Applied Time Series Analysis
FS 2011 – Week 13

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Spectral Analysis

Idea: Time series are interpreted as a combination of cyclic components, and thus, a linear combination of harmonic oscillations.

Why: As a descriptive means, showing the character and the dependency structure within the series.

What: It is in spirit, but also mathematically, closely related to the correlogram

Where: - engineering
  - economics
  - biology/medicine

Regression Model & Periodogram

We write a time series with a regression equation containing sine and cosine terms at the fourier frequencies $\nu_k = k/n$.

$$Y_t = \alpha_0 + \sum_{k=1}^{m} \alpha_k \cos(2\pi \nu_k t) + \beta_k \sin(2\pi \nu_k t) + E_t$$

The most important frequencies are the ones which lead to a pronounced decrease in goodness-of-fit when omitted.
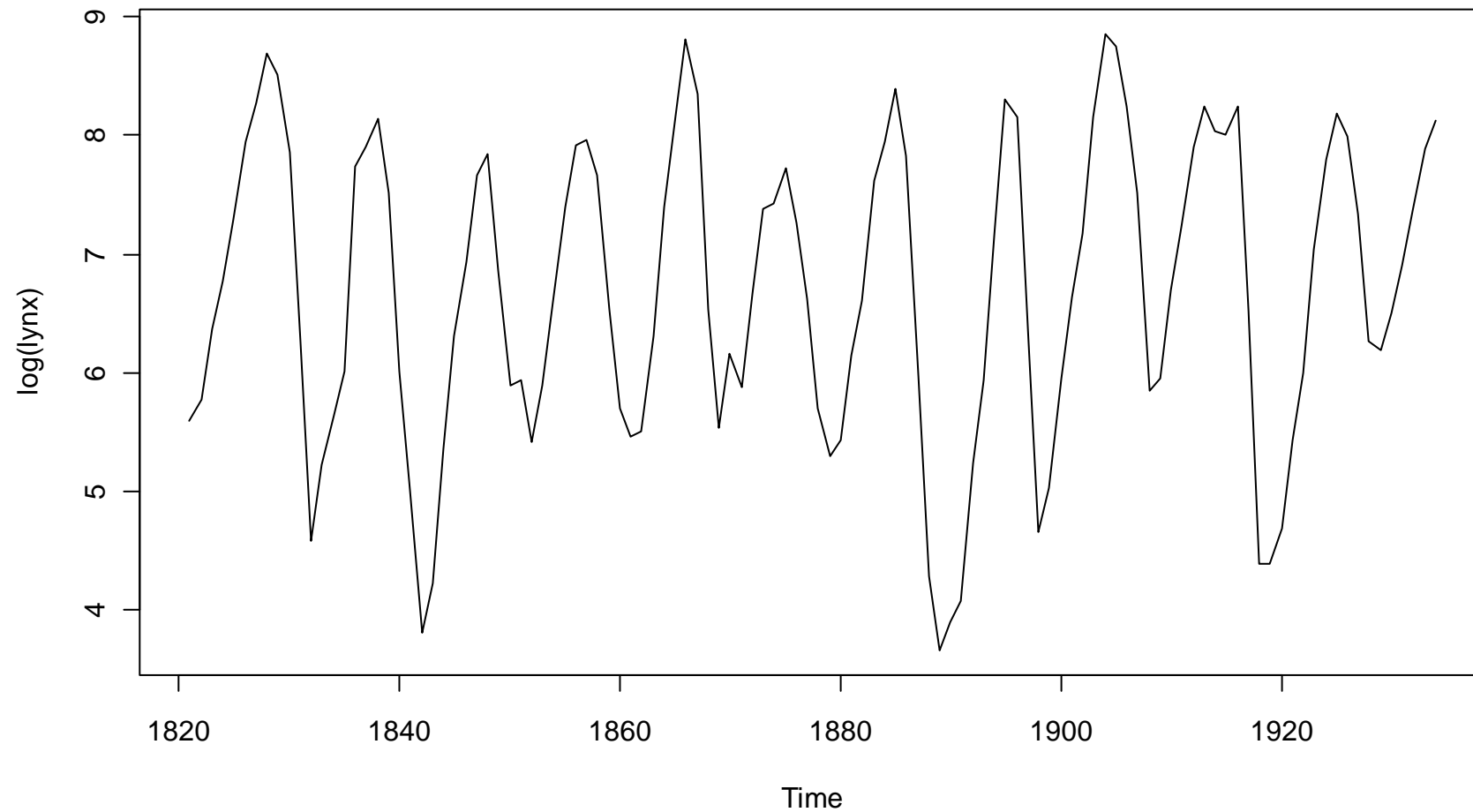
- The periodic analysis is limited to frequencies between 0 and 0.5, i.e. things we observe at least twice.

