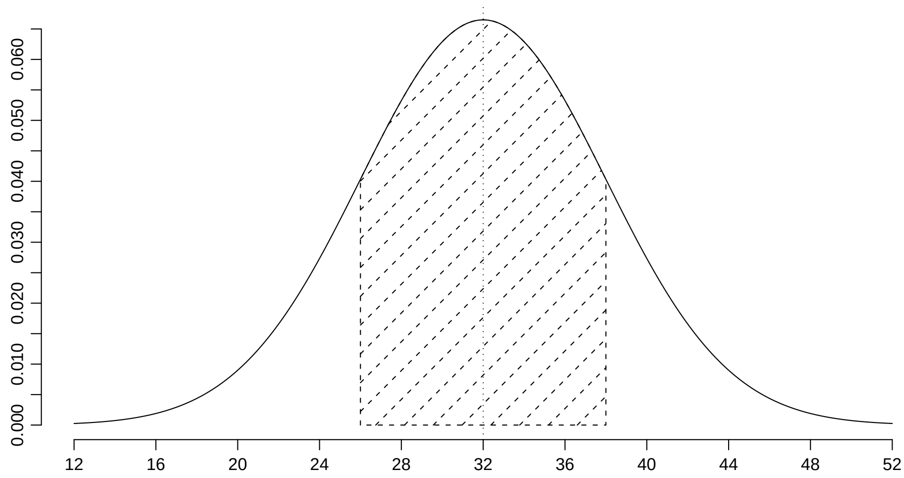
Solutions to Exercise Sheet 6

1. a) Sketch:



b) Let X denote the lead content of lettuces. We have:

$$X \sim \mathcal{N}\left(\mu, \sigma^2\right) \qquad \text{where } \mu = 32 \text{ and } \sigma^2 = 6^2.$$

If we are not using a computer, we should work with the standardized random variable $Z = (X - \mu)/\sigma$ for practical reasons (table). We obtain: $Z \sim \mathcal{N}(0,1)$.

$$\mathrm{P}\left[X \leq 40\right] = \mathrm{P}\left[Z \leq \frac{40-32}{6}\right] = \mathrm{P}\left[Z \leq 1.33\right] = \Phi(1.33) = 0.9082$$

c)

$$\mathrm{P}\left[X \leq 27\right] = \mathrm{P}\left[Z \leq -0.83\right] = \Phi(-0.83) = 1 - \Phi(0.83) = 0.2033.$$

d)

$$\mathrm{P}\left[X \leq c\right] = 0.975 = \mathrm{P}\left[Z \leq \frac{c-32}{6}\right] = \Phi(\frac{c-32}{6}).$$

Using the table, we find that $\Phi(1.96) = 0.975$. Thus:

$$\frac{c-32}{6} = 1.96 \text{ and therefore } c = 32 + 1.96 * 6 = 43.76\ .$$

e) From the table: $\Phi(1.28) = 0.9$ and $\Phi(-1.28) = 1 - 0.9 = 0.1$. Thus

$$c = 32 - 1.28 * 6 = 24.31\ .$$

f)

$$\Phi(1) - \Phi(-1) = 2 * \Phi(1) - 1 = 2 * 0.8413 - 1 = 0.6826 \ .$$

2. **a)** We associate the random variable X to the i-th person, setting $X_i = 1$ if they buy something and $X_i = 0$ if they do not. We already know that

$$\mathrm{P}\left[X_i = 1\right] = 0.3.$$

Let Y denote the total number of sales, i.e.

$$Y = \sum_{i=1}^{10} X_i.$$

Thus $Y \sim \mathrm{Bin}\,(10, 0.3)$.

b) No-one buys anything:

$$\mathrm{P}\left[Y = 0\right] = (1 - 0.3)^{10} = 0.7^{10} \approx 0.028.$$

At least 2 people buy something:

$$\mathrm{P}\left[Y \geq 2\right] = 1 - \mathrm{P}\left[Y < 2\right] = 1 - \mathrm{P}\left[Y = 0\right] - \mathrm{P}\left[Y = 1\right]$$

We have $\mathrm{P}\left[Y = 1\right] = 10 \cdot 0.3 \cdot 0.7^9 \approx 0.121$. Thus

$$\mathrm{P}\left[Y \geq 2\right] \approx 1 - 0.028 - 0.121 \approx 0.85.$$

c) Normal approximation:

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right),$$

where $\mu = 200 \cdot 0.3 = 60$, $\sigma^2 = 200 \cdot 0.3 \cdot 0.7 = 42$.
We now standardize to obtain

$$\mathrm{P}\left[50 \leq Y \leq 60\right] = \mathrm{P}\left[-1.54 \leq Z \leq 0\right] = \Phi(0) - \Phi(-1.54) = 0.4382,$$

using the fact that $\Phi(-1.54) = 1 - \Phi(1.54) = 1 - 0.9382 = 0.0618$ (see the table).