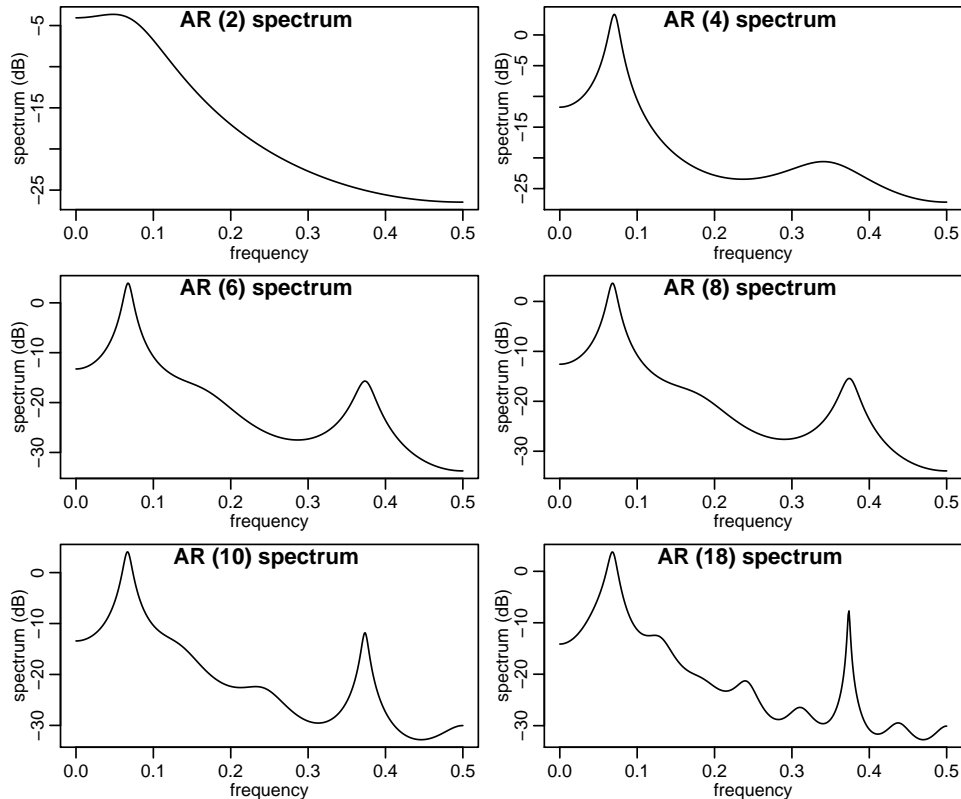


## Solution to Series 7

1. a) The theoretical spectra of the AR processes look as follows:

```
> par(mfrow=c(3,2))
> for (p in c(2, 4, 6, 8, 10, 18))
  spec.ar(ts.force, method = "burg", order = p, plot = TRUE, log = "dB")
```



Note that the length of an observation period is  $\Delta t = 0.15$  s. This means, e.g., that  $\nu = 0.25$  corresponds to a period of  $\frac{\Delta t}{\nu} = 0.6$  s.

For an order of  $p = 4$  and higher, there are two maxima, namely for  $\nu \approx 0.07$  (main maximum) and for  $\nu \approx 0.37$ .