- This idea is used as a proxy for the periodogram, which is an approximation to the spectrum.

Lynx Data

Log Lynx Data

Periodogram of the Log Lynx Data
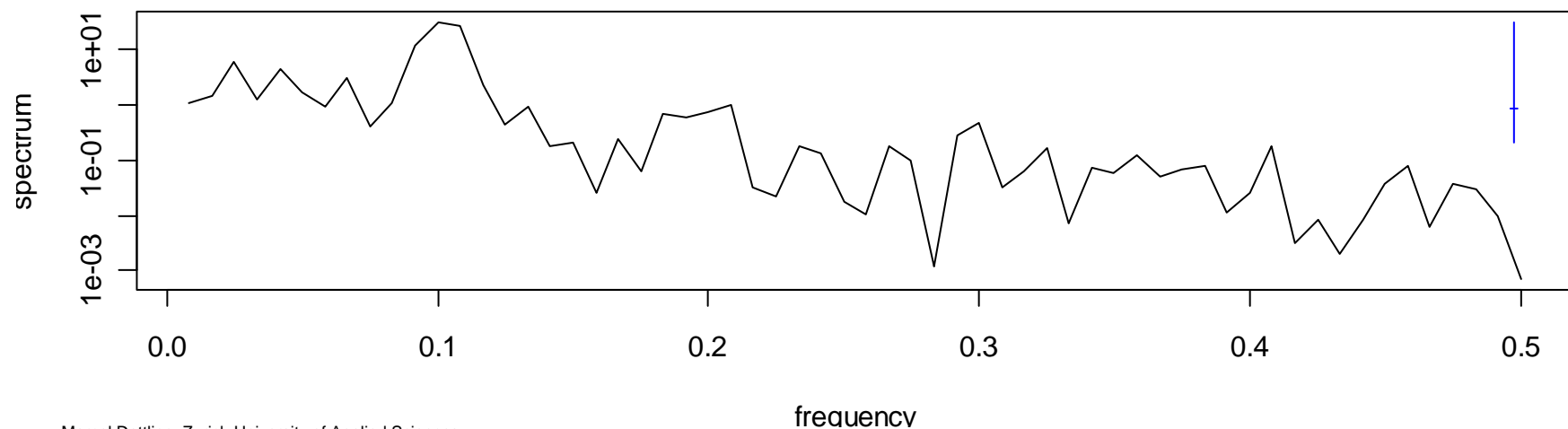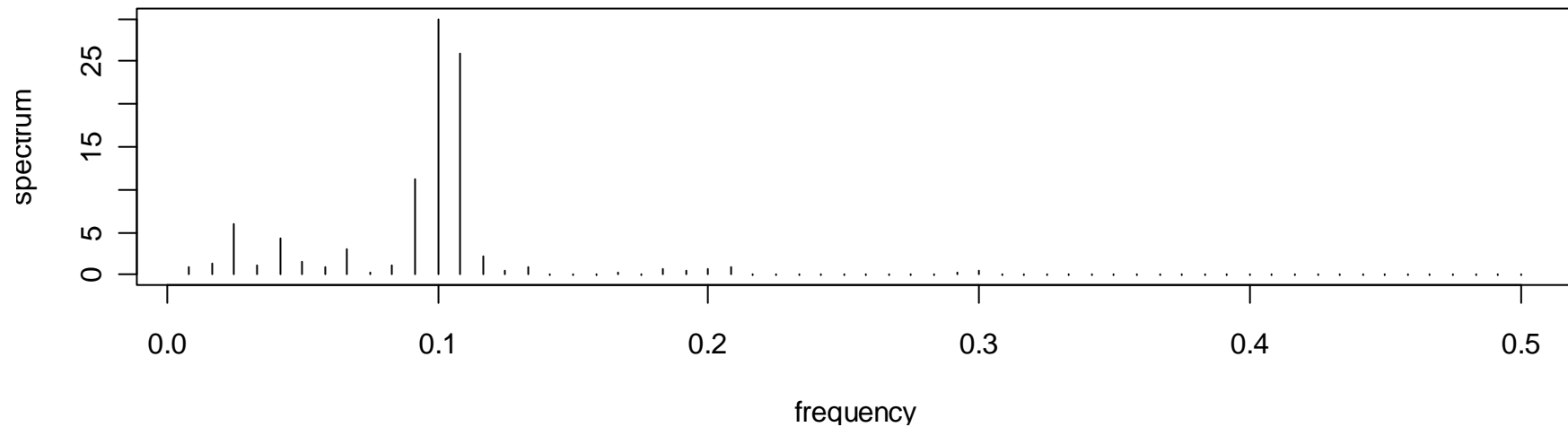
Statistical Properties of the Periodogram

