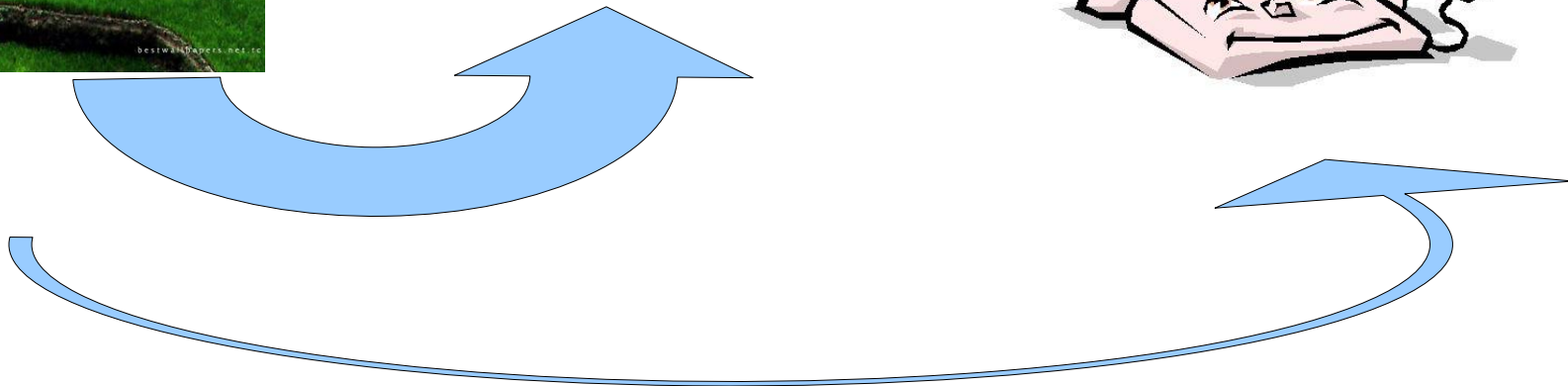
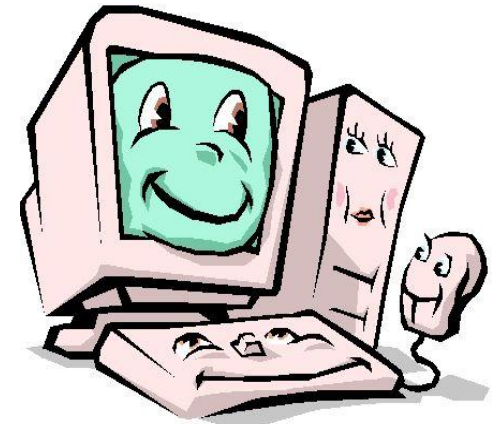
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Postprocessing can never add information

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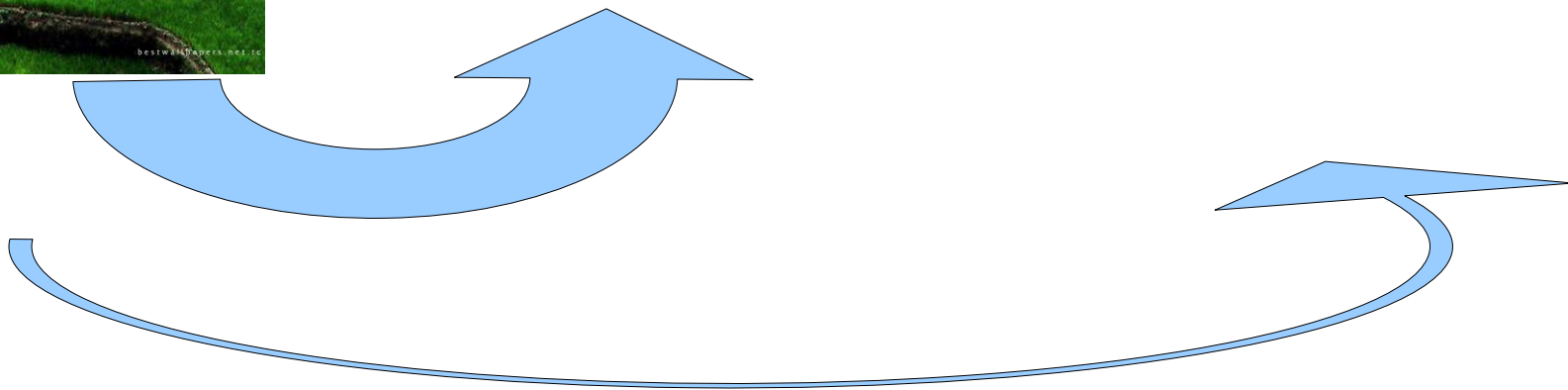


Data with
missing values

A	B	C
1.3	5.4	7.2
3.2	?	?
?	8.3	?

