Solution to Series 1

1. a)
```
> hstart <- read.table("ftp://stat.ethz.ch/Teaching/Datasets/WBL/hstart.dat")
> hstart <- ts(hstart[, 1], start = 1966, frequency = 12)
```

The plot of the time series is shown below (Part b)); it is simply produced by the function call

```
> plot(hstart, xlab = "Year")
```

The time series is non-stationary (see the plot for Part b)). Its non-stationary properties consist of a trend and deterministic seasonal fluctuations (by month).
We can decompose this time series into its trend, its seasonal fluctuations (month effects) and the remainder:

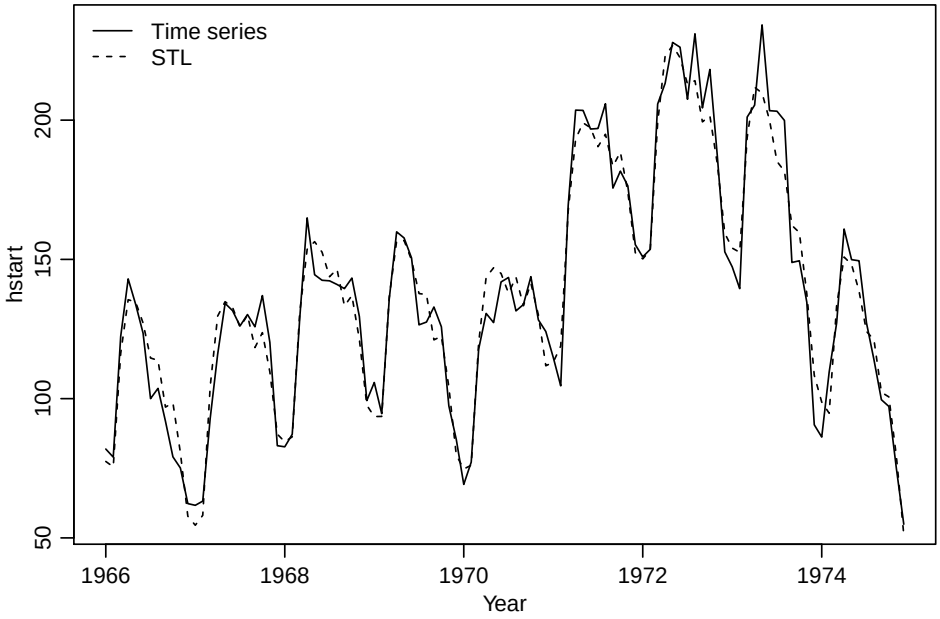
$$X_t = m_t + \alpha_{i\langle t\rangle} + E_t \ .$$

b) Decomposition of the time series, with averaging and by manually choosing a smoothing parameter:

```
> H.stl <- stl(hstart, s.window = "periodic")
> H.predict <- H.stl$time.series[, "trend"] + H.stl$time.series[, "seasonal"]
> H.stl.var <- stl(hstart, s.window = 15)
```

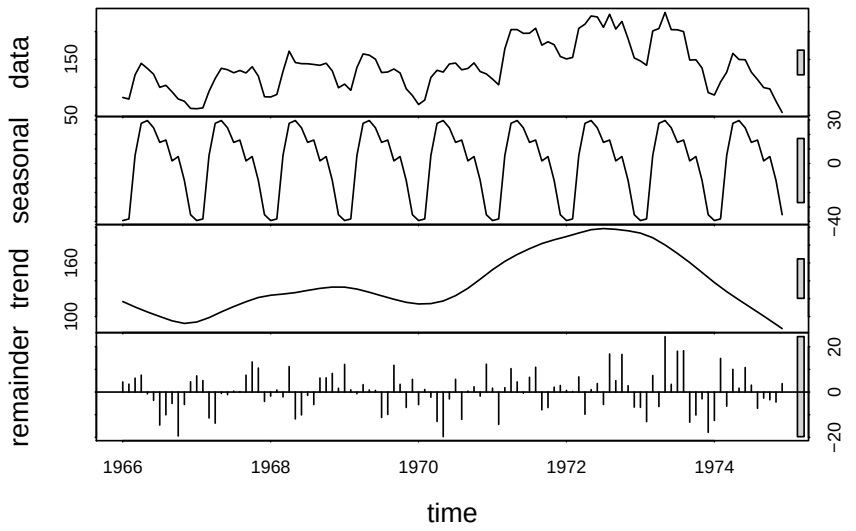
Plotting of time series and decomposition:

```
> plot(hstart, xlab = "Year")
> lines(H.predict, lty = 2)
> legend("topleft", lty = c(1, 2), legend = c("Time series", "STL"), bty = "n")
```



