Exercise Sheet 3

1. A company that manufactures insulating materials changes the way it produces a certain type of plate, with the aim of reducing the number of carcinogenic fibres contained in such plates. The old production method yielded plates that contained an average of 3 such fibres per square millimetre. After modifying the process, five samples were taken and analysed; the results were (number of carcinogenic fibres per square millimetre):

1 0 2 1 3

Now the manufacturer would like to know whether the new procedure does indeed reduce the number of carcinogenic fibres.

Hint: Assume that the X_i, the number of fibres in a sample, follows a Poisson distribution with parameter λ, and that the X_i are independent of each other: $X_i \sim Poisson\langle\lambda\rangle$, independent. Use the following fact: If $X_i \sim Poisson\langle\lambda\rangle$ are independent and identically distributed Poisson random variables, then $S = \sum_{i=1}^{n} X_i$ follows a Poisson distribution with parameter $\widetilde{\lambda} = n\lambda$.

a) State the null hypothesis H_0 and the alternative H_A. Should a one-sided or a two-sided test be carried out?

b) Sketch the distribution of S under H_0.

c) Determine the rejection region at level 5% and mark it on the above sketch.

d) Mark the *value* of S in the above sketch. Is there a significant difference between the old and the new method of production?

e) Quantify the probability of a Type II error if the new method leads to an average of 2 carcinogenic fibres per square millimetre (A Type II error consists in keeping H_0 despite $\lambda = 2 \in H_A$ being true.)?

In the setup of Problem 1, compute a confidence interval for each of λ and $\tilde{\lambda}$.

2. **a)** Two-sided interval by means of an appropriate approximation.

b) Qualitatively in the situation, where a one-sided test is required. Does the interval look like $[0, c]$, or like $[c, \infty]$?