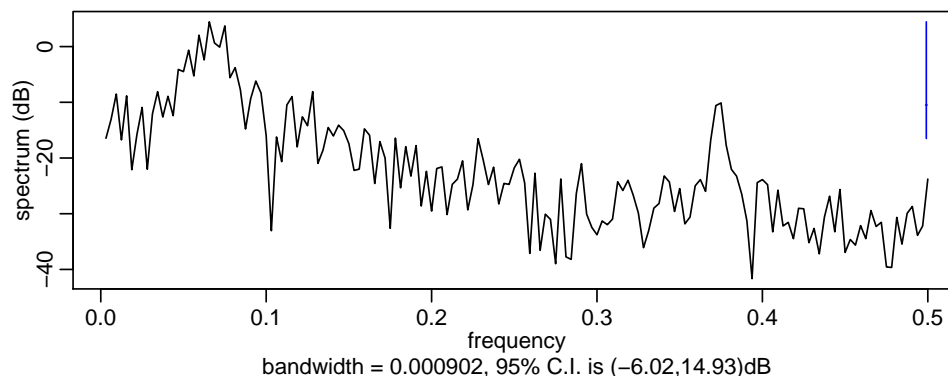
For the main maximum, the frequency of  $\nu \approx 0.07$  corresponds to a period of  $\frac{\Delta t}{0.07} \approx 2.1$  s. It is the period of approximately 2 seconds mentioned in the description of the time series.

For the secondary maximum, the frequency of  $\nu \approx 0.37$  corresponds to a period of  $\frac{0.15}{0.37} \approx 0.41$  s. This high frequency is visible as a superposition in the time series plot.

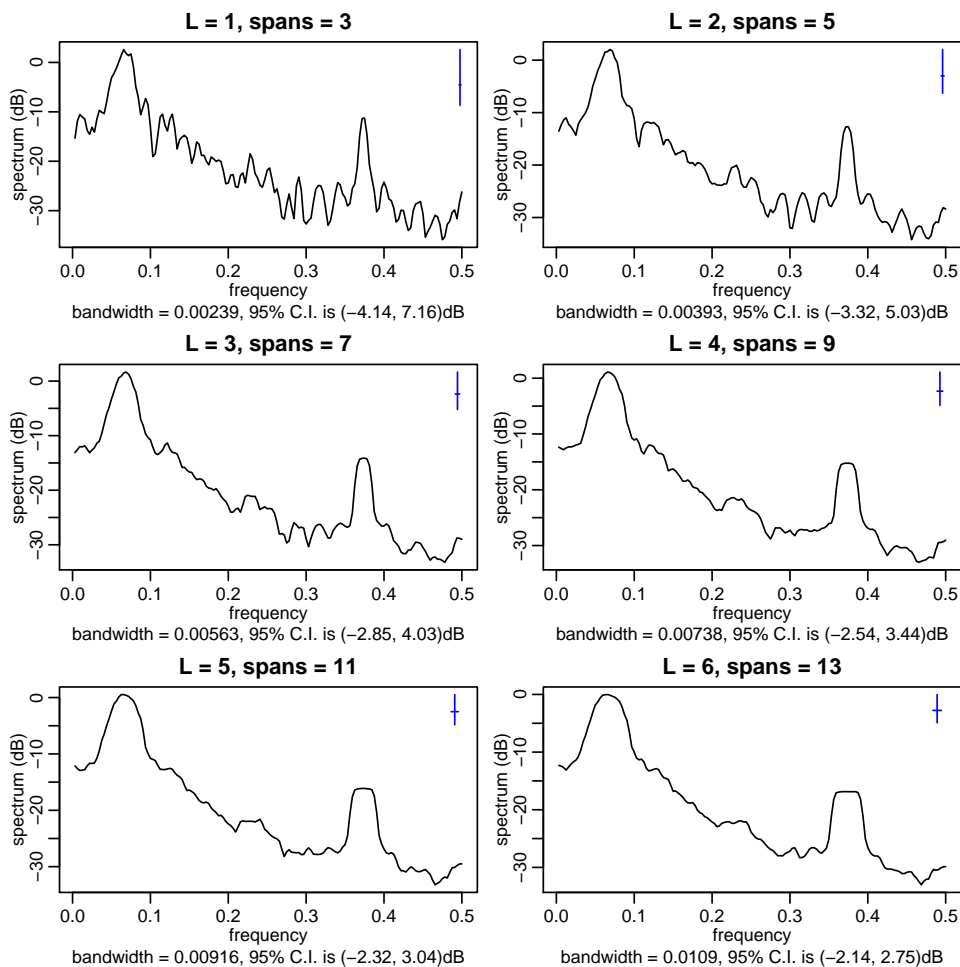
Due to the plots, an AR(6) model seems to be sufficient, since the spectra does not change significantly any more for higher orders. Note that in Exercise 1 of Sheet 4, we took an AR(9) model to get rid of correlations in the residuals. Using AIC for model selection (parameter `aic = TRUE` in the call of `ar.burg()`), one even gets an AR(18) model.

