Solution Sheet 7

1. a) Carrying out the test:

Model assumptions:	X_i: $i-$th measurement of ammonia. X_i i.i.d. $\mathcal{N}\left(\mu, \sigma^2\right)$ where $\sigma = 10$.
Null hypothesis H_0:	X_i i.i.d. $\mathcal{N}(\mu_0, \sigma^2)$ where $\mu_0 = 200, \sigma = 10$
Alternative hypothesis H_A:	X_i i.i.d. $\mathcal{N}(\mu, \sigma^2)$ where $\mu > 200, \sigma = 10$ (one-sided)
Rejection set:	From the table of the normal distribution, we obtain $\mathcal{K} = \{z : \Phi(z) > 0.95\} =]1.64, \infty]$. (This corresponds to the rejection set $]204.1, \infty]$ for $\overline{X}$.)
Test statistic:	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{204.2-200}{\sigma/\sqrt{16}} = 1.68$
Outcome:	$1.68 \in \mathcal{K}$. Thus the null hypothesis is rejected by this test. We have statistical evidence that the limit is exceeded here.

b) From a) we conclude that the null hypothesis can be rejected if the average of all measurements exceeds 204.1

$$\overline{X} > 204.1.$$

To compute the probability that we can prove the limits are being exceeded (i.e., that we can reject H_0), we switch back to a standardized normal random variable. For $\mu_A = 205$ and $\sigma = 10$, we get

$$\begin{aligned} P[\overline{X} > 204.1] &= P[\frac{\overline{X} - \mu_A}{\sigma/\sqrt{n}} > \frac{204.1 - \mu_A}{\sigma/\sqrt{n}}] \\ &= P[\frac{\overline{X} - \mu_A}{\sigma/\sqrt{n}} > -0.36] \\ &= P[Z > -0.36] \end{aligned}$$

This is the probability that a standard normal random variable

$$Z \sim \mathcal{N}(0, 1),$$

exceeds -0.36. Due to symmetry, this probability is equal to

$$P[Z \leq 0.36] = 0.6406 \ ,$$

as is evident from the table. The probability we are looking for (the power of the test) is appproximately 64%.

c) This is precisely the level of the test and is set at 5%.

2. The plot given below show that the distribution of the mean of independent random variables approaches the normal distribution even if the variables themselves are not normally distributed. Looking at the x-axis, we can even see the variance become ever smaller.