The (raw) periodogram is not a good estimator for the spectrum.

Why?

→ **see the follwing 3 slides & the blackboard...**
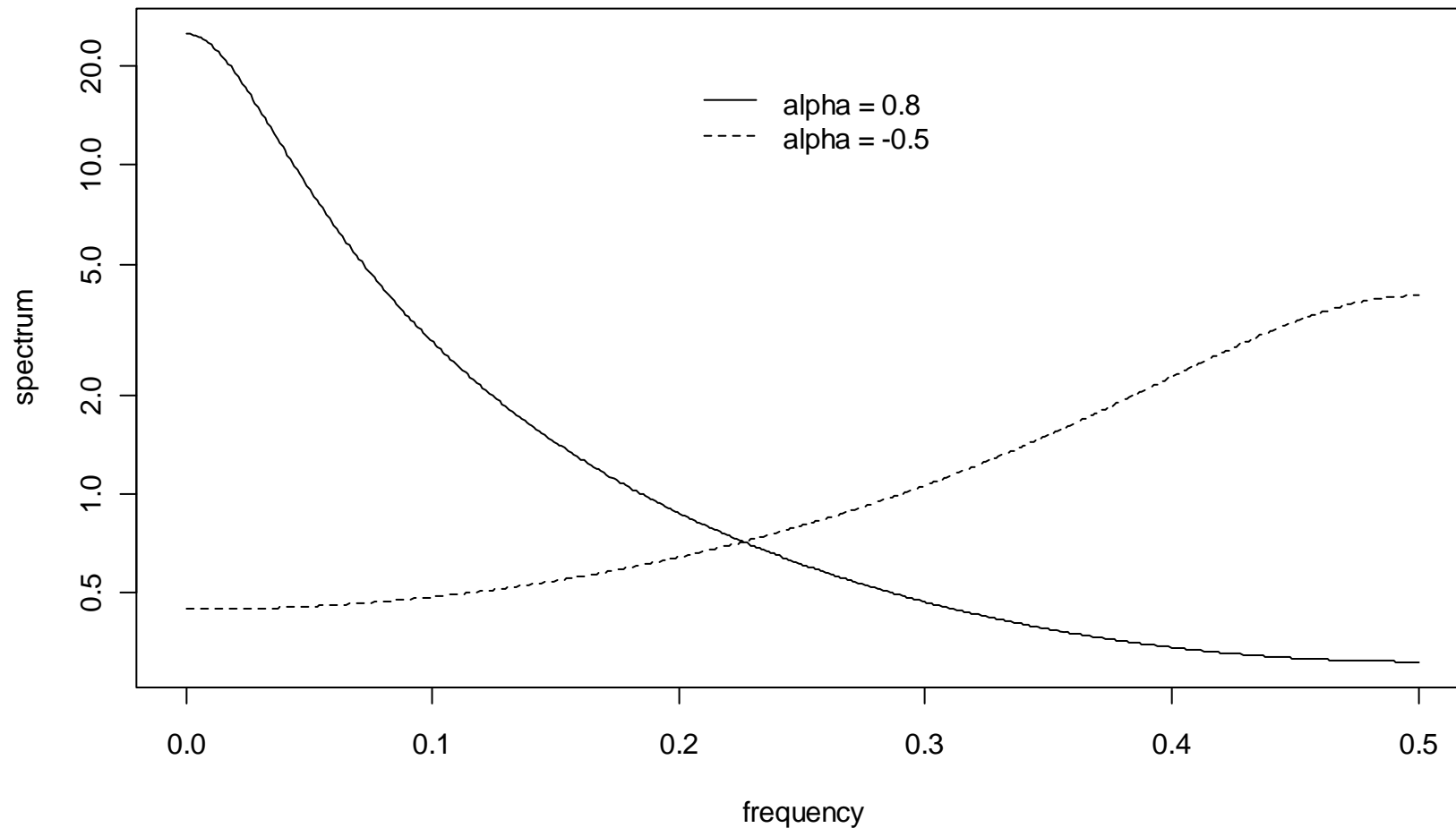
Spectrum of AR(1)-Processes