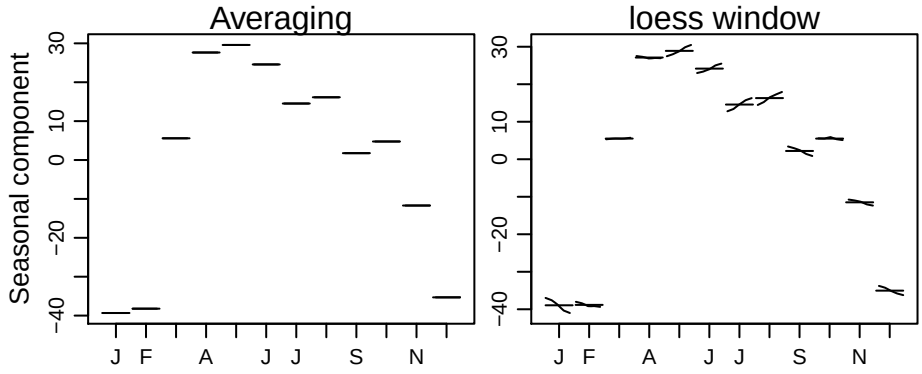
c) In the following plot, we have the original data, additive seasonal effects, the trend curve and the residuals (remainder) of the STL decomposition, drawn in order from top to bottom.

```
> plot(H.stl)
```



The next plot compares the monthly effects of `H.stl` to those of `H.stl.var`, where the smoothing parameter was chosen by hand. We see that the monthly effects of `H.stl` remain constant over the years, whereas those of `H.stl.var` vary: the effects of May to August increase with the years, while the effects of January, September and December decrease.

```
> par(mfrow = c(1, 2), cex = 0.7, mar = c(2, 2, 0.1, 0.1), oma = c(0, 3, 1.5, 0))
> monthplot(H.stl$time.series[, "seasonal"], ylab = "")
> mtext("Averaging", side = 3)
> mtext("Seasonal component", side = 2, line = 2, cex = 0.8)
> monthplot(H.stl.var$time.series[, "seasonal"], ylab = "")
> mtext("loess window", side = 3)
```



d) The trend line from the special filter is somewhat less smooth than the trend line stemming from the STL decomposition. However, the smoothness of the STL trend line can be steered by the smoothness parameter `t.window` of the function `stl`, e.g. setting this parameter to 25 (cf. R hint for Part b)).

```
> H.filt <- filter(hstart, c(1, rep(2, 11), 1)/24)
```

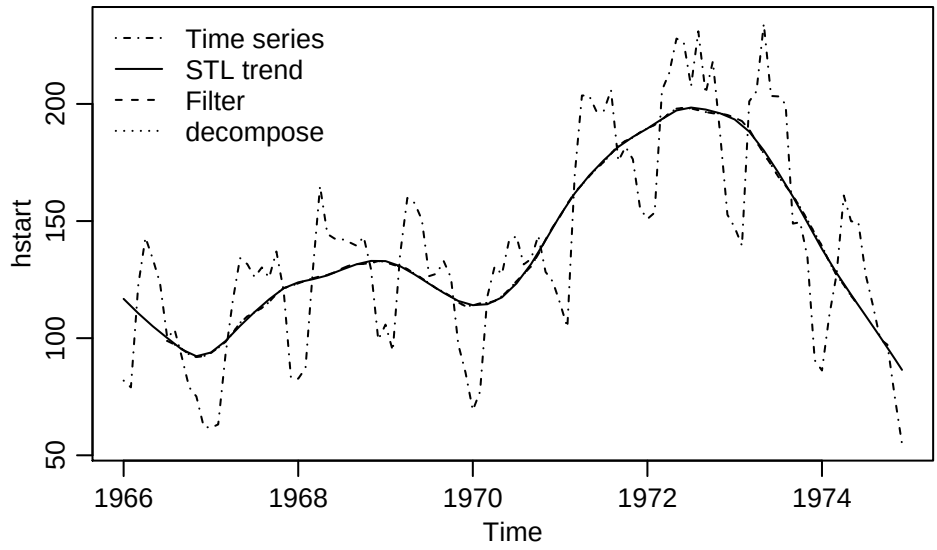
e) Calculating the decomposition:

```
> H.decomp <- decompose(hstart, type = "additive")
```