- b) The two maxima at  $\nu \approx 0.07$  and  $\nu \approx 0.37$  are also visible in the periodogram. By “smoothing by eye”, we get a periodogram that resembles the ones from part a).

Series: ts.force  
Raw Periodogram



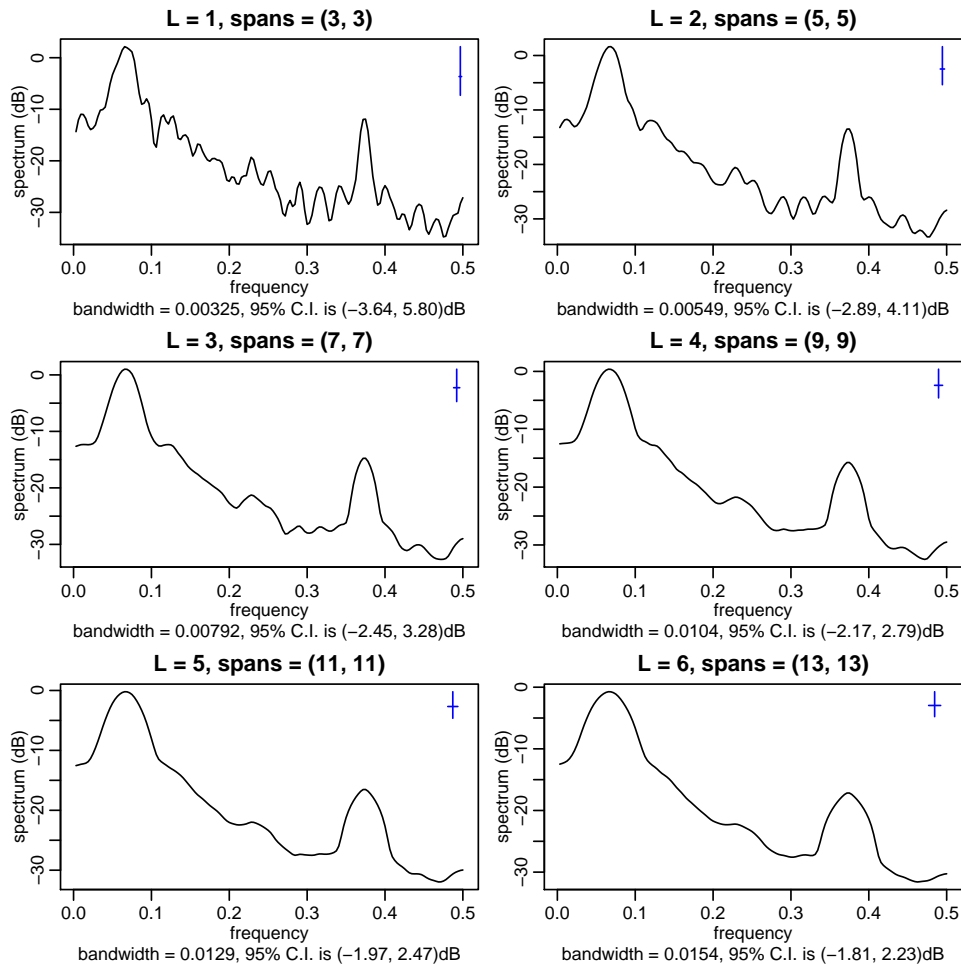
```
c) > par(mfrow = c(3, 2), mar = c(3.5, 3, 2, 0.1))
> for (L in 1:6)
  spec.pgram(ts.force, taper = 0, spans = 2*L + 1, detrend = FALSE, demean = TRUE,
    plot = TRUE, log = "dB", main = sprintf("L = %d, spans = %d", L, 2*L + 1))
```



