Exercise Sheet 2

1. Smarties are produced in a variety of colours. Different colours of Smarties are then mixed and sold in packets of 1000.

Their manufacturer claims that each packet contains 10 purple Smarties on average. You doubt this claim and suspect that there are in fact fewer purple Smarties in an average packet.

To test this in a neutral way, you buy an arbitrary packet of Smarties in a shop, and you then only find only five purple ones inside. Does this "prove" that you were right – that there are less than 10 purple Smarties in an average packet?

a) What is the null hypothesis here, and what is the alternative?

To convince other people that the manufacturer's claim is false (and thus the null hypothesis can be rejected), you must show that the event observed is very improbable under the null hypothesis.

b) Why is it not enough to just show that finding *exactly* five instead of 10 Smarties in a packet is very improbable? Which probability must be computed instead?

c) We can assume that the number of purple Smarties in a packet follows a Poisson distribution (why?). What then is the probability (under the null hypothesis) of finding at most five purple Smarties in a packet?

d) Can the manufacturer's claims thus be disproven?

2. We shall revisit the example of "quality control for test tubes" regarded in Problem 2 of Problem Sheet 1. To ensure the quality of their produce, the manufacturer takes a random sample of fifty items from a large batch of test tubes. Of these fifty, 3 are deficient.
The problem for the manufacturer is to decide whether they can safely assume the whole batch has an acceptable proportion of deficient glasses (that is, $< 10\%$), or if the low proportion observed is entirely down to chance (and the true proportion of deficient glasses is 10%).

a) What null hypothesis is being tested here?

b) What is the alternative hypothesis?

c) Compute the acceptance and rejection regions.

d) What does the test conclude? What should the manufacturer's decision be?