After dealing with
missing values
somehow

A	B	C
1.3	5.4	7.2
3.2	7.2	5.6
8.1	8.3	8.2



Information Theory on dealing with missing values

- The information is lost!
You cannot retrieve it just from the data!
- Try to avoid missing values where possible!
- When dealing with the data, don't waste even more information!
Use clever methods!

Get an overview of missing values in data

- R: Function “md.pattern” in package “mice”

Types of missing values

- Missing Completely At Random (MCAR)
- Missing At Random (MAR)
- Missing Not At Random (MNAR)

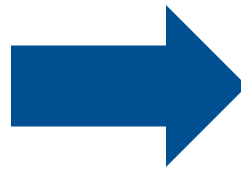
OK

PROBLEM

Distribution of Missingness

Complete data Y_{com}

A	B	C
1.3	2.5	6.3
2.0	3.6	5.4
1.6	2.3	4.3



Some values are missing

Y_{obs}

A	B	C
1.3	2.5	
2.0		5.4
1.6		4.3

Y_{mis}

A	B	C
		6.3
	3.6	
	2.3	

R

A	B	C
1	1	0
1	0	1
1	0	1

Example: Blood Pressure

- 30 participants in January (X) and February (Y)
- MCAR: Delete 23 Y values randomly
- MAR: Keep Y only where $X > 140$ (follow-up)
- MNAR: Record Y only where $Y > 140$ (test everybody again but only keep values of critical participants)

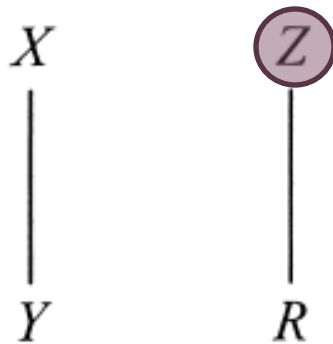
X	Y			
	Complete	MCAR	MAR	MNAR
Data for individual participants				
169	148	148	148	148
126	123	—	—	—
132	149	—	—	149
160	169	—	169	169
105	138	—	—	—
116	102	—	—	—
125	88	—	—	—
112	100	—	—	—
133	150	—	—	150
94	113	—	—	—
109	96	—	—	—
109	78	—	—	—
106	148	—	—	148
176	137	—	137	—
128	155	—	—	155
131	131	—	—	—
130	101	101	—	—
145	155	—	155	155
136	140	—	—	—
146	134	—	134	—
111	129	—	—	—
97	85	85	—	—
134	124	124	—	—
153	112	—	112	—
118	118	—	—	—
137	122	122	—	—
101	119	—	—	—
103	106	106	—	—
78	74	74	—	—
151	113	—	113	—

Distribution of Missingness

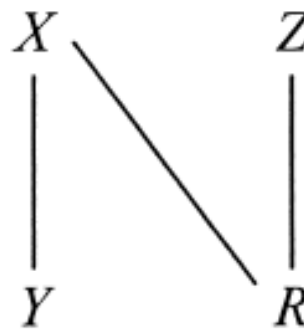
- MCAR $P(R|Y_{com.}) = P(R)$
Missingness does not depend on data
- MAR $P(R|Y_{com.}) = P(R|Y_{obs})$
Missingness depends only on observed data
- MNAR $P(R|Y_{com.}) = P(R|Y_{obs})$
Missingness depends on missing data

Distribution of Missingness: Intuition

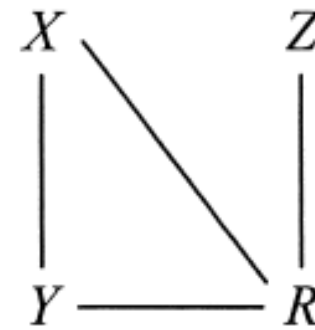
Some unmeasured
variables not related to
 X or Y



(a) MCAR



(b) MAR



(c) MNAR

Problems in practice

- Type is not testable.
- Pragmatic:
 - Use methods which hold in MAR
 - Don't use methods which hold only in MCAR

Dealing with missing values

- Complete-case analysis - **valid for MCAR**
- Single Imputation - **valid for MAR**
- (Multiple Imputation – **valid for MAR**)