Spectrum of Simulated AR(1)-Processes

Simulated AR(2)-Process
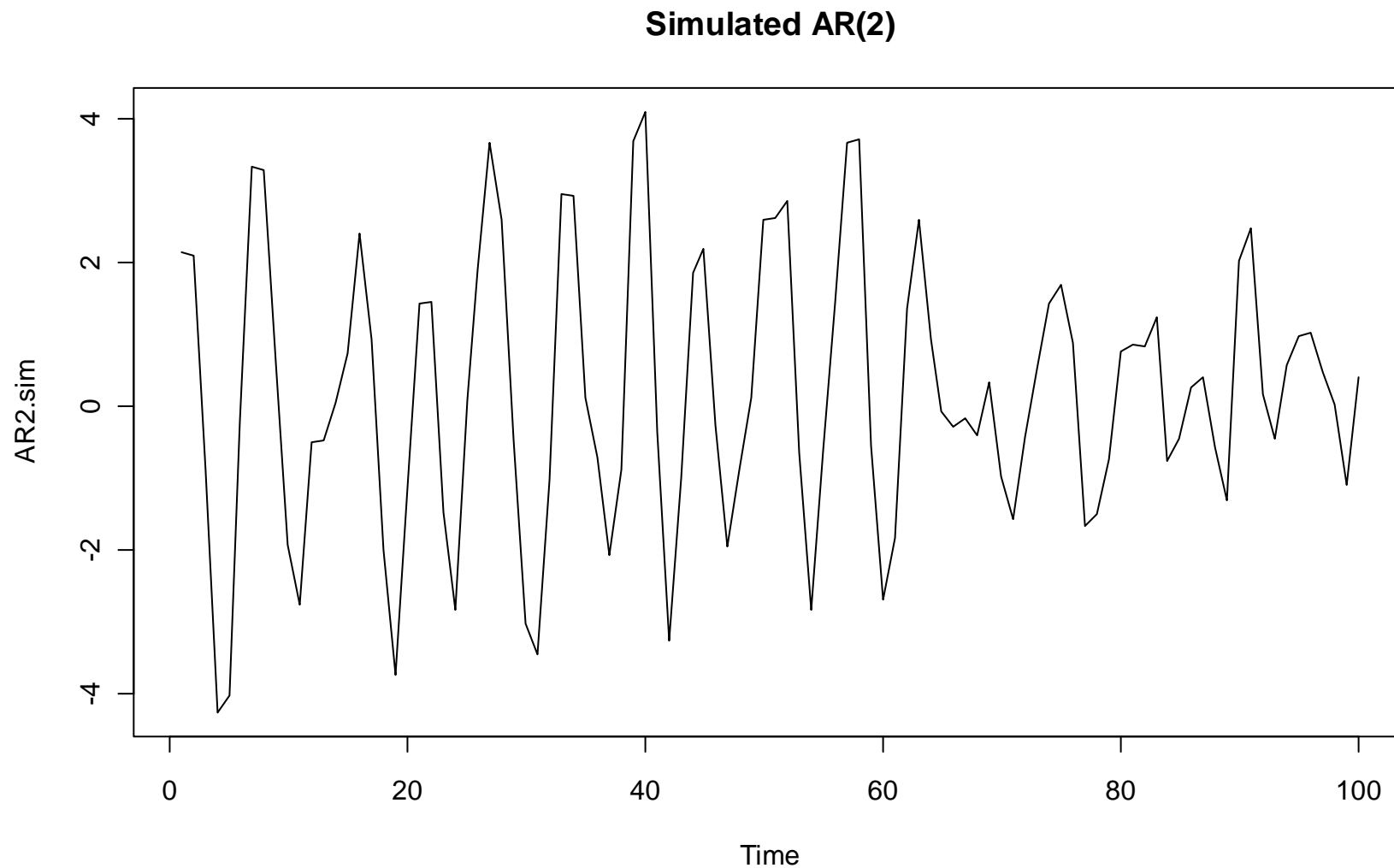
Simulated AR(2)