With growing  $L$ , the periodogram gets smoother (less wiggly). The plots suggest to take a smoothing window with approximately  $L = 4$  (i.e., spans = 9). For very large  $L$ , the maxima broaden and loose weight.

```
d) > par(mfrow = c(3, 2), mar = c(3.5, 3, 2, 0.1))
> for (L in 1:6)
```

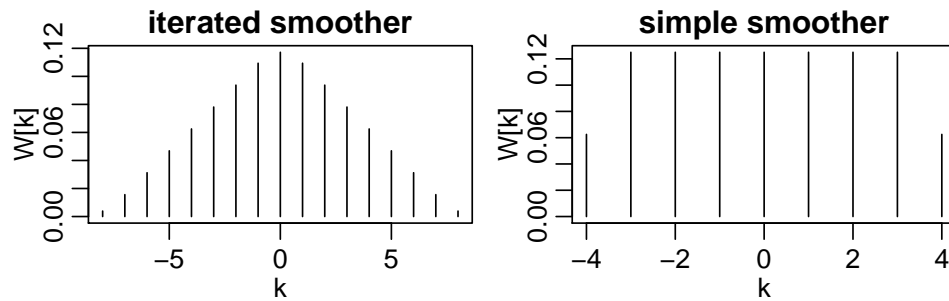
```
spec.pgram(ts.force, taper = 0, spans = c(2*L + 1, 2*L + 1), detrend = FALSE,
demean = TRUE, plot = TRUE, log = "dB",
main = sprintf("L = %d, spans = (%d, %d)", L, 2*L + 1, 2*L + 1))
```



With growing  $L$ , the periodograms again get smoother. In contrast to task c), the maxima are not flattened too much any more; this is the reason why one would in tendency prefer the iterated Daniell smoother instead of the simple one.

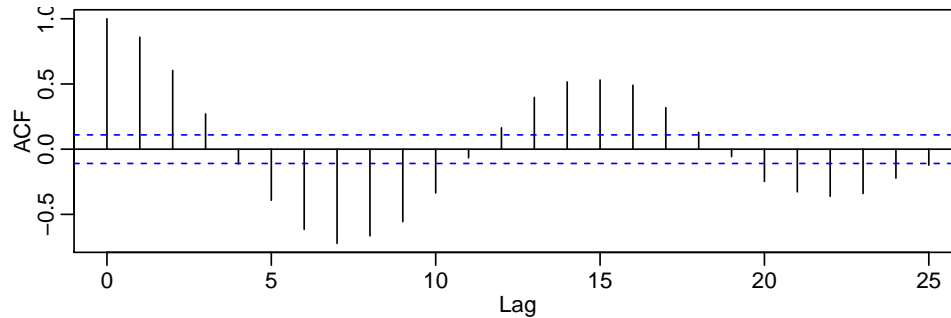
A parameter of  $L = 4$  (i.e.,  $\text{spans} = c(9, 9)$ ) again seems appropriate. We determine the weights of the corresponding smoother and compare them to the weights of the simple Daniell smoother:

```
> par(mfrow = c(1, 2), mar = c(3, 3, 1.5, 0.1))
> L <- 4
> pg <- spec.pgram(ts.force, taper = 0, spans = c(2*L + 1, 2*L + 1),
  detrend = FALSE, demean = TRUE, plot = FALSE, log = "dB")
> plot(pg$kernel, main = "iterated smoother", ylim = c(0, max(pg$kernel$coef)))
> pg <- spec.pgram(ts.force, taper = 0, spans = 2*L + 1,
  detrend = FALSE, demean = TRUE, plot = FALSE, log = "dB")
> plot(pg$kernel, main = "simple smoother", ylim = c(0, max(pg$kernel$coef)))
```



Whereas the weights of the simple Daniell smoother are “flat” (right plot), the weights of the iterated Daniell smoother are triangular (left plot).