Finally, we plot the time series and three different estimates of the trend into one plot:

```
> plot(hstart, lty = 4)
> lines(H.stl$time.series[, "trend"], lty = 1)
> lines(H.filt, lty = 2)
> lines(H.decomp$trend, lty = 3)
```

```
> legend("topleft", legend = c("Time series", "STL trend", "Filter", "decompose"),
+   lty = c(4, 1, 2, 3), bty = "n")
```

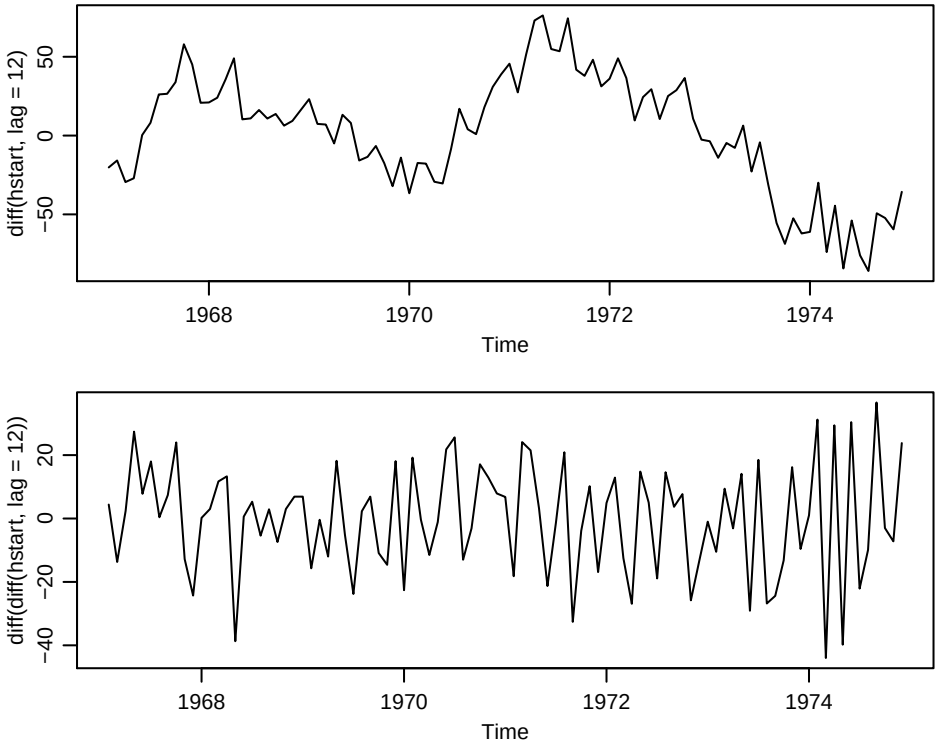


f) First we remove the seasonal fluctuations by computing $Y_t = X_t - X_{t-12}$. In the next plot, we then see that a trend nonetheless remains.
Taking first-order differences of Y_t, i.e.

$$Z_t = Y_t - Y_{t-1} = (X_t - X_{t-12}) - (X_{t-1} - X_{t-13}) \ ,$$

finally gives us a stationary series.

```
> par(mfrow = c(2,1), cex = 0.7)
> plot(diff(hstart, lag = 12))
> plot(diff(diff(hstart, lag = 12)))
```



2. **a)** The series is non-stationary. There is a nonlinear trend, mostly increasing. There is no seasonal component.

b) The series is probably stationary; the trend from before was removed by differencing. We achieved an approximately constant variance by taking the logarithm (however, the variance is perhaps still a bit augmenting after 1997; this is hard to see by eye).

c) The series is non-stationary. There is an (exponentially) increasing trend and a (multiplicative) seasonal component with period 1 year.

d) Dito, as in Task c).

e) The series is non-stationary. There is no trend, but a seasonal effect with period 1 year.

f) The series is probably stationary. There are no trend and seasonal effects, but a (non-seasonal!) periodicity with a period of approximately 11 years. Maybe, the variance could be varying over time.

g) The series is probably stationary. There is no clear trend (maybe a slight one), and no seasonal effect visible.