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ACF/Spectrum of Simulated AR(2)-Process
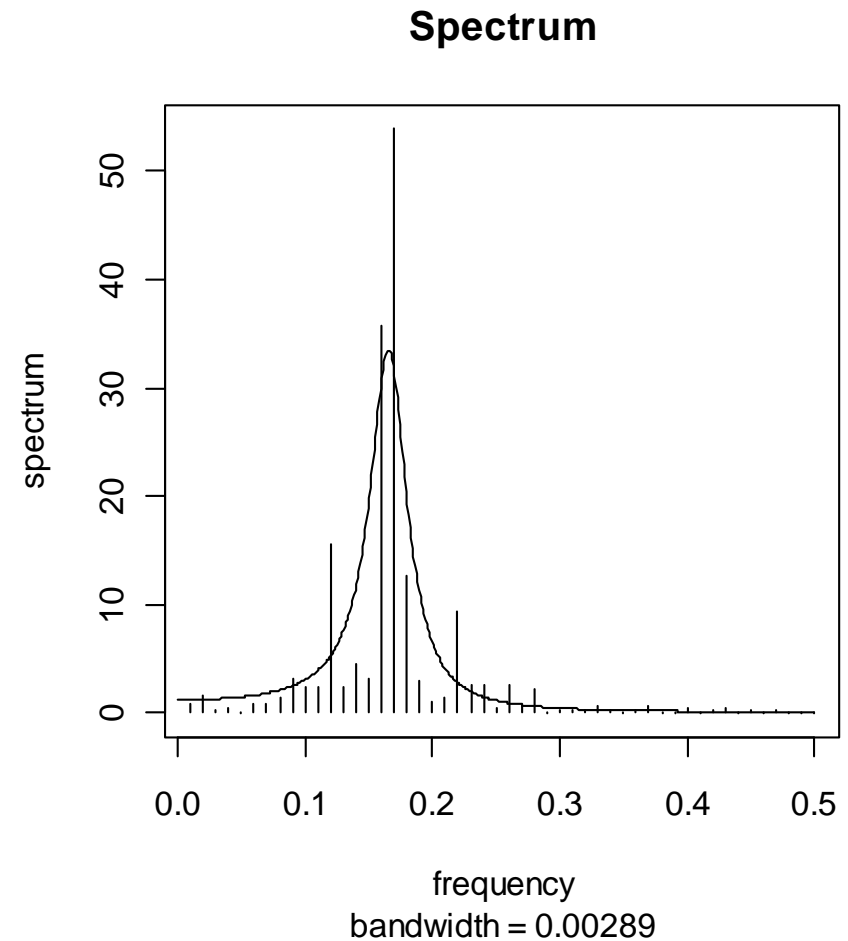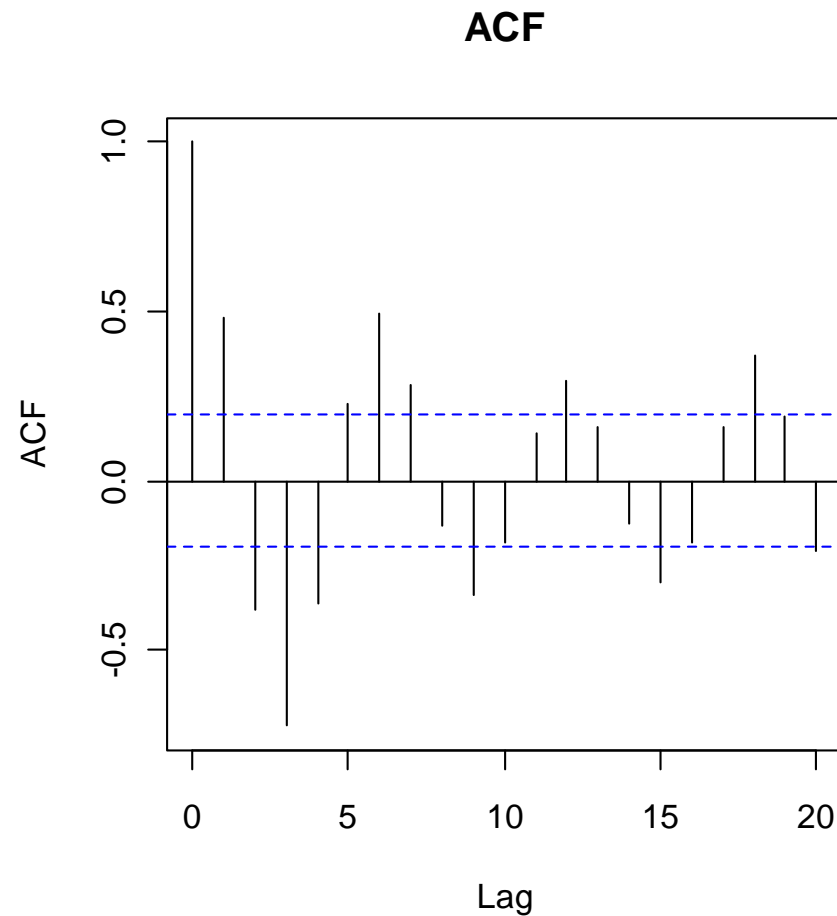
Improving the Raw Periodogram