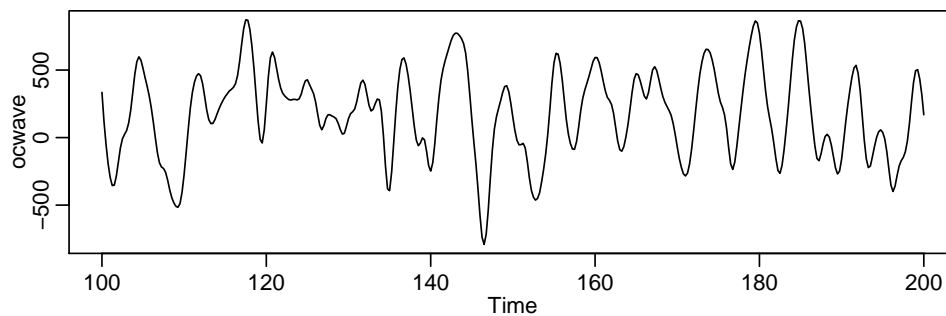
e) The autocorrelations look as follows:



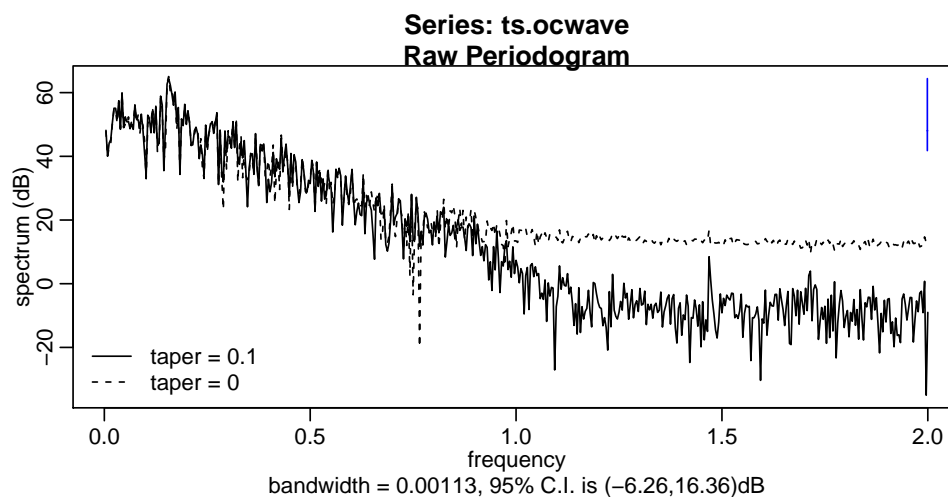
The period of approximately 2 seconds is also visible in the autocorrelations plot: the distance between two maxima of the damped cosine curve is approximately 14 lags, which corresponds to a period of  $14\Delta t \approx 2.1$  s. The period of the higher frequency visible in the periodogram is not visible in the correlogram.

2. a) In the plot of the time series, oscillations with a period of approximately 6.5 s are apparent. To determine the period, it may be necessary to plot only a short region of the complete time series, see below.

```
> plot(window(ts.ocwave, start = 100, end = 200), ylab = "ocwave")
```

