Exercise Sheet 8

1. We shall revisit the setup of Exercise Sheet 7, Problem 1 (sample mean $\bar{x} = 204.2\ \mu g/l$). This time, however, we assume that the standard deviation (10 μg/l) was estimated using the 16 samples. Thus a t-test (of the null hypothesis $\mu_o = 200\ \mu g/l$) is applicable, not a z-test.

a) Should we expect that proving a breach of the limit is easier, just as difficult or more difficult using the t-test than if we were using the z-test?

b) Perform the t-test.
Hint: You will find a percentile table for the t distribution at the end of this exercise sheet.

c) Which prerequisite assumption for the t-test might be violated here, thus giving the t-test bad power?

2. Assume – as in Exercise Sheet 6, Problem 1 – that the lead content X of lettuces follows a normal distribution,

$$X \sim \mathcal{N}(\mu, \sigma^2).$$

a) We take the standard deviation ($\sigma = 10$ ppb) to be known; our interest lies in the expected value μ. Measuring the lead contained in 10 lettuces, we obtain an average value of 31 ppb. Compute a 99% confidence interval for μ.
Hint: For the cumulative distribution function of the standard normal distribution, we have
$\Phi(2.58) = 0.995$.

b) How many observations do we need to reduce the width of the confidence interval by half?
How many (independent) measurements of lead content should we plan for, if we are to estimate the lead content "to a precision of 1 ppb", i.e. if the width of the 99 % confidence interval must not exceed 1 ppb?

c) In most situations, the standard deviation σ is not priorly known. If we estimate σ from the data, what factor is the width of the confidence interval in a) scaled by?

3. The construction of a new road is feared to damage surrounding buildings by the sheer amount of vibrations it causes. To monitor this, the width of various cracks in adjacent buildings is measured before and after the construction work is carried out. The table below lists the (fictitious) measurements.

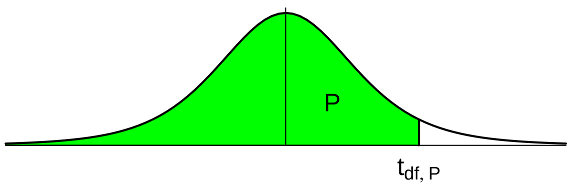
Measurement	1	2	3	4	5	6	7	8	9	10	11	12
before	1.25	2.60	3.10	2.10	3.30	3.75	1.95	2.70	3.20	2.50	3.30	2.75
after	1.20	3.35	2.80	2.90	3.70	4.85	2.55	2.50	3.80	2.55	3.70	3.10

Perform a sign test at level 5% using these data.

a) Formulate the null hypothesis H_0 and the alternative hypothesis H_A.
Hint: Think about the likely nature of "changes" in the cracks, and what the term "feared" means in the description of the problem. Then decide whether a one-sided or a two-sided test is appropriate.
Moreover, determine the rejection set.

b) What is the value of the test statistic? What is the result of the sign test?

Tabulated values of the t distribution



Example: $t_{9;\ 0.975} = 2.262$

df	$t_{0.60}$	$t_{0.70}$	$t_{0.80}$	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
31	0.255	0.530	0.853	1.309	1.696	2.040	2.452	2.744
32	0.255	0.530	0.853	1.309	1.694	2.037	2.449	2.738
33	0.255	0.530	0.853	1.308	1.693	2.035	2.445	2.733
34	0.255	0.529	0.852	1.307	1.691	2.032	2.441	2.728
35	0.255	0.529	0.852	1.306	1.690	2.030	2.438	2.724
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
90	0.254	0.526	0.846	1.291	1.662	1.987	2.368	2.632
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576