1) **Smoothing with a running mean**

+ simple approach

- choice of the bandwith

2) **Smoothing with a weighted running mean**

+ choice of the bandwith is less critical

- difficulties shift to the choice of weights

3) **Weighted plug-in estimation**

+ weighted Fourier trsf. of estimated autocovariances

- choice of weights

4) **Piecewise periodogram estimation with averaging**

+ can serve as a check for stationarity, too

Improving the Raw Periodogram

5) Spectrum of an estimated model

+ fundamentally different from 1)-4)

- only works for „small" orders p

6) Tapering

+ further modification of periodogram estimation

+ reduces the bias in the periodogram

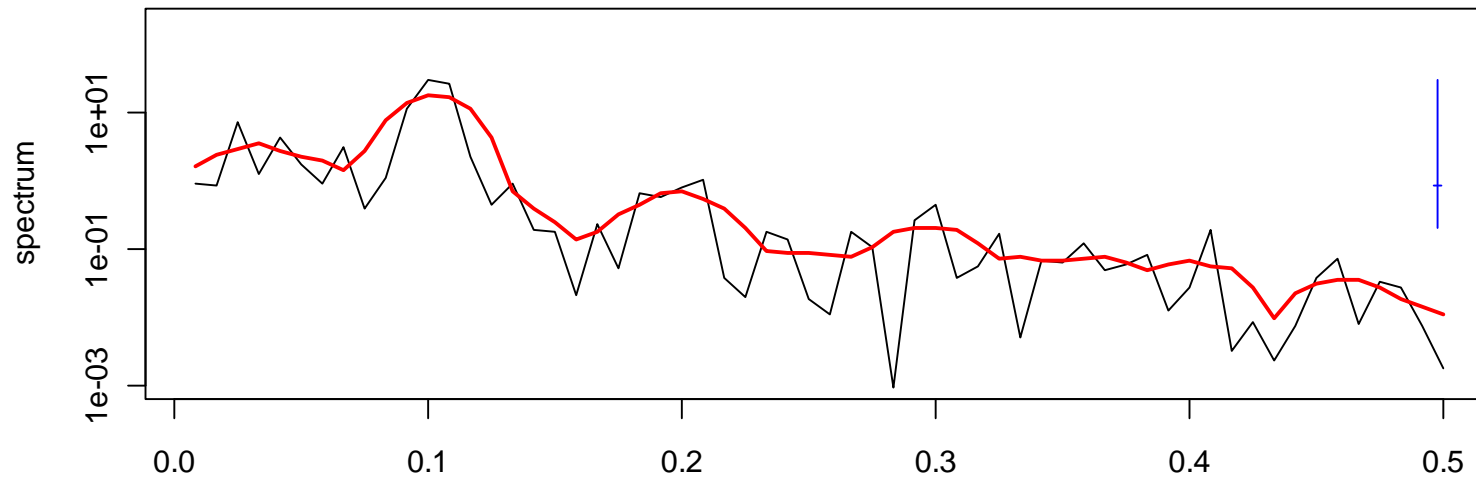+ should always be applied

7) Prewhitening and Rescoloring

+ model fit and periodogram estimation on residuals

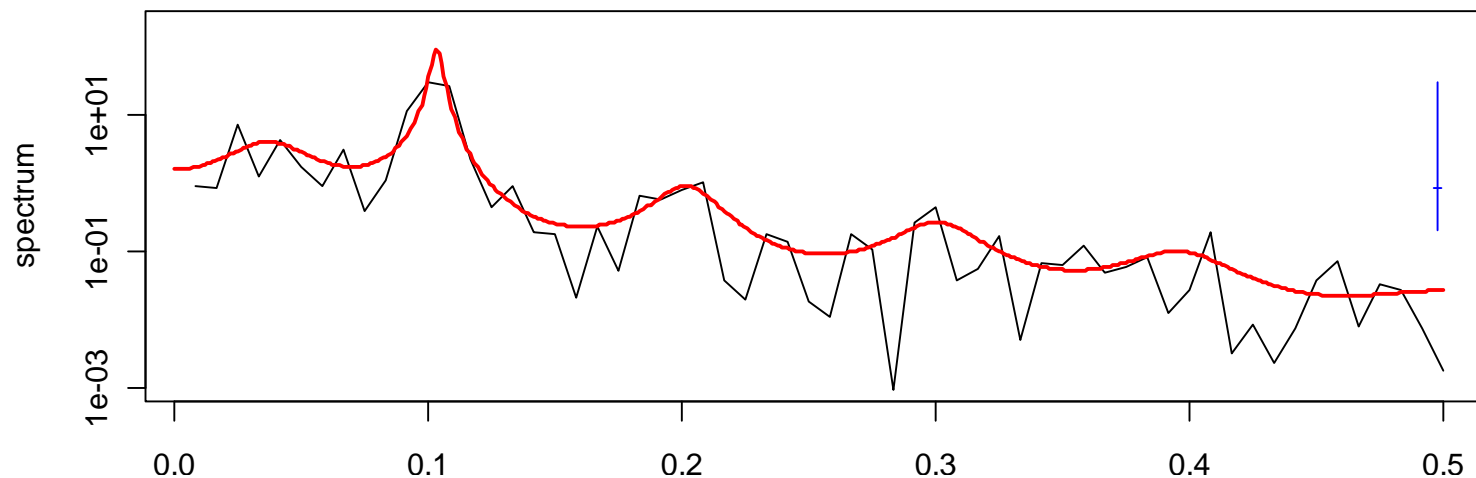+ the effect of the model will be added again

Modified Periodogram of log(Lynx) Data

Modified Periodogram of log(Lynx) Data

Piecewise periodogram of ocean wave data