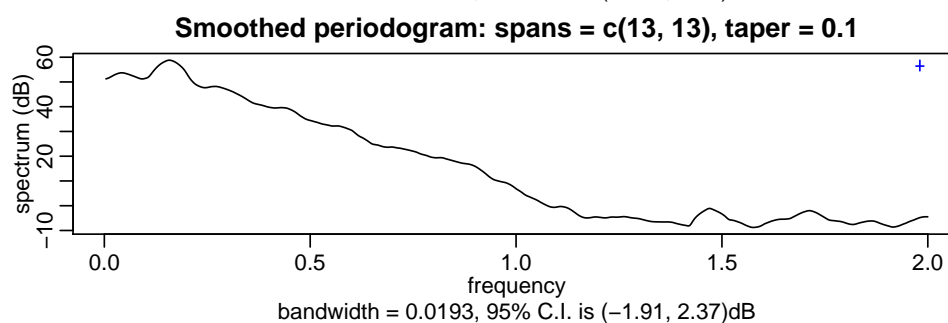
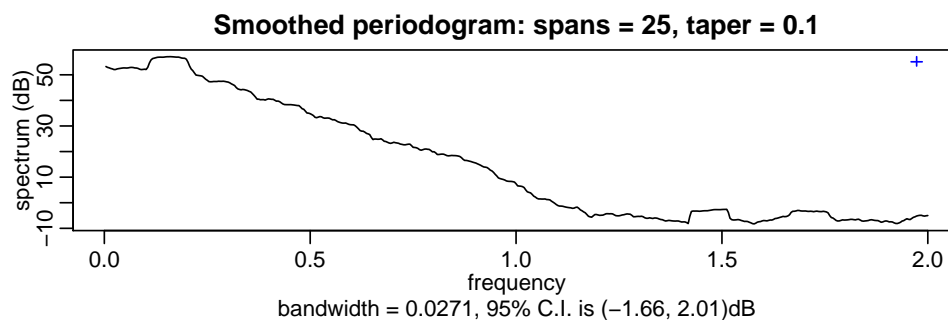


- b) The raw periodogram (without tapering) overestimates the spectrum at high frequencies. The bias is reduced by tapering. The strongly reduced variance in the range  $[1, 2]$  compared to the range  $[0, 1]$  in the raw periodogram is a hint for the problems in estimating the spectrum without tapering: we know from theory that the estimation accuracy in logarithmic scale should be constant over varying frequencies. This is indeed the case in the tapered periodogram.

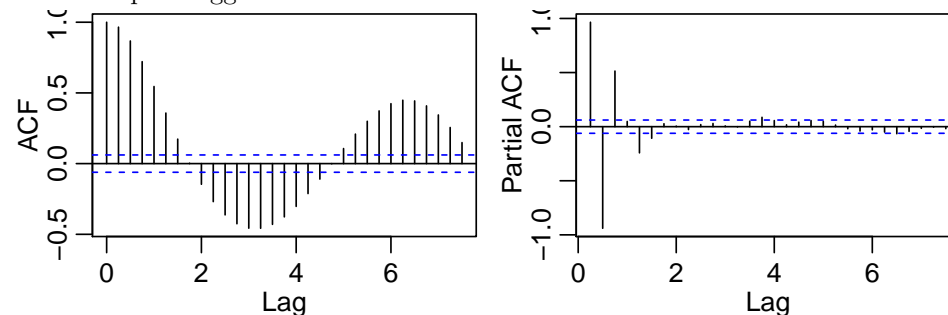


Note: the frequency range goes from 0 to 2, since the Nyquist frequency is  $\frac{1}{2\Delta t}$ . Hence the unit of the frequency scale is Hz.

- c) The spectrum is maximal for the frequency  $\nu \approx 0.16$  Hz, corresponding to a period of  $T = \frac{1}{\nu} \approx 6.2$  s. This is the same period we found in the time series plot in task a).



- d) The PACF plot suggests to choose an order of 6:



The Burg estimator for order 6 has a prediction variance of 149, the Yule-Walker estimator a much larger one of 797.

The respective spectra show clear differences. The method of Burg yields a better estimate of the spectrum as the method of Yule-Walker: as we have seen in part b), the spectrum should shrink

to low values in the high frequency range.

```
> spec.ar(ocwave.burg, log = "dB")  
> spec.ar(ocwave.yw, log = "dB", add = TRUE, lty = 2)  
> legend("bottomleft", legend = c("Burg", "Yule-Walker"), lty = 1:2, bty = "n")
```

