Series 7

1. The time series in the file `kraft.dat` contains measurements of the vertical forces acting on a cylinder in a tank of water. A total of 320 measurements were taken at intervals of .15 seconds. It is also known that at the time of the experiment, waves were present with a (randomly fluctuating) periodicity of around 2 seconds (cf. Exercise 1 of Sheet 4).

Load the data as follows:

```
> d.force <- read.table("http://stat.ethz.ch/Teaching/Datasets/WBL/kraft.dat",
+   header = FALSE)
> ts.force <- ts(d.force[, 1])
```

a) Fit AR models of order $p = 2, 4, 6, 8$ and 10 for the time series `ts.force`. For each order, draw the corresponding (theoretical) spectrum on a logarithmic scale to base 10 ("decibel scale").

- How do these spectra differ?
- To which periods do the maxima of the spectra correspond?
- Which order p would you choose on the basis of these plots?

R hints: Compute the spectrum by means of the function `spec.ar(..., log="dB")`. Use the Burg algorithm (argument: `method="burg"`) to estimate the AR coefficients. Specify the desired order p by setting the argument `order`. If p is not specified, it is chosen so as to minimize the AIC (cf. `order.max` and `aic=FALSE/TRUE` in `ar()`).

b) Draw the raw periodogram of the time series and compare the outcome with part a).

R hints:

```
> spec.pgram(ts.force, taper = 0, detrend = FALSE, demean = TRUE, plot = TRUE,
+   log = "dB")
```

c) Smooth the raw periodogram using the *modified Daniell Smoother*, averaging over $(2L+1)$ adjacent Fourier frequencies using the weights

$$w_{-L} = \frac{1}{2 \cdot (2L)},\ \underbrace{w_{-L+1} = \frac{1}{2L}, \ldots, w_{L-1} = \frac{1}{2L}}_{2L-1},\ w_L = \frac{1}{2 \cdot (2L)}\,.$$

Which L appears to be most adequate to this task?

R hints:

- Use the argument `spans = 2*L + 1` of the function `spec.pgram()` to apply a Daniell smoother as described above.
- The upper right corner of the periodogram contains a cross; its vertical length corresponds to the 95% confidence interval for $10 \cdot \log_{10}(f(\nu))$ and its horizontal length represents the smoothing bandwidth. This confidence interval should help distinguish true effects from random ones.

d) For the same values of L as you chose in task c), smooth the raw periodogram by applying the modified Daniell smoother *twice*. Compare the smoothed periodograms to the ones you got in task c). Which smoother would you prefer?

Which L seems to be appropriate this time? Compare the smoothing weights w_i of this iterated Daniell smoother to the ones of the simple Daniell smoother you chose in task c), and describe them in words.

R hints:

- To apply the Daniell smoother described in task c) twice, use the argument `spans = c(2*L + 1, 2*L + 1)` of the function `spec.pgram()`.

- The smoothing weights w_i used by function `spec.pgram()` are stored in the entry named `kernel` of the returned list object; therefore you can determine them as follows:

```
> pg <- spec.pgram(ts.force, taper = 0, spans = ..., ...)
> pg$kernel
```

e) Plot the correlogram. Are the principal frequencies both recognizable from this diagram?

2. In this exercise we shall look at the height of oceanic waves as measured at intervals of .25 seconds. The data can be found in `ocwave.dat`. Load them as follows:

```
> ts.ocwave <- ts(scan("http://stat.ethz.ch/Teaching/Datasets/WBL/ocwave.dat"),
+   start = 1, frequency = 4)
```

a) Make a time series plot and comment on it.

b) Compute the periodogram with and without taper. Comment on your results.
R hints:

```
> spec.pgram(ts.ocwave, taper = ..., detrend = FALSE, demean = TRUE, log = "dB")
```

c) Smooth the tapered periodogram and describe the spectrum in words.

d) Fit an AR(p) model with a suitable order p. Use both the Yule-Walker and Burg methods, and choose the order on the basis of the PACF plot. Compare the prediction variances to the AR spectra.
R hints:

```
> ocwave.yw <- ar.yw(ts.ocwave, aic = FALSE, order = ...)
> ocwave.burg <- ar.burg(ts.ocwave, aic = FALSE, order = ...)
```

The prediction variances can be found under `...$var.pred`. To facilitate the comparison, the spectra can be plotted in the same diagram:

```
> spec.ar(ocwave.burg, plot = TRUE, log = "dB")
> spec.ar(..., add = TRUE)
```

Preliminary discussion: Monday, May 23.

Deadline: Thursday, May 26.