Complete-case analysis

- Delete all rows, that have a missing value
- Problem:
 - waste of information; **inefficient**
 - introduces **bias if MAR**
- OK, if 95% or more complete cases
- R: Function “complete.cases” in base distribution

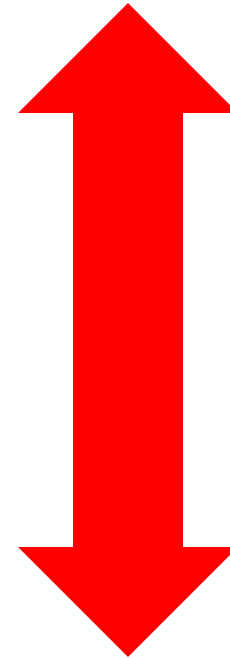
A	B	C	D
NA	3	4	6
3	2	3	NA
2	NA	5	4
5	7	NA	5
6	NA	9	2

- 25% missing values
 - ZERO complete cases
- Complete-case analysis is useless

Single Imputation

- Unconditional Mean
- Unconditional Distribution
- Conditional Mean
- Conditional Distribution

Easy / Inaccurate



Hard / Accurate

Unconditional Mean: Idea

A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	NA

Mean = 4.75



A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	4.75

Unconditional Distribution: Hot Deck Imputation

A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	NA

Randomly select
observed value
in column



A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	6.3

Conditional Mean: E.g. Linear Regression

A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	NA

Estimate $\text{Im}(C \sim A + B)$
or something similar

Apply to predict C

Conditional Mean: E.g. Linear Regression

A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	NA

Prediction of
linear regression



A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	8

Conditional Distribution: E.g. Linear Regression

- Start with Conditional Mean as before
- Add randomly sampled residual noise

A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	NA

Prediction of
linear regression

PLUS NOISE



A	B	C
2.1	6.2	3.2
3.4	3.7	6.3
4.1	4.5	8.3

Being pragmatic:

Conditional Mean Imputation with missForest

- Use Random Forest (see later lecture) instead of linear regression
- Good trade-off between ease of use / accuracy
- Works with **mixed data** types (categorical, continuous and mixed)
- Estimates the **quality of imputation**
OOBError: Imputation error as percentage of total variation
close to 0 - good
close to 1 - bad

Idea of missForest

A	B	SEX
2.1	NA	M
3.4	3.7	F
4.1	4.5	NA

Idea of missForest

A	B	SEX
2.1	3.0	M
3.4	3.7	F
4.1	4.5	F

Fill in random values

Idea of missForest: Step 1

A	B	SEX
2.1	3.0	M
3.4	3.7	F
4.1	4.5	F

Apply $B \sim A + \text{SEX}$

Learn $B \sim A + \text{SEX}$
with Random Forest

Idea of missForest: Step 1

A	B	SEX
2.1	3.2	M
3.4	3.7	F
4.1	4.5	F

Apply $B \sim A + \text{SEX} \rightarrow$ update value

Learn $B \sim A + \text{SEX}$
with Random Forest

Idea of missForest: Step 2

A	B	SEX
2.1	3.2	M
3.4	3.7	F
4.1	4.5	F

Learn $SEX \sim A + B$
with Random Forest

Apply $SEX \sim A + B \rightarrow$ update

Repeat steps 1 & 2 until some stopping criterion is reached
(no real convergence;
stop if updates start getting bigger again)

Measuring quality of imputation

- Normalized Root Mean Squared Error (NRMSE):

$$NRMSE = \sqrt{\frac{\text{mean}(Y_{com} - Y_{imputed})^2}{\text{var}(Y_{com})}}$$

- Proportion of falsely classified entries (PFC) over all categorical values

$$PFC = \frac{\textit{nmb. missclassified}}{\textit{nmb. categorical values}}$$

Pros and Cons of missForest

- Effects are OK, if MAR holds
- Easily available: Function “missForest” in package “missForest”
- Estimation of imputation error
- Accuracy might be too optimistic, because
 - imputed values have no random scatter
 - model for prediction was taken to be the true model, but it is just an estimate
- Solution: Multiple Imputation

Concepts to know

- Data Processing Inequality and connection to missing values
- Distributions of missing values
- Case-wise deletion
- Methods for Single Imputation
- Idea of missForest; error measures for imputed values

R functions to know

- `md.pattern`
- `complete.cases`